Dispersive atomic evolution in a dissipative-driven cavity

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We study the dynamics of a collection of two-level atoms interacting with a single mode of a quantized field in a dissipative cavity in the dispersive regime. The quantized mode is driven by a classical driving field. The steady-state density matrix is obtained. The influence of the driving field on the quantized driven field and on atomic properties in both the dissipative and the lossless cases is studied. The atomic decoherence time in the dissipative-driven case is obtained. We show that the external driving field strongly suppresses the atomic coherence (in comparison with the dissipative cavity without the driving field).

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I. INTRODUCTION

Recently, interesting experiments in the optical domain of cavity quantum electrodynamics (QED) have been performed. In particular, the dynamics of resonant cold atoms crossing a cavity was studied for the case when the interaction energy is greater than the kinetic energy of the atom [1]. The quantum output field provided information about the atomic evolution during the atomic passage time. In this experiment an additional pumping field from a classical source was included to compensate for cavity energy loss. Experimental data have been compared both with a semiclassical theory and with a quantum description (the center of mass motion was treated classically). In the microwave regime of cavity QED, an unusual experiment studying the reversible decoherence of superpositions of coherent states was performed several years ago [2], where superposition of coherent states was produced by making use of the dispersive interaction of the quantum field with a single two-level atom [3]. Due to atom-field coupling, the initial field splits into two coherent states. These states lately lose their mutual coherence due to cavity losses. In this experiment the cavity was driven by a pulsed microwave source to create the initial coherent state.

The above-mentioned systems can be well described by quantum-optical Hamiltonians. It is well known that only a few nonlinear models of quantum optics, such as a Jaynes-Cummings model and evolution in the Kerr medium, admit an exact solution. It is even more difficult to determine analytically the behavior of the quantum system when both dissipation and a classical driving field are present. On the other hand, only a restricted numerical analysis usually can be carried out to describe the interaction of a quantum field with a collection of atoms in the presence of a driving field, due to the rapid increase of the dimension of the composed system. Here we shall make use of analytical solutions for understanding the long-time system dynamics.

The dynamics of the Dicke model in the dispersive limit has been studied for the generation of superpositions of atomic [4,5] and field [6,7] states. On the other hand, an approximate analytic solution for the atomic system interacting with a strong quantum field in an ideal cavity has been found in Ref. [8] and was generalized to include cavity losses in Ref. [9].

Following the above-mentioned experiments, in this work we study the dispersive limit of the atom-field interaction in a dissipative-driven cavity, in a case of A atoms interacting with a quantum field. Since in the dispersive regime there is no energy transfer between atomic levels, only field and atomic coherence can be affected in the course of evolution. The main goal of this work is to study the effect of a classical pump field and of cavity losses on atomic coherence. We note that loss of field coherence in a dissipative and driven cavity (in the absence of atoms) has been studied in a set of papers [10] (see also Ref. [3]). The dynamics of field and atomic decoherence in quantum nondemolition measurements during a dispersive evolution of a single atom in the context of the Lindblad approach was analyzed in Ref. [11]. This has been also studied in a recent work [12], where the influence of field dissipation on atomic coherence properties was emphasized.

This work is organized as follows: In Sec. II we find an exact solution for the approximate master equation describing the dispersive atom-field interaction in the presence of both an external driving field and field dissipation. In Sec. III we describe the main features of quantum dynamics in unitary nondriven evolution. In Sec. IV we find the steady state of this system and study the influence of the atomic system on the field properties both in the dissipative and the lossless cases. Here we concentrate on the modified atomic evolution due to the presence of the driving field. In particular, we analyze the loss of atomic coherence induced by field dissipation, which is essentially enhanced due to the presence of the classical driving field.

II. MASTER EQUATION AND ITS SOLUTION

We consider cavity losses as a mechanism of dissipation. The corresponding master equation for the atom-field density

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matrix, describing a collection of A two-level atoms interacting with a quantum field in a dissipative cavity in the presence of a driving field, has a standard form ($\hbar=1$)

$$\partial_t\rho=-i[H,\rho]+L\rho, \tag{2.1}$$

with

$$L\rho=\frac{\gamma}{2}(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a). \tag{2.2}$$

Here, H is the driven Dicke Hamiltonian which, in the dipole and rotating wave approximations, takes the form ($\hbar=1$)

$$H=\omega_a S_z+\omega_f a^\dagger a+g(aS_+ +a^\dagger S_-)+\Omega(ae^{i\omega_c t}+a^\dagger e^{-i\omega_c t}), \tag{2.3}$$

where S_+, S_-, S_z are operators from an $A+1$ dimensional representation of su(2) algebra with commutation relations

$$[S_+,S_-]=2S_z,\ [S_\pm,S_z]=\mp S_z. \tag{2.4}$$

a ($a^\dagger$) is the common bosonic annihilation (creation) operator, i.e., $[a,a^\dagger]=1$ and Ω is the Rabi frequency of the classical driving field.

We consider the far-off resonance limit for atom-field interaction (dispersive interaction), i.e., $A\sqrt{\bar{n}}g/\Delta\ll 1$, where $\Delta=\omega_a-\omega_f$ and $\bar{n}$ is the average number of photons in the field. In this limit the Hamiltonian H goes to an effective Hamiltonian for describing the unitary contribution to the dynamics in Eq. (2.1) which reads as (see the Appendix)

$$H_{eff}=\Delta_1 S_z+\delta a^\dagger a+\eta(2a^\dagger a+1)S_z+\eta(S^2-S_z^2)+\Omega(a+a^\dagger), \tag{2.5}$$

where $\eta=g^2/\Delta$ is the effective atom-field coupling in the dispersive limit, $\delta=\omega_f-\omega_c$ and $\Delta_1=\Delta+\delta$. This Hamiltonian is nonlinear in the atomic space and, as was shown in Ref. [4], leads to a number of collective effects such as atomic Schrödinger cat generation and atomic squeezing.

The effective Hamiltonian is further simplified by the following transformation of the density matrix:

$$\tilde{\rho}=e^{i[\Delta_1 S_z+\eta(S^2-S_z^2)]t}\rho e^{-i[\Delta_1 S_z+\eta(S^2-S_z^2)]t}, \tag{2.6}$$

and takes the form

$$\tilde{H}_{eff}=\delta a^\dagger a+\eta(2a^\dagger a+1)S_z+\Omega(a+a^\dagger). \tag{2.7}$$

This transformed version of the Hamiltonian (2.7) is linear in the field space and diagonal in the atomic space, ($[H,S_z]=0$). Hence, the master equation can be easily solved by applying the dynamical symmetry method proposed in Ref. [13]. We introduce the following notations for operators that appear in the master equation:

$$N_-\rho=a\rho a^\dagger,\quad N_l\rho=a^\dagger a\rho,\quad N_r\rho=\rho a^\dagger a,$$
$$A_l\rho=a\rho,\quad A_{+l}\rho=a^\dagger\rho,\quad A_r\rho=\rho a,\quad A_{+r}\rho=\rho a^\dagger. \tag{2.8}$$

It is clear that all the left operators commute with all the right operators. The operators N_l, N_r, and N_- form a soluble subalgebra,

$$[N_l,N_-]=[N_r,N_-]=-N_-,\quad [N_l,N_r]=0. \tag{2.9}$$

A_r, A_{+r}, 1 and A_l, A_{+l}, 1 are the two Heisenberg-Weyl subalgebras. The rest of the commutators are

$$[N_l,A_l]=-A_l,\quad [N_l,A_{+l}]=A_{+l},$$
$$[N_-,A_l]=0,\quad [N_-,A_{+l}]=A_{+r},$$
$$[N_r,A_r]=A_r,\quad [N_r,A_{+r}]=-A_{+r},$$
$$[N_-,A_r]=A_l,\quad [N_-,A_{+r}]=0. \tag{2.10}$$

In the bare atomic basis, which is defined by

$$S_z|k\rangle_{\rm at}=(k-A/2)|k\rangle_{\rm at},$$

the master equation takes a linear form with respect to this set of operators,

$$\partial_t\tilde{\rho}_{pq}=[\gamma N_- +v_p N_l+\bar{v}_q N_r+i\Omega(A_r+A_{+r}-A_l-A_{+l})]\tilde{\rho}_{pq}, \tag{2.11}$$

where $\tilde{\rho}_{pq}={}_{\rm at}\langle p|\rho|q\rangle_{\rm at}$, $v_p=-i\delta-2i\eta\lambda_p-\gamma/2$ and $\lambda_p=p-A/2$. Note that $\tilde{\rho}_{pq}$ continue to be operators in the field space. The solution of Eq. (2.11) is given by

$$\tilde{\rho}_{pq}(t)=e^{\mu_{pq}}e^{\bar{v}_q tN_r}e^{v_p tN_l}e^{f_{pq}N_-}\times e^{\bar{b}_p A_{+l}}e^{c_{pq}A_l}e^{b_q A_r}e^{\bar{c}_{qp}A_{+r}}\tilde{\rho}_{pq}(0), \tag{2.12}$$

where $\tilde{\rho}_{pq}(0)$ is the initial density matrix and

$$\frac{\mu_{pq}(t)}{\Omega^2}=\chi_{pq}t+\frac{\gamma}{v_p\bar{v}_q}\frac{(1-e^{-v_p t})(1-e^{-\bar{v}_q t})}{v_p+\bar{v}_q}-\left(1+\frac{\gamma}{v_p+\bar{v}_q}\right)\left(\frac{e^{v_p t}-1}{v_p^2}+\frac{e^{\bar{v}_q t}-1}{\bar{v}_q^2}\right), \tag{2.13}$$

$$\chi_{pq}=-\frac{2\eta}{|v_p|^2|v_q|^2}\{\gamma\eta(\lambda_p-\lambda_q)^2-i(\lambda_q-\lambda_p)[\gamma^2/4+(\delta+2\eta\lambda_p)(\delta+2\eta\lambda_q)]\},$$

$$f_{pq}(t)=\frac{\gamma}{v_p+\bar{v}_q}[e^{(v_p+\bar{v}_q)t}-1],\quad b_q(t)=-i\frac{\Omega}{\bar{v}_q}(e^{-\bar{v}_q t}-1), \tag{2.14}$$

$$c_{pq}(t)=-i\Omega\left[\left(1+\frac{\gamma}{v_p+\bar{v}_q}\right)\frac{e^{v_p t}-1}{v_p}+\frac{\gamma}{(v_p+\bar{v}_q)\bar{v}_q}(e^{-\bar{v}_q t}-1)\right].$$

Now, provided with the general solution (2.12), we shall concentrate on studying different limits in this model. As mentioned before, we are interested in the effects of the driving field and of dissipation on atomic coherence.

III. NONDRIVEN LOSSLESS CASE

In the lossless case and in the absence of a driving field, the dynamics of the Dicke model in the far-off resonance case is described by the effective Hamiltonian (2.5) with $\Omega=0$ and $\delta=0$:

$$H_{eff}^{(0)}=\omega_a S_z+\omega_f\hat{n}+\eta(2\hat{n}+1)S_z+\eta(S^2-S_z^2), \quad (3.1)$$

where $\hat{n}=a^{\dagger}a$. A similar effective atomic Hamiltonian was derived by Agarval *et al.* [4] considering the evolution of a collection of A two-level atoms in a dissipative cavity in the presence of thermal photons. The proposed Hamiltonian was independent of the field operators and has the form of Eq. (3.1) with substitution $\hat{n}\rightarrow\bar{n}$, with $\bar{n}$ being the average number of thermal photons in the field. It was noted that the effective atomic Hamiltonian, being a nonlinear form on the generators of su(2) algebra [14], leads to the generation of a superposition of atomic coherent states (atomic Schrödinger cats) from an initial atomic coherent state. In contrast to Ref. [4], where only the atomic dynamics has been considered and the effect of the field on the atomic state is reduced to a pure phase factor, the Hamiltonian (3.1) leads to the evolution of both field and atomic subsystems.

Moreover, due to the joint atom-field evolution, the behavior of the atomic subsystem essentially depends on the initial field state and, in general, cannot be described by any effective atomic Hamiltonian. For instance, in the case of an arbitrary initial atomic

$$|in\rangle_{\rm at}=\sum_{k=0}^{A}c_p|p\rangle_{\rm at} \quad (3.2)$$

and the field being initially in a coherent state $|in\rangle_f=|\alpha\rangle_f$, the atomic density matrix reduces to the following form:

$$\rho_{\rm at}(t)=\sum_{p,q=0}^{A}c_p\bar{c}_q|p\rangle_{\rm at\,at}\langle q|\exp(-\bar{n}\{1-\exp[2it\eta\times(q-p)]\})\exp\{it[\eta(q+p+1)+it\omega_a](q-p)\}, \quad (3.3)$$

where $\bar{n}=\alpha^2$.

It follows from Eq. (3.3) that the initial atomic coherent state is reconstructed at times $t=m\pi/\eta$, with $m=1,2,\ldots$. At these instants the atomic and field systems become disentangled, i.e., the whole system is in a factorized state. At any other time the atomic system remains in a mixed state due to its entanglement with the field.

Thus, the atom-field coupling, even in the dispersive regime, drastically modifies the atomic coherence properties in the course of evolution due to contributions from different field number states. In the following, we shall concentrate on studying atomic coherence properties including a classical driving field and cavity losses.

IV. DRIVEN AND DISSIPATIVE CASE

Using Eq. (2.12) we can study evolution of an arbitrary atomic-field initial condition. Let us start with driven evolution in a perfect cavity, i.e., where the system is described by the effective Hamiltonian (2.5). From the general solution (2.12) and (2.13) we obtain that for an arbitrary initial atomic state (3.2) and vacuum initial field state, the field and atomic density matrices, respectively, take the following forms:

$$\rho_{\rm f}=\sum_{p=0}^{A}|c_p|^2|\alpha_p(t)\rangle_{\rm f\,f}\langle\alpha_p(t)|, \quad (4.1)$$

$$\rho_{\rm at}=\sum_{p,q=0}^{A}c_p'\bar{c}_q'e^{\alpha_p\bar{\alpha}_q}|p\rangle_{\rm at\,at}\langle q|. \quad (4.2)$$

We have used the following definitions:

$$\alpha_p(t)=\frac{\Omega}{\delta_p}(1-e^{it\delta_p}), \quad (4.3)$$

$$c_p'=c_p\exp\left[it\left(\lambda_p(\omega_a+\eta)-\lambda_p^2\eta+\frac{\Omega^2}{\delta_p}\right)-\frac{\Omega}{\delta_p}\bar{\alpha}_p\right], \quad (4.4)$$

where $\delta_p=\delta+2\eta\lambda_p$.

As was expected from the diagonal structure of the effective Hamiltonian (2.5) in atomic space, the field density matrix, as in the nondriven case, has the form of an incoherent superposition of coherent states $|\alpha_p(t)\rangle_{\rm f}$ which are generated by the classical current from the initial vacuum state. The atomic density matrix conserves its initial diagonal matrix elements. The modified coefficients c_p' have a nonlinear phase factor and an amplitude modulation, which essentially affect the atomic coherence. One can show that the amplitude of nondiagonal matrix elements decays by a factor

$$\exp\left(-\frac{\Omega^2}{2}\left[\frac{\sin\delta_p t/2}{\delta_p}\mp\frac{\sin\delta_q t/2}{\delta_q}\right]^2\right). \quad (4.5)$$

Thus we can conclude that, due to the presence of the driving field, the initial pure atomic state will never be reconstructed.

Here we have to distinguish between two cases that can appear, depending on the relation between frequencies of the classical driving and quantum driven fields. In the absence of atoms, one can transfer energy from the classical to the quantum one if the resonance condition $\omega_f=\omega_c$ is satisfied. In the presence of atoms, the field frequency is shifted due to the atom-field coupling. Thus, the resonance condition is modified and takes the form

$$\omega_f-\omega_c+2\eta\lambda_{k_0}=0. \tag{4.6}$$

In this case only the resonant term with $k=k_0$ (if it exists) will increase its amplitude, i.e., the coherent state corresponding to index k_0 grows with time: $\alpha(t)=-i\Omega t$. For long times only this state will essentially contribute to the average photon number, i.e., the field energy will quadratically grow with time according to $\langle\hat{n}\rangle\sim|c_{k_0}|^2(\Omega t)^2$. However, the photon number fluctuations for large enough interaction times depend on the initial atomic state. If the atomic population is distributed among different atomic states, the photon number fluctuations are of the order of the average photon number $\Delta\hat{n}\sim|c_{k_0}|^2(1-|c_{k_0}|^2)(\Omega t)^2$. Nevertheless, if only the resonant, satisfying condition (4.6), atomic energy level is excited, we obtain $\Delta\hat{n}\sim\Omega t$.

If the resonance condition (4.6) is not satisfied for any atomic state, the amplitudes of the coherent states $|\alpha_k(t)\rangle_{\rm f}$ oscillate between zero and $2\Omega/\delta_k$. Nevertheless, due to the appearance of pumping field dependent factors in the atomic density matrix, the initial atomic state will never be perfectly reconstructed—even at the moments $2\pi m/\eta$—in contrast to the nondriven case. Thus, in some sense, the classical pumping field introduces a mechanism of decoherence in atomic dynamics. This fact is still more pronounced when we take into account (in addition to the driving field) the effect of dissipation on the system dynamics.

Now, let us consider a purely dissipative case: $\Omega=0$. Then, starting from the initial field coherent state and an arbitrary initial atomic state, we obtain from Eq. (2.12) the system density matrix

$$\tilde{\rho}(t)=\sum_{p,q=0}^{A}c_p\overline{c_q}|p\rangle_{\rm at\,at}\langle q|\otimes|\alpha e^{v_pt}\rangle_{\rm f\,f}\langle\alpha e^{v_qt}| \times\exp[|\alpha|^2(f_{pq}+e^{-\gamma t}-1)], \tag{4.7}$$

where f_{pq} is defined in Eq. (2.13). One can observe that for large times ($t\to\infty$) the atomic density matrix takes the form

$$\tilde{\rho}_{\rm at}(t)=\sum_{p,q=0}^{A}c_p\overline{c_q}|p\rangle_{\rm at\,at}\Big\langle q\Big| \times\exp\left[-2i\eta\Big|\alpha\Big|^2\frac{\lambda_p-\lambda_q}{\gamma+2i\eta(\lambda_p-\lambda_q)}\right], \tag{4.8}$$

while the field evolves into the vacuum state: $\rho_{\rm f}\to|0\rangle_{\rm f\,f}\langle 0|$). This means that atomic coherence is only partially lost in the presence of field dissipation, and is improving when γ grows [12].

Considering the evolution of the off-diagonal elements of the atomic density matrix to this steady state, we may note that the factor $e^{-\gamma t}$ determines the envelope of the oscillations with the frequency $2\eta(\lambda_p-\lambda_q)$, while the first oscillation has a Gaussian envelope $\exp[-2t^2\eta^2|\alpha|^2(\lambda_p-\lambda_q)^2]$ which does not depend on γ. For the appropriate relation between the parameters $\eta|\alpha|$ and γ it can happen that only the oscillation survives. In such a case we are faced with a Gaussian decay, with the decay time being independent of γ (while the steady state depends on γ).

Finally, in the presence of both driving field and dissipation, we obtain that, starting with the initial coherent field state $\rho_f(0)=|\alpha\rangle_{\rm f\,f}\langle\alpha|$ and an arbitrary atomic state (3.2), the atomic density matrix for the characteristic times $t\sim\gamma^{-1}$ acquires a form similar to Eq. (4.8):

$$\tilde{\rho}_{\rm at}(t)=\sum_{p,q=0}^{A}c_p\overline{c_q}|p\rangle_{\rm at\,at}\langle q|\exp[\Omega^2\chi_{pq}t]\tilde{\rho}_{\rm at}^{\rm pq}, \tag{4.9}$$

where

$$\tilde{\rho}_{\rm at}^{\rm pq}=\exp\left[\left(\Omega^2\frac{v_p^2+\overline{v_q^2}+v_p\overline{v_q}}{(v_p\overline{v_q})^2}-\Big|\alpha\Big|^2\right)\left(1+\frac{\gamma}{v_p+\overline{v_q}}\right)\right] \times\exp\left[-2\Omega^2{\rm Im}\left\{\frac{\alpha}{v_p}\left(1+\frac{\gamma}{v_p+\overline{v_q}}\right)\right\}\right], \tag{4.10}$$

and the time-dependent factor will be discussed below. It is easy to see that $\tilde{\rho}_{\rm at}^{\rm pp}=1$, which means that no population transfer is produced (as is to be expected in the dispersive regime).

The reduced field density matrix, as well as in the lossless case, takes (in the initial reference frame) the form of an incoherent superposition of coherent states:

$$\rho_{\rm f}(t)=\sum_{p=0}^{A}|c_p|^2|z_p(t)e^{-i\omega_ct}\rangle_{\rm f\,f}\langle z_p(t)e^{-i\omega_ct}|, \tag{4.11}$$

where

$$z_p(t)=[\alpha+\overline{b_p}(t)]e^{v_pt}, \tag{4.12}$$

and $b_p(t)$ is defined in Eq. (2.14).

The steady state ($t\to\infty$) field density matrix acquires the following form:

$$\rho_{\rm f}^{ss}=\sum_{p=0}^{A}|c_p|^2|z_p^{ss}e^{-i\omega_ct}\rangle_{\rm f\,f}\langle z_p^{ss}e^{-i\omega_ct}|, \tag{4.13}$$

with $z_p^{ss}=i\Omega/v_p$. Thus the average photon number in the steady state is

$$\langle n\rangle={\rm Tr}(a^\dagger a\rho_{\rm f}^{ss})=\Omega^2\sum_{p=0}^{A}\frac{|c_p|^2}{(\delta+2\eta\lambda_p)^2+\gamma^2/4}. \tag{4.14}$$

Note that even in the far-off resonant regime, where there is no energy transfer between the field and the atomic system, the field steady state depends on the atomic indices. We illustrate the quality of the approximations in Fig. 1, where we plot the average photon number, $\langle n(t)\rangle$, as a function of interaction time. We can observe that the approximation gives us the evolution of $\langle n(t)\rangle$ in perfect agreement with exact numerical simulations. Here, small oscillations appear due to energy exchange between different atomic states for finite detuning. The agreement is better for large values of

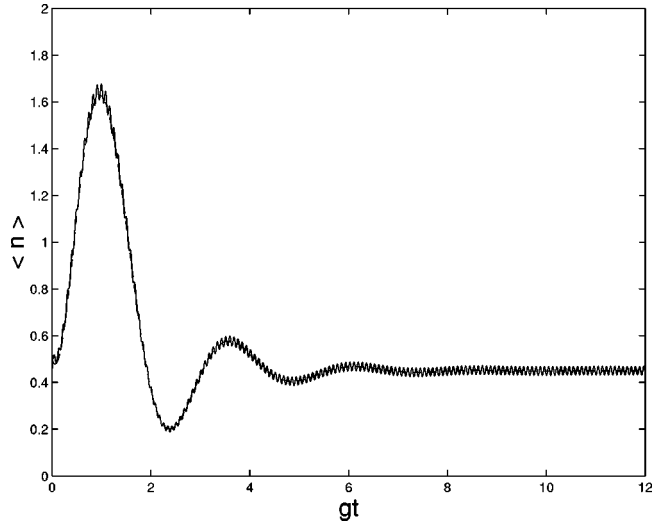

FIG. 1. Average photon number as a function of the adimentional time gt. The field was assumed to be initially prepared in a coherent state $|\alpha=\sqrt{0.5}\rangle$. The collection of $A=2$ atoms was assumed initially to be in a state $|in\rangle_{at}=(|0\rangle_{at}-|2\rangle_{at})/\sqrt{2}$. We have considered $\Omega/g=1.75$, $\gamma/g=1.5$, $\delta/g=2.5$, $\Delta/g=75$. Solid line: exact evolution. Dashed line: analytical approximation.

detuning. In Fig. 2 we show the evolution of atomic entropy. This quantity is much more sensitive to any approximation because it depends on all atomic density matrix elements. However, it is still well described by the approximation and we can see the existence of two different decoherence time scales as we discuss below.

As we can observe from expression (4.9), the atomic density matrix for typical times of dissipative decoherence $t \sim \gamma^{-1}$ still depends on time through the factor

$$\exp[\Omega^2 \chi_{pq} t], \tag{4.15}$$

which goes to unity for $p=q$ and decreases when $p \neq q$. This leads to complete coherence loss among different atomic components for times

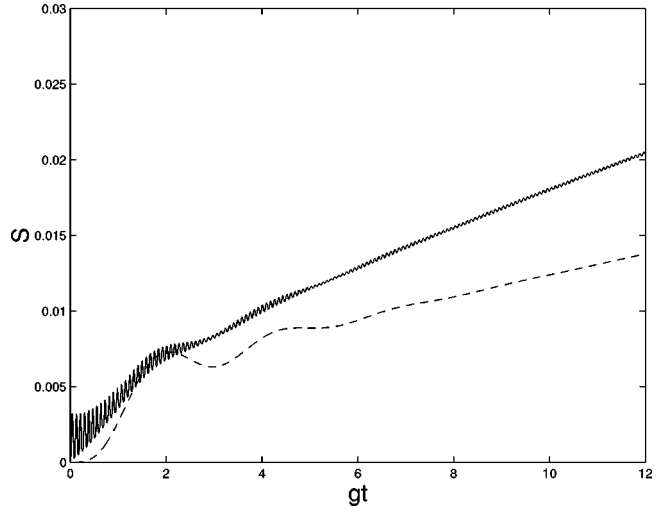

FIG. 2. Atomic entropy as a function of the adimentional time gt. All parameters are the same as in Fig. 1. Solid line: exact evolution. Dashed line: analytical approximation.

$$t_{\mathrm{dec}}^{pq} \sim \frac{|v_p|^2 |\overline{v_q}|^2}{\gamma \Omega^2 \eta^2 (\lambda_p - \lambda_q)^2}, \quad p \neq q. \tag{4.16}$$

For typical values of the system variables $\eta \ll \gamma$, in resonance $\delta=0$, the decoherence time is of order

$$t_{\mathrm{dec}} \sim \frac{\gamma^3}{\Omega^2 \eta^2}, \tag{4.17}$$

which is usually much longer than the dissipative decoherence time $\sim \gamma^{-1}$. This means that for times $t \sim t_{\mathrm{dec}}$ the atomic system reaches a purely mixed state:

$$\rho_{\mathrm{at}}(t \geqslant t_{\mathrm{dec}}) \simeq \sum_{p=0}^{A} |c_p|^2 |p\rangle_{\mathrm{at\,at}}\langle p|. \tag{4.18}$$

Thus collective effects like atomic squeezing and atomic Schrödinger cats generation [4,6], coming from the atomic nonlinearity of the effective Hamiltonian (2.5), have a transitional nature and will completely disappear after a time t_{dec}.

V. CONCLUSIONS

We obtain and solve the effective master equation describing Dicke model dynamics in the dispersive regime in the presence of driving field and dissipation. We are especially interested in the influence of the driving field on atomic coherence.

For a generic initial field state in the absence of driving field and in an ideal cavity, the initial atomic state is reconstructed at times $t=m\pi/\eta$, with m integer, when the composed system becomes disentangled. The presence of an external driving field essentially affects the system dynamics and introduces an additional mechanism for decaying of nondiagonal atomic density matrix elements. This has as a consequence, for instance, the impossibility of a perfect reconstruction of the initial atomic state even in the lossless situation. In the dissipative case the driving field leads to complete loss of coherence in the atomic system.

Let us stress that dissipation alone does not produce the complete loss of atomic coherence. It is the simultaneous influence of the driving field and dissipation which produces the decay of the off-diagonal atomic density matrix elements. This feature is in contrast to the usual action expected from the driving field—coherence induction to a single quantum system, as happens, for example, in the case of field evolution in a driven dissipative cavity [10].

Thus, one can distinguish between two temporal scales while the atomic coherence is being lost. At first, atomic coherence is partially lost for times of order γ^{-1} due to field dissipation. Then, for times t_{dec} [Eq. (4.17)] the atomic density matrix acquires a diagonal form with the initial atomic population distribution (compare with the situation of field coherence loss [10]). One should note that terms which have been neglected in obtaining the efective Hamiltonian (2.3) also produce coherence loss. In particular, apart from decay

of nondiagonal elements of the atomic density matrix, these terms lead to randomization of the population distribution.

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APPENDIX

In the interaction picture the Hamiltonian (2.3) takes the form

$$H=\Delta S_z+g(aS_+ +a^\dagger S_-)+\Omega(ae^{it\delta}+a^\dagger e^{-i\delta t}), \quad \text{(A1)}$$

where $\Delta=\omega_a-\omega_f$, $\delta=\omega_f-\omega_c$.

In the large detuning limit we can adiabatically eliminate the transitions among different eigenstates of S_z. To achieve this goal, we apply to the Hamiltonian (A1) a set of unitary transformations, which correspond to small rotations in the SU(2) group with an operator parameter,

$$H_1=U_2U_1HU_1^\dagger U_2^\dagger, \quad \text{(A2)}$$

where

$$U_1=\exp\left(i\frac{\sqrt{2}g}{\Delta}pS_x\right), \quad U_2=\exp\left(i\frac{\sqrt{2}g}{\Delta}qS_y\right), \quad \text{(A3)}$$

and

$$q=\frac{1}{\sqrt{2}}(a+a^\dagger), \quad p=\frac{i}{\sqrt{2}}(a^\dagger-a), \quad \text{(A4)}$$

$$S_x=\frac{1}{2}(S_+ +S_-), \quad S_y=-\frac{i}{2}(S_+ -S_-). \quad \text{(A5)}$$

Keeping terms up to first order in $\sqrt{2}g/\Delta\ll 1$, we get (in the frame rotating with the external field frequency ω_c)

$$H_1=\Delta_1 S_z+\delta a^\dagger a+\frac{g^2}{\Delta}[(2a^\dagger a+1)S_z+S^2-S_z^2] +\Omega\left[a+a^\dagger+\frac{2\sqrt{2}g}{\Delta}S_x\right]+\frac{g^2}{2\Delta}(S_+^2e^{it\delta}+S_-^2e^{-i\delta t}), \quad \text{(A6)}$$

where $\Delta_1=\omega_a-\omega_c$, $S^2=(A/2+1)$, $A/2$ is the Casimir operator of the su(2) algebra and $\hat{n}$ is the photon number operator.

It is easy to see that the term $\sqrt{8}\Omega gS_x/\Delta$ in the above equation can be eliminated by applying to H_1 the transformation

$$U_2=\exp\left(-i\frac{2\sqrt{2}\Omega g}{\Delta_1^2}S_y\right), \quad \text{(A7)}$$

which does not affect the rest of the Hamiltonian (in the given accuracy and for the not very strong external field, when $\Omega g\ll\Delta_1^2$).

Also, the last term in Eq. (A6) does not essentially affect the system dynamics and can be removed using a rotation such as in the rotating wave approximation. Actually, if we consider this term as a perturbation to the rest of the Hamiltonian (after the elimination of the $\Omega gS_x/\Delta$ term), one can note that the first-order correction to the eigenvalues vanishes and the second-order correction is of order $(g/A\Delta)^3$.

Finally, the system Hamiltonian takes the form

$$H_{eff}=\Delta_1 S_z+\delta a^\dagger a_z+\eta(2a^\dagger a+1)S_z+\eta(S^2-S_z^2) +\Omega(a+a^\dagger), \quad \text{(A8)}$$

where $\eta=g^2/\Delta$. One can easily see that the dynamics of any observable will not be affected by the transformations U_1, U_2, and U_3, due to the fact that these transformations are time independent and would only introduce small corrections to the coefficients of the Heisenberg operators. The time range where the Hamiltonian H_2 describes the system dynamics well is defined by the order of the neglected terms and one can show that it is of the order

$$gt\ll(g/A\Delta)^3. \quad \text{(A9)}$$

To find the effective master equation which rules the system in the dissipative case, we have to apply the transformations U_1, U_2, and U_3 to the non-Hamiltonian part (2.2) of the master equation. Nevertheless, it is easy to see that these transformations result in adding terms of order $O(\gamma g/\Delta)$ which can be neglected in the case of weak dissipation ($\gamma\ll g$). Finally, the master equation for the driven Dicke model in the dispersive limit takes the form of Eq. (2.1) where the Hamiltonian is given by Eq. (A8).

[1] C.J. Hood, M.S. Chapman, T.W. Lynn, and H.J. Kimble, Phys. Rev. Lett. **80**, 4157 (1998).

[2] J.M. Raimond, M. Brune, and S. Haroche, Phys. Rev. Lett. **79**, 1964 (1997).

[3] L. Davidovich, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A **53**, 1295 (1997).

[4] G.S. Agarval, R.R. Puri, and R.P. Singh, Phys. Rev. A **56**, 2249 (1997).

[5] A.B. Klimov and C. Saavedra, Phys. Lett. A **247**, 14 (1998).

[6] Christofer C. Gerry and Rainer Grobe, Phys. Rev. A **56**, 2390 (1997).

[7] A. Delgado, A.B. Klimov, J.C. Retamal, and C. Saavedra, Phys. Rev. A **58**, 655 (1998).

[8] S.M. Chumakov, A.B. Klimov, and J.J. Sánchez-Mondragón, Phys. Rev. A **49**, 4972 (1994).

[9] C. Saavedra, A.B. Klimov, S.M. Chumakov, and J.C. Retamal, Phys. Rev. A **58**, 4078 (1998).

[10] A.H. Castro, Neto, and A.O. Caldeira, Phys. Rev. A **42**, 6884 (1990); C.M. Savage and D.F. Walls, *ibid.* **40**, 1071 (1989); M.H.Y. Moussa and D. Otero, Phys. Lett. A **180**, 224

(1993); M.H.Y. Moussa, S.S. Mizrahi, and A.O. Caldeira, *ibid.* **221**, 145 (1996).
[11] R. Onofrio and L. Viola, Phys. Rev. A **58**, 69 (1998).
[12] J.G. Peixotto de Faria and M.C. Nemes, Phys. Rev. A **59**, 3918 (1999).
[13] M. Ban, J. Math. Phys. **33**, 3213 (1992); L.M. Arevalo-Aguilar and H. Moya-Cessa, Quantum Semiclassic. Opt. **10**, 671 (1998).
[14] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).