Effect of four-wave mixing on induced waveguiding

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We study the waveguiding induced in a weak probe propagating in a medium of two-level atoms driven by a copropagating strong pump. We show that when the transverse profile of the pump exhibits solitonlike oscillations, similar oscillations can be induced in the transverse profile of the probe. When the probe waist size is initially narrower than that of the pump, the probe adjusts its width after a short propagation distance, to that of the pump. We show that the effect of the weak beam, which is created during propagation by four-wave mixing, on the waveguiding of the probe, becomes important when the medium is optically thick. This suggests that the importance of four-wave mixing in modifying induced waveguiding should be considered in other waveguiding schemes.

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I. INTRODUCTION

We have previously shown that under suitable conditions, a weak nondegenerate probe beam propagating in a Doppler-broadened two-level atomic medium can exhibit spatial solitonlike behavior due to the effect of a strong copropagating resonant pump beam [1]. This behavior persists as long as the Rabi frequency of the pump remains strong enough to maintain lossless propagation of the probe. However, the pump beam experiences reshaping during propagation, mainly due to nonlinear absorption, thereby limiting the solitonlike propagation of the probe in the medium. Manassah and Gross [2] have also investigated induced focusing of a weak probe. They studied a three-level Λ system, in which a weak probe interacting with one leg of the system experienced focusing due to the interaction of a strong pump with the second leg of the system. More recently, Truscott *et al.* [3] demonstrated experimentally the waveguiding of a weak probe, interacting with one transition of atomic rubidium, due to the influence of an intense pump centered on a different atomic transition. The waveguiding was achieved by creating a transverse refractive index profile for the probe similar to that of an optical fiber. The atomic system was modeled as a three-level V system. In order to create the steep-sided refractive index for the probe, a pump beam with a doughnut intensity profile, and peak intensity of 280 times the saturation intensity of the rubidium vapor was required.

In this paper, we investigate the waveguide effect created by a strong detuned pump beam, propagating in a medium of two-level atoms, on a copropagating detuned weak probe. In this scheme, the pump acts as a spatial soliton in the sense that its transverse intensity profile displays oscillating spatial behavior due to the interplay of self-focusing and diffraction. The conditions for realizing such a spatial soliton have been extensively explored [4–7]. One major concern is the effect of absorption, which is generally present in experiments and tends to damp the predicted effects. In the system considered here the pump is hardly absorbed since it is far from resonance with the atomic transition. We show that an incident weak wave copropagating with this strong pump may experience waveguiding because of the influence of the pump beam. Moreover, we show that this waveguiding is robust: an incident Gaussian probe beam, of width smaller than that of the pump profile, exhibits a pattern on propagation which almost completely follows the oscillations carried out by the pump profile. This occurs after an initial propagation distance through the medium in which the probe profile adjusts its width so that it almost matches that of the pump profile. However, an analysis of the waveguide effect that is similar to the description given by Gross *et al.* [2] and Truscott *et al.* [3] for other systems, misses the contribution of an important effect, namely four-wave mixing. It is well known [8–10] that in a copropagating configuration, a weak beam is created by the process of four-wave mixing, in addition to the incident strong pump and weak probe beams. The influence of this weak generated beam, which copropagates through the atomic medium with the incident pump and probe beams, on the waveguiding of the probe has not been considered before (see, however, Ref. [11]). We show here that the effect of the wave created by four-wave mixing is significant, *even* for configurations with large phase mismatch. We find that whereas the waveguide effect is essentially unlimited when the effect of four-wave mixing is ignored, it becomes severely limited when the effect of four-wave mixing becomes dominant. This occurs when the probe is enhanced because of parametric coupling to the weak generated wave, to the extent that the probe intensity becomes of the order of the intensity of the pump, thereby violating the basic assumption of the probe being much weaker than the pump. We show that by a proper choice of parameters, the weak generated wave can be forced to remain small during most of the propagation distance through the medium, so that the waveguide effect can be clearly discerned.

II. DYNAMICS OF PUMP-PROBE PROPAGATION

We consider the interaction between an incident cw electromagnetic field of the form

$$\tilde{\mathbf{E}}(\mathbf{r},t)=\sum_{Q=L,P}\tilde{\mathbf{E}}_Q(\mathbf{r},t)=\sum_{Q=L,P}\mathbf{E}_Q(\mathbf{r})e^{-i(\omega_Q t-\mathbf{k}_Q\cdot\mathbf{r})}+\text{c.c.} \tag{1}$$

and a medium consisting of two-level atoms. $\mathbf{E}_L(\mathbf{r})$ and $\mathbf{E}_P(\mathbf{r})$ represent the slowly varying envelope amplitude of the strong pump field and the weak probe field, respectively, and ω_L and ω_P are the pump and probe frequencies with wave vectors $\mathbf{k}_L$ and $\mathbf{k}_P$. The two incident laser beams propagate in the z direction through the atomic medium. The dynamics of these two beams as they propagate in a nonlinear medium is determined by solving the coupled Maxwell-Bloch equations. These equations have been used to describe various phenomena, such as the induced focusing of a weak probe in an atomic Λ system due to the influence of a strong pump [2], and the induced focusing and deflection of a weak beam in a self-defocusing medium, due to the presence of a strong pump beam [12,13]. However, these descriptions neglected the possible influence on the propagation of the weak probe due to four-wave mixing. It is well known that on propagation, a new weak beam is created by the process of four-wave mixing, which copropagates with the incident beams through the medium. Here, $\mathbf{E}_M(\mathbf{r})$ represents the slowly varying envelope of the new weak beam of frequency $\omega_M = 2\omega_L - \omega_P$ and wave vector $\mathbf{k}_M$. Boyd *et al.* [8] found that this weak created wave can be significantly amplified during propagation under appropriate circumstances. The influence of this generated weak wave on the propagation of the incident probe is large when phase matching conditions are met. When the mismatch is large, the propagation of the two weak beams, that is, the incident probe and the generated weak beam, is usually considered to be decoupled [9]. However, we have found that *even* for a large phase mismatch, the coupling between the probe and the generated beam cannot be ignored when the beams propagate through long distances in the medium. We therefore infer that it is not possible to assume *a priori* that four-wave mixing has a negligible effect on the propagation of the probe, even for large phase mismatch, and include terms that take this effect into account in the dynamical equations.

Having established the importance of the effect of the generated beam $\mathbf{E}_M$ on the characteristics of the system, we now turn to the dynamical equations. The intensity of the three beams as they propagate is determined by solving the coupled Maxwell-Bloch equations [9]

$$-\nabla^2 \tilde{\mathbf{E}}_J(\mathbf{r},t) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\mathbf{E}}_J(\mathbf{r},t) = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\mathbf{P}}_J(\mathbf{r},t), \quad (2)$$

where $\tilde{\mathbf{P}}_J(\mathbf{r},t)$, with $J = L,P,M$, is the induced polarization of the medium which can be calculated from the density operator $\hat{\rho}$

$$\tilde{\mathbf{P}} = N\langle\tilde{\mu}\rangle = N\langle \mathrm{Tr}(\hat{\rho}\hat{\mu})\rangle, \quad (3)$$

where N is the atomic number density and $\hat{\mu}$ is the dipole moment operator. The medium consists of two-level atoms with lower level $|a\rangle$ and upper level $|b\rangle$ and resonance frequency ω_0. For this two-level system, the density matrix elements are calculated from the steady-state Bloch equations (see, for example, Ref. [14]). Assuming the paraxial approximation, the coupled amplitude equations take the form

$$\frac{\partial}{\partial z}U_L = \frac{i}{4L_D}\nabla_r^2 U_L + \frac{i}{L_{\mathrm{NL}}}\alpha_L(|U_L|^2)U_L, \quad (4)$$

$$\frac{\partial}{\partial z}U_P = \frac{i}{4L_D}\nabla_r^2 U_P + \frac{i}{L_{\mathrm{NL}}}\alpha_P(|U_L|^2)U_P + \frac{i}{L_{\mathrm{NL}}}\kappa_P(U_L^2)U_M^*, \quad (5)$$

$$\frac{\partial}{\partial z}U_M = \frac{i}{4L_D}\nabla_r^2 U_M + \frac{i}{L_{\mathrm{NL}}}\alpha_M(|U_L|^2)U_M + \frac{i}{L_{\mathrm{NL}}}\kappa_M(U_L^2)U_P^*, \quad (6)$$

where we have introduced the normalized variables

$$\xi = x/\sqrt{2}w_{0L}, \qquad U_{L,P,M}(\xi,z) = A_{L,P,M}(\xi,z)/A_{0L}. \quad (7)$$

Here A_{0L}^2 is the dimensionless peak intensity of the incident pump, and $2A_{L,P,M} = \mu T_2 E_{L,P,M}/\hbar$ are the dimensionless pump and weak beam Rabi frequencies, respectively, T_2 is the transverse decay time, and μ is the transition dipole moment. In Eqs. (4)–(6), the parameter $L_{\mathrm{NL}} = \hbar/\pi k N \mu^2 T_2$ is a characteristic length indicating the strength of the nonlinear term. Note that the definition of the parameter L_{NL} is different from that defined for a Kerr-type medium [12], since it does not depend on the pump laser intensity. $L_D = k w_{0L}^2$ is the diffraction length with w_{0L} being the initial spot size of the pump transverse profile. Note that we have restricted our analysis to one transverse dimension only [13], that is, we have considered diffraction effects only in the x direction. The wave vector is given by $k = |\mathbf{k}_J| \simeq \omega_L/c$ and the coefficients α_J and $\kappa_{P,M}$ are given by (see Ref. [9] for full expressions)

$$\alpha_J = \rho_{ba}(\omega_J)/A_J, \quad J = L,P,M, \quad (8)$$

$$\kappa_P = \rho_{ba}(\omega_P = 2\omega_L - \omega_M)/A_M^*, \quad (9)$$

$$\kappa_M = \rho_{ba}(\omega_M = 2\omega_L - \omega_P)/A_P^*. \quad (10)$$

The coefficients α_J account for self-induced absorption and refraction of the pump, and for cross-induced (pump-induced) absorption and refraction of the probe and the created weak beams. The terms $\kappa_{P,M}$ provide the coupling for parametric mixing of the weak beams U_P and U_M. These complex coefficients determine the propagation conditions of the pump, probe, and the four-wave mixing beams. In particular, the choice of parameters such as pump and probe detunings Δ_L and Δ_P, the rate of collisional dephasing $1/T_2$ and the pump Rabi frequency A_L, determines the transverse variation of α_J and $\kappa_{P,M}$. The aim of the present work is to show that under appropriate conditions, a self-focusing pump beam, which undergoes solitonlike cycles of focusing and spreading along its propagation path in the medium, can induce a waveguide effect in a copropagating probe. Moreover, we show that the weak beam that is generated by four-wave mixing plays *a non-negligible role* in this system, and has to be taken into account. We find that as long as its behavior is not dominated by parametric enhancement, the probe follows the behavior of the pump and exhibits similar cycles of focusing and spreading.

In a previous publication [1], we showed that for a judicious choice of parameters, a nondegenerate probe can become confined along the propagation path through the medium, due to the effect of a resonant pump beam. This effect can be obtained when two requirements are met. First, the probe absorption, which is proportional to $\mathrm{Im}\alpha_P$, is small and, second, the refractive index, which is proportional to $\mathrm{Re}\,\alpha_P$, leads to a converging wave front by inducing focusing of the probe. The first requirement was fulfilled by exploiting the unique properties of the dead zone, which is obtained for Doppler-broadened atoms when the Doppler width is larger than the generalized pump Rabi frequency $(\Delta_L^2+4V_L^2)^{1/2}$ with $V_L=A_L/T_2$, so that the stimulated emission of atoms with Doppler-shifted detunings smaller than the Rabi frequency is cancelled by the absorption of Doppler-shifted atoms with detunings larger than the Rabi frequency. These cancellation effects do not exist for the nonlinear probe refractive index so that it survives Doppler broadening. Thus, the dead zone has the unique advantage of almost zero absorption of the probe accompanied by relatively large values of its nonlinear refractive index [15,16].

The second requirement concerns the induced focusing of the probe. This induced focusing is essential for probe confinement, as well as for waveguiding of the probe which is described in this paper. We therefore discuss this subject in detail. The induced focusing is determined by the nonlinear refractive index, which is usually written in the form

$$n_P=n_0+n_2I, \tag{11}$$

where n_2 is a constant for the case of a Kerr medium [17] and I is the intensity. The Kerr coefficient n_2 determines the spatial behavior of the beam. Thus a beam whose transverse intensity profile decreases monotonically from the center to the periphery is focused if $n_2>0$ and defocused if $n_2<0$. Equation (7) is only valid for weak fields but can readily be generalized for more intense fields by replacing n_2 by dn_P/dI [18,19]. When n_P was plotted as a function of the square of the pump Rabi frequency $(2V_LT_2)^2$ for a pump in resonance with a two-level system [18,19], we found that an extremum is obtained when the generalized $2V_L$ is equal to the absolute value of the pump-probe detuning $\delta=\omega_P-\omega_L$. This extremum is a maximum for $\delta>0$, and a minimum for $\delta<0$. Thus the generalized Kerr coefficient dn_P/dI of the probe is itself strongly nonlinear near $2V_L=|\delta|$: for $\delta<0$ and $2V_L<|\delta|, dn_P/dI<0$ and the probe is defocused, whereas for $\delta<0$ and $2V_L>|\delta|, dn_P/dI>0$ and the probe is focused. The extrema coincide with the low-intensity limit of the dead zone. These observations led us to suggest that in an experimental setup in which the probe is red-detuned, and the transverse intensity profile is chosen such that $2V_L>|\delta|$ throughout most of the profile, the pump would lead to induced focusing and hence confinement of the probe beam.

We have shown that when both the conditions are fulfilled, the interplay between pump-induced focusing and diffraction-induced spreading of the probe beam leads to the confinement of the probe along its propagation path in the medium.

When the pump is detuned from the atomic resonance, that is $\Delta_L\neq 0$, the behavior of the nonlinear refractive index n_P is more complex. Plotting n_P as a function of the square of the pump Rabi frequency $(2V_LT_2)^2$, we found that for $|\Delta_L|<|\delta|$, an extremum appears at a pump Rabi frequency such that $(\Delta_L^2+4V_L^2)\approx\delta^2$ when δ and Δ have opposite signs. When $|\Delta_L|<|\delta|$ and δ and Δ have the same signs, the refractive index has a dispersive line shape centered near the pump Rabi frequency given by $(\Delta_L^2+4V_L^2)\approx\delta^2$. These features disappear for detunings satisfying $|\Delta_L|>|\delta|$. For this case, the generalized Kerr coefficient dn_P/dI of the probe is negative for all values of pump Rabi frequency when $\Delta_L>0$ and the probe is defocused, whereas for $\Delta_L<0, dn_P/dI>0$ and the probe is focused. These observations lead us to suggest that a self-focusing pump with $\Delta_L<0$ would lead to focusing throughout the transverse profile of a probe whose frequency satisfies $\delta>\Delta_L$.

III. PUMP PROPAGATION AND PROBE PROPAGATION

It is well known that a strong pump whose frequency is tuned to the high-frequency side of the atomic transition experiences self-focusing [4,6,7]. When the self-focusing effect of the refractive index exactly compensates the effect of diffraction, the beam becomes self-trapped and is called a spatial soliton. An incident beam which deviates from the self-trapped solution may exhibit cycles of focusing and spreading, which may be called *solitonlike oscillations*. This behavior is similar to the solitonlike propagation of a beam in a Kerr medium. In an ideal Kerr medium the pump beam is strong but does not saturate the medium. Moreover, absorption is negligible and the soliton propagates without being attenuated. The strong pump beam in our system propagates in the medium without being affected by the weak probe, as determined by Eq. (4). In this sense its characteristics resemble those of a single strong beam propagating in a medium composed of two-level atoms. The pump beam here is chosen to be far off-resonance with the atomic transition so that $\Delta_L\gg 1/T_2$. In addition, the pump Rabi frequency employed is highly saturating. For this large detuning and high beam intensities, absorption is small compared to the effect of the refractive index. However, absorption at the low intensity wings of the pump can still lead to reshaping of the pump, which in turn modifies the focusing effects experienced by the probe on propagation. We have found that this effect can be reduced by introducing dephasing collisions which decrease T_2 and hence the value of $\mathrm{Im}\alpha_L$. Note that the effect of Doppler broadening is negligible for this system, since the detunings are much larger than the typical values of Doppler width. Under the above conditions, a blue-detuned self-focusing pump, propagating in a medium of atomic two-level system, exhibits solitonlike behavior, similar to that observed for a Kerr medium.

We now consider propagation of the probe. An incident blue-detuned weak probe copropagating in an atomic medium with an incident strong blue-detuned pump beam is influenced by several effects. For a probe whose frequency satisfies $\delta > \Delta_L$, the pump-induced generalized Kerr coefficient dn_P/dI is positive for all values of pump Rabi frequency, leading to probe focusing. Pump-induced absorption is significant for low pump intensities and smaller at high pump intensities due to saturation effects. Therefore, a probe whose initial transverse intensity profile coincides mostly with the high intensity part of the pump transverse profile, should not be much absorbed. Note that self-induced effects of absorption or refractive index are not taken into account since we have assumed that the probe is weak compared to the pump. Another influence, which we have already mentioned and is often underestimated when pump-probe propagation is analyzed, comes from the generation of a weak beam by the process of four-wave mixing. This weak beam copropagates in the medium with the incident pump and probe beams. From Eqs. (5) and (6) one can see that on propagation the weak incident probe and generated beam become coupled. It has been shown that if a beam of frequency $\omega_M = 2\omega_L - \omega_P$ is incident upon the medium, it can be amplified because of this coupling [8]. The coupling is maximal when there is perfect phase matching and small when the phase mismatch is large. When the magnitude of phase mismatch is very large, the coupled Eqs. (5) and (6) decouple and Eqs. (2) and (3) of Ref. [1] are recovered [9]. The phase mismatch can be made very large by arranging the incident beams such that the angle between them is very large. In general, however, one cannot assume that the propagation of probe and generated beams is decoupled.

For a copropagating configuration the phase mismatch is determined by

$$\Delta\varphi = k(2n_L - n_P - n_M), \tag{12}$$

where n_J is the refractive index of the corresponding beam given by

$$n_J^2 = 1 + \frac{4}{kL_{NL}} \mathrm{Re}\,\alpha_J, \quad J = L, P, M. \tag{13}$$

According to Eqs. (12) and (13), the phase mismatch is mainly determined by the choice of the parameters, such as pump and probe detunings Δ_L and Δ_P, the rate of collisional dephasing $1/T_2$ and the pump Rabi frequency A_L. Note that, due to pump reshaping, the phase mismatch $\Delta\varphi$ is modified during propagation. Here, our aim is to study the waveguiding of the probe due to the influence of the pump, and the investigation of the effect of four-wave mixing on this waveguiding. We find that whereas the waveguide effect is ideally unlimited when the effect of four-wave mixing is ignored, the model fails when the effect of four-wave mixing becomes dominant. This results when the probe is enhanced due to parametric coupling to the generated weak wave, to the extent that the probe intensity becomes of the order of the intensity of the pump, thus violating the assumption of the probe being much weaker than the pump. By properly choosing these parameters, one can minimize the coupling between the probe and mixing beams so that in a configuration where no incident wave with frequency $\omega_M = 2\omega_L - \omega_P$ is present, the beam created at frequency ω_M remains very weak during most of the propagation distance in the medium. In this case, the probe is unaffected by the weak created beam throughout most of its propagation. Under these conditions, the propagation of the probe is mainly governed by the interplay of the pump-induced focusing effect and the spreading due to diffraction. As a result of these combined effects, the probe exhibits solitonlike oscillations provided that the weak created beam remains small.

We assume that the transverse profiles of the two incident beams are initially Gaussian,

$$U_L(\xi,0) = \exp[-\xi^2], \tag{14}$$

$$U_P(\xi,0) = \left(\frac{A_{0P}}{A_{0L}}\right) \exp\left[-\xi^2 \cdot \frac{w_{0L}^2}{w_{0P}^2}\right], \tag{15}$$

where w_{0L} and w_{0P} are the initial pump and probe waist sizes. We choose the probe waist to be smaller than that of the pump so that the probe profile is initially within the transverse region where the pump intensity is large so that the pump-induced absorption is small. Then, under the conditions described above, the pump performs solitonlike oscillations due to the interplay between self-focusing and diffraction, and induces similar oscillations on the probe, by means of the competing effects of pump-induced focusing and diffraction. We solve Eqs. (4)–(6) numerically by a procedure based on the discretization of the transverse Laplacian in terms of second-order differences and the integration of the first-order differential equations by the fourth-order Runge-Kutta method.

IV. RESULTS AND DISCUSSION

In the results presented here, the pump is detuned to the high-frequency side of the atomic resonance with $\Delta_L T_2 = -70$ and the pump-probe detuning is $\delta T_2 = -25$ with $T_2 = 0.1 T_1$. The initial dimensionless pump and probe Rabi frequencies are $2A_{01} = 100$ and $A_{02} = 10^{-2} A_{01}$. The waist size of the probe is initially half that of the pump, that is, $w_{0L} = 2w_{0P}$, and thus almost all the probe beam is contained in the region where absorption is negligible. In order to emphasize the effect of four-wave mixing, we first present the numerical results under the approximation that the propagation of the probe and the created beams are completely decoupled, that is, we deliberately drop the coupling terms in Eqs. (4)–(6). This approximation is often made in propagation schemes where the phase mismatch is large. Under these circumstances, the propagation of the probe is solely determined by the mutual effects of pump-induced refractive index and absorption, and diffraction. In Fig. 1 we show the transverse intensity profiles of the probe and the pump as a function of the propagation distance, for the case of decoupled probe and generated beams. Here the nonlinear and diffraction lengths are given by $L_{\mathrm{NL}} = 1 \times 10^{-3}$ cm and $L_D = 2$ cm. The pump exhibits sequences of focusing and broadening, exactly in the same way that would be expected from the propagation of a single intense beam through the atomic medium. In the initial stages of propagation the pump

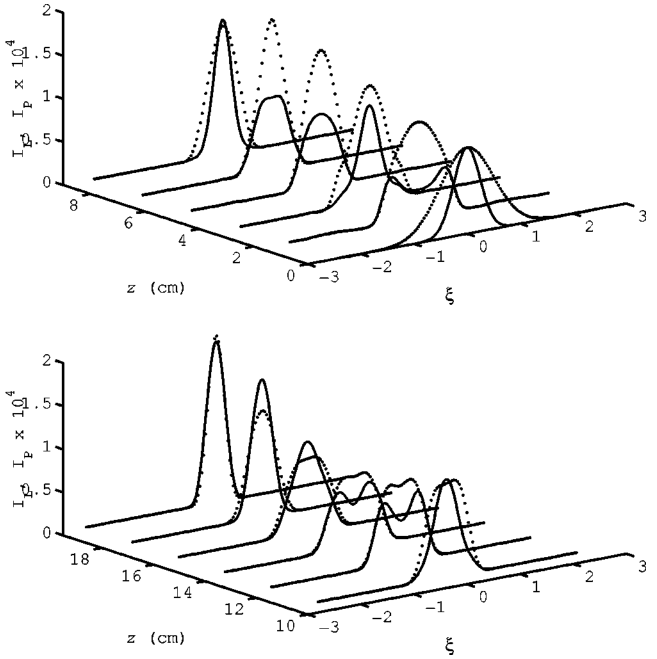

FIG. 1. The transverse intensity profile of the pump I_L, and the transverse intensity profile of the probe I_P, as a function of the propagation distance *z*, for decoupled probe and generated beams. The solid line indicates the probe while the dotted line illustrates the pump. The pump and probe parameters are $A_{0L}=50,\Delta_L T_2=-70,\delta T_2=-25,T_2=0.1T_1,L_D=2$ cm,$L_{NL}=10^{-3}$ cm.

experiences self-focusing, which is followed by broadening due to diffraction. On further propagation, the pump exhibits reshaping of its intensity profile due to the interplay between the nonlinear refractive index and diffraction, which gives rise to a successive focusing and defocusing. We now turn to the propagation of the probe, also shown in Fig. 1. The initially narrow probe promptly adjusts its transverse profile to follow the sequences of focusing and broadening performed by the pump, and the two beams exhibit simultaneous solitonlike oscillations. Note that after propagating a short distance in the medium, during which the probe transverse profile adapts its width to fit the width of the pump transverse profile, the two beams proceed to propagate with almost equal widths. This ''almost matching'' of the widths of the two beams occurs because changes in the pump transverse profile induce changes in the refractive index of the probe. At the very early stages of propagation (not shown in the figure), the probe is almost unaffected by the pump, and only broadens due to diffraction, since its waist size is small and the refractive index induced by the much wider pump is mainly significant outside the probe transverse profile. On further propagation, the probe starts being influenced by the pump. When the pump is focused and its transverse profile is pulled towards the center, the induced probe refractive index becomes more pronounced near the center. The transverse profile of the probe then ''feels'' the effect of the induced refractive index and is focused. After this initial adjusting of the widths, the pump and probe propagate for a long distance in the medium.

In Fig. 2, we plot the transverse intensity profiles of the

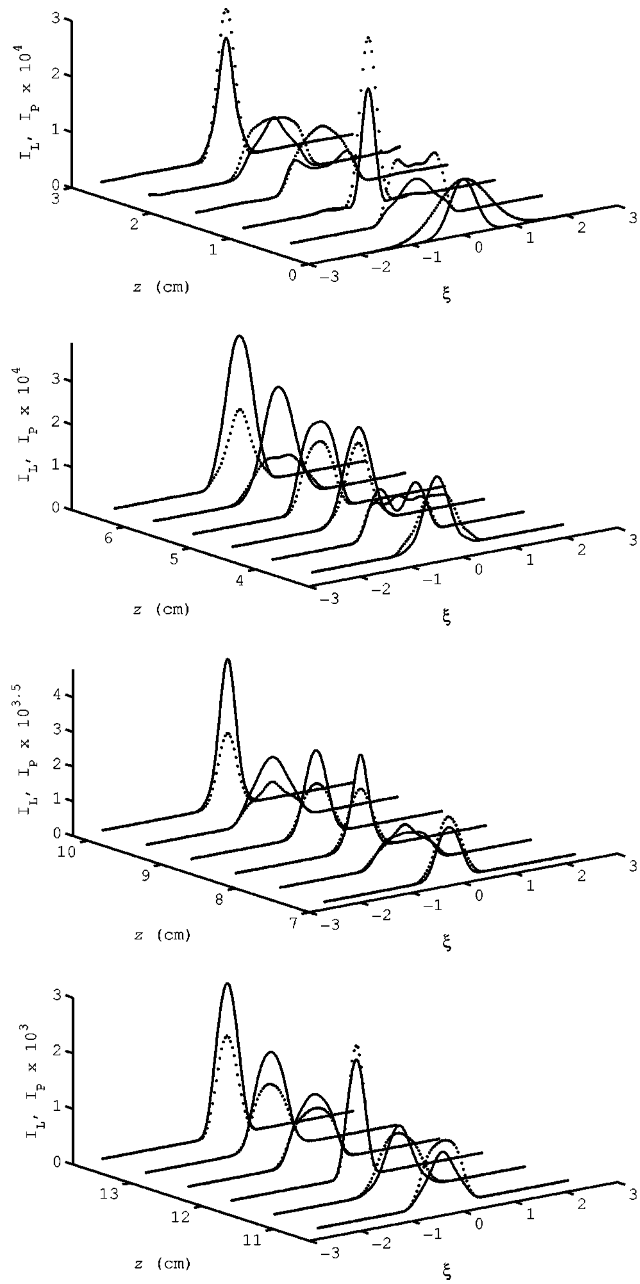

FIG. 2. The transverse intensity profile of the pump I_L, and the transverse intensity profile of the probe I_P, as a function of the propagation distance *z*, for coupled probe and generated beams. The parameters are the same as in Fig. 1 but the effect of four-wave mixing is included.

probe and the pump as a function of the propagation distance for the same parameters used in Fig. 1 but now including the effect of four-wave mixing. Note that there are three beams copropagating in the medium, the incident pump and probe beams, and the generated weak beam, but only the transverse profiles of the pump and the probe are displayed in Fig. 2. The propagation of the strong pump is, as expected within the assumptions made here, unaffected by the effect of four-wave mixing, and its transverse profile performs the same oscillations as in Fig. 1. In the first stages of propagation, the transverse profile of the probe exhibits cycles of focusing

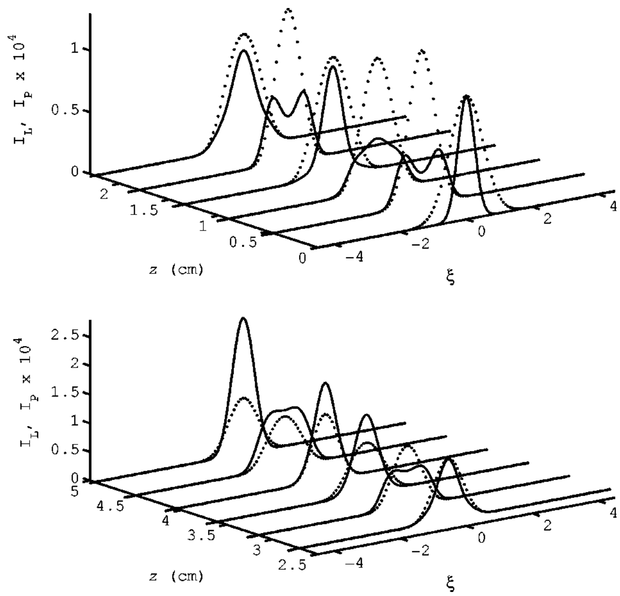

FIG. 3. The transverse intensity profile of the pump I_L, and the transverse intensity profile of the probe I_P, as a function of the propagation distance z, for coupled probe and generated beams. The parameters are the same as in Fig. 2 but with $L_D = 0.4$ cm.

and defocusing in much the same way as shown in Fig. 1, indicating that the generated beam is very weak and has no significant effect. On further propagation, as the weak beam starts growing and the phase matching conditions change, the probe is affected by this new beam. The transverse intensity of the probe increases due to the combined effects of pump-induced focusing and the parametric gain induced by the coupling to the weak beam. For example, at $z \sim 8$ cm, the probe intensity is five times greater than in the absence of four-wave mixing. This propagation distance corresponds to an optical path of $\alpha_0 L \sim 16\,000$, where L is the length of the medium (here taken to be 8 cm), and α_0 is the weak-field, line center, absorption coefficient given by $\alpha_0 = 4\pi N k \mu^2 T_2/\hbar$. After much further propagation, the assumption of a weak probe becomes invalid due to the effect of four-wave mixing.

We now show that the waveguiding of the probe due to the effect of the pump is robust. We reduce the size of the waists of the pump and probe profiles, and show that as before, the probe profile adjusts its width and follows the profile of the pump. In Fig. 3 we plot the transverse intensity profiles of the probe and the pump as a function of the propagation distance for the same parameters used in Fig. 2 but with $L_D = 0.4$ cm. One can see that by $z \sim 0.3$ cm, the probe transverse profile has a width comparable to the width of the transverse profile of the pump. For these parameters, the effect of the generated weak wave is pronounced already at $z \sim 5$ cm, where the probe transverse intensity increases due to parametric gain.

The waveguiding of a weak probe by the influence of a strong pump can be experimentally tested in a medium consisting of atomic vapor, for example, sodium atoms. The $3^2S_{1/2}$-$3^2P_{3/2}$ transition of sodium with transition wavelength $\lambda_0 = 589$ nm forms an effective two-level system. We expect that it should be possible to observe the behavior of the probe shown in Fig. 2 with $N \sim 10^{13}$ atoms/cm^3 in the medium and a pump laser with initial intensity $I \sim 50$ kW/cm^2. With an initial pump waist $w_{0L} \sim 44$ μm, the diffraction length is $L_D \sim 2$ cm and the nonlinear coefficient is $L_{NL} \sim 10^{-3}$ cm. The transverse decay time T_2 is reduced due to collisions, with $T_2 \sim 0.1 T_1$. Under these conditions, a probe beam with $w_{0L} \sim 22$ μm should exhibit an oscillating transverse profile following similar behavior of the pump beam. This behavior can be explained within the assumptions that have been discussed in this work.

It is clear that the effect of four-wave mixing on the waveguiding of a probe is significant in thick optical media. In a configuration where only a pump beam and a single probe beam are incident upon the atomic medium, a weak beam is generated by four-wave mixing and builds up during propagation. After having propagated for some distance in the medium, this created weak beam becomes strong. The probe is then enhanced due to parametric coupling with the created weak beam and eventually becomes as intense as the pump. At this point one has to resort to a full scale analysis, where all beams participating in the system are treated to all orders.

In conclusion, we have discussed the waveguiding of a probe in a medium of two-level system, due to the influence of a strong pump. When the pump transverse profile exhibits solitonlike oscillations, the probe transverse profile follows a similar pattern. We have shown that the effect of a weak beam which is created during propagation, on the waveguiding of the probe, becomes important when the medium is optically thick. This suggests that the effect of this weak beam which is created by the process of four-wave mixing should be considered in other waveguiding schemes.

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