Coherent dynamics of Bose-Einstein condensates in high-finesse optical cavities

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We study the mutual interaction of a Bose-Einstein condensed gas with a single mode of a high-finesse optical cavity. We show how the cavity transmission reflects condensate properties, and calculate the self-consistent intracavity light field and condensate evolution. Solving the coupled condensate-cavity equations we find that while falling through the cavity, the condensate is adiabatically transfered into the ground state of the periodic optical potential. This allows time-dependent nondestructive measurements on Bose-Einstein condensates, with intriguing prospects for subsequent controlled manipulation.

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Since the first experimental realizations of Bose-Einstein condensation in dilute gases [1–3], the properties and possible applications of condensates in various situations have been investigated. Most recently, attention has been drawn to the study of condensates in optical lattices [4–8], as have been intensely used in the context of laser cooling and trapping of clouds of noninteracting atoms [9]. But whereas the occupation of the lattice sites in an optical molasses for a cloud of laser-cooled atoms at best is one atom per ten wells, the atomic densities found in a condensate allow for multiple occupation of each single well, which gives rise to a variety of new phenomena.

In this paper we investigate the case of a condensate falling through a driven high-finesse optical cavity. The strong coupling of the condensed atoms to the cavity mode changes the resonance frequency of the cavity, which hence is shifted into or out of resonance with the driving field. Consequently, the intracavity field intensity is modified, and this can easily be measured by detecting the cavity output field. We show that according to the collective nature of the condensate this gives a measurable effect even for very low-field intensities and for detunings from the atomic resonance frequency so large that the spontaneous scattering of photons is negligible. The proposed system should allow us to perform nondestructive measurements on the condensate. Similar systems have been used recently to predict amplification of matter waves [10] and the appearance of dressed condensates [11,12].

Let us first introduce our model system in more detail, which is similar to that used recently to study the effect of a dynamically changing cavity field on the motion of a single atom [13–16]. We consider a Bose-Einstein condensate consisting of N two-level atoms of resonance frequency ω_a and spontaneous decay rate Γ falling through an optical cavity. The atom-cavity coupling is

$$g(x,t)=g_0\cos(kx)e^{-(v_zt)^2/(2w^2)} \tag{1}$$

for a cavity mode in the form of a standing wave in the longitudinal direction and a Gaussian with waist w transversally. The condensate is assumed to fall with constant velocity v_z, meaning that we neglect the transverse light forces on the atoms and the gravitational acceleration in the interaction region. Two further assumptions have been made at this point. First, the spatial extension of the condensate has to be small compared to the waist w of the cavity in order to allow a quasi-one-dimensional treatment. This condition must be fulfilled during the complete interaction time, which limits the maximum condensate density. For densities that are too high, the atom-atom repulsion will lead to a fast expansion of the condensate in the transverse direction. However, as we will see below, the transition times required for our scheme are of the order of the recoil time, which means time scales of 0.1 ms. This should be short enough to allow reasonably high densities without too much transverse expansion. Second, we assume that the induced resonance frequency shift of the cavity [17,18] is much smaller than the longitudinal mode spacing, so that we can restrict the model to a single longitudinal mode with wave number k. The cavity with resonance frequency ω_c and cavity decay rate κ is externally driven by a laser of frequency ω with pump amplitude η and is treated classically; that is, the intracavity field is described by a (complex) field amplitude α.

As we are interested in the limit where the condensate is not destroyed by the light field, we will assume a large detuning $\Delta_a=\omega-\omega_a\gg\Gamma$ of the driving laser from the atomic resonance such that the saturation parameter $s=g_0^2/\Delta_a^2\ll 1$. Moreover, we want the cavity decay to dominate over the spontaneous decay of *all* atoms, and thus impose the condition

$$\kappa\gg N\Gamma s. \tag{2}$$

In this realistically achievable limit we are not only allowed to adiabatically eliminate the excited state of the atoms but also to completely omit the effect of atomic decay.

Hence we obtain the equation of motion for the field amplitude

$$\dot{\alpha}(t)=[i\Delta_c-iN\langle U(\hat{x},t)\rangle-\kappa]\alpha(t)+\eta \tag{3}$$

where $\Delta_c=\omega-\omega_c$ is the cavity-pump detuning, $U(x,t)=g(x,t)^2/\Delta_a$ the optical potential per photon, and "$\langle\cdots\rangle$" denotes the expectation value taken with respect to the condensate wave function $|\psi(t)\rangle$ at this time. This term describes the action of the condensate on the cavity field: the refractive index of the condensate shifts the resonance fre-

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quency of the cavity by an amount of $N\langle U(\hat{x},t)\rangle$. (With very high finesse optical cavities this effect has already been observed even for a single atom [16,19,20]; that is, for $N=1$.) If we require that this effect significantly changes the intracavity field intensity in order to yield a measurable difference in the cavity output, then the maximum frequency shift must be of the order of or larger than the cavity line width κ, which implies

$$Ng_0^2/\Delta_a \geqslant \kappa. \tag{4}$$

From this we obtain an order of magnitude for the required detuning Δ_a which we insert into Eq. (2) to obtain the following condition for the cavity parameters:

$$\frac{Ng_0^2}{\Gamma\kappa} \gg 1. \tag{5}$$

The condensate wave function itself obeys a nonlinear Schrödinger equation, known as the Gross-Pitaevskii equation (GPE)

$$i\frac{d}{dt}\psi(x,t)=\Bigg\{\frac{\hat{p}^2}{2m}+|\alpha(t)|^2U(x,t) + Ng_{coll}|\psi(x,t)|^2\Bigg\}\psi(x,t), \tag{6}$$

where the last term describes two-particle collisions between the condensed atoms and is related to the s-wave scattering length a by $g_{coll}=4\pi\hbar^2 a/m$. The GPE and Eq. (3) for the cavity field form a set of coupled nonlinear equations describing the dynamics of the compound system formed by the condensate and the optical cavity.

In this work we will only consider the special case where the cavity decay rate κ is much larger than the oscillation frequency of bound atoms in the optical potential of the cavity. In this limit the intracavity field amplitude adiabatically follows the condensate wave function, and hence at any time is given by

$$\alpha(t)=\frac{\eta}{\kappa - i[\Delta_c - N\langle U(\hat{x},t)\rangle]}. \tag{7}$$

Thus, the light intensity of the cavity output, which is proportional to $|\alpha|^2$, provides information about the condensate wave function. In the following we will investigate this effect in certain special parameter limits.

Let us first consider the simple case where the cavity field is weak enough and the interaction time $\tau=w/v_z$ is short enough such that the condensate wave function remains essentially unperturbed (flat on the length scale of an optical wavelength). In this limit the cavity field can be evaluated analytically by inserting the frequency shift per atom,

$$\langle U(\hat{x},t)\rangle=\frac{U_0}{2}e^{-(v_z t)^2/w^2}, \tag{8}$$

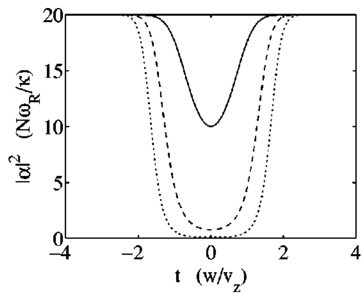

FIG. 1. Cavity photon number $|\alpha|^2$ vs time for a condensate falling through the cavity without being perturbed by the cavity field (see the text for details). The optical potential depth is given by $NU_0=2\kappa$ (solid line), 10κ (dashed), and 30κ (dotted), respectively; $\Delta_c=0$ and $\eta^2=20N\kappa\omega_R$.

into Eq. (7), where we have introduced $U_0=g_0^2/\Delta_a$. In Fig. 1 the resulting mean cavity photon number $I=|\alpha|^2$ is plotted as a function of time for different atom numbers in the condensate or, equivalently, for different optical potentials. For the parameters chosen in Fig. 1, the empty cavity is in resonance with the driving field but is shifted out of resonance by the presence of the condensate. Hence the condensate is detected by the *absence* of light, which further reduces the action of the cavity onto the condensate. Therefore the cavity provides a nonperturbative method of detection. The maximum resolution of the detection is limited, however, by the cavity waist w, and is not good enough to detect fine structures such as condensate interference fringes. It might be useful, however, to measure the output of an atom laser, as has recently been realized experimentally [21,22]. Note also that this detection scheme only relies on the density of the condensate, not on its coherence, and thus in principle works as well with an incoherent cloud of atoms.

Let us now consider the opposite limit of a condensate falling very slowly through the cavity. We will find that under such conditions the condensate is adiabatically transfered into the lowest bound state of the optical potential, and hence is strongly localized. In this case we must use the position and time-dependent optical potential

$$U(\hat{x},t)=U_0\cos^2(k\hat{x})e^{-(v_z t)^2/w^2}, \tag{9}$$

so that the condensate at all times ''feels'' a periodic optical potential with periodicity λ/2. The condensate wave function is conveniently described in terms of Bloch states,

$$\psi_{n,q}(x)=e^{iqx}\phi_{n,q}(x), \tag{10}$$

where the functions $\phi_{n,q}$, $n\geqslant 0$, are periodic, and the Bloch momentum $\hbar q$ is confined to the interval $[-\hbar k,\hbar k]$. Note that the coherent interaction of the condensate with the cavity light only couples Bloch states of different n, but leaves states of different q decoupled, thus q is a conserved quantity. We are interested in the adiabatic limit corresponding to small transverse velocities $v_z \ll w/\tau_R$, where τ_R is the inverse of the recoil frequency $\omega_R=\hbar k^2/(2m)$. In this limit the coherent time evolution associated with a potential of the form of Eq. (9) maps the Bloch energy bands n onto the

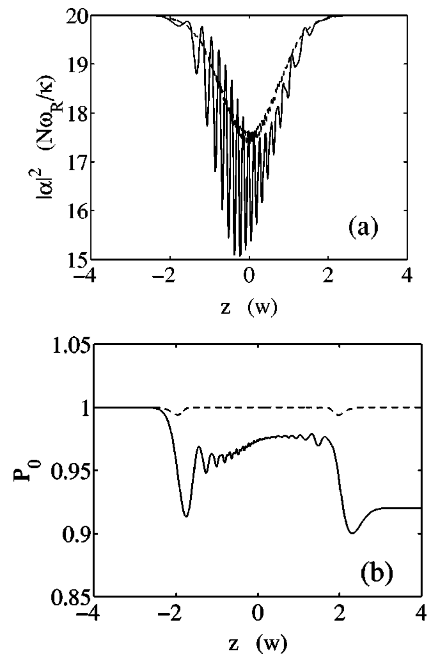

FIG. 2. Adiabatic transfer of the condensate wave function to the bound ground state. $g_{coll}=0$, $\Delta_c=0$, $NU_0=10\kappa$, and $v_z=w/(3\tau_R)$ (dashed curves), respectively, and $v_z=w/\tau_R$ (solid curves). (a) Intracavity photon number $|\alpha|^2$. (b) Population of instantaneous ground state corresponding to the photon number at any time. The initial condensate wave function is taken as the zero-momentum eigenstate $|p=0\rangle$.

free-space momentum intervals $[-(n+1)\hbar k,-n\hbar k]$ and $[n\hbar k,(n+1)\hbar k]$. This phenomenon is known, and has been exploited in the context of laser cooling [23,24]. The same effect can be used in our model to transfer a falling condensate with a transverse momentum distribution confined in $[-\hbar k,\hbar k]$ into the lowest-energy band $(n=0)$ of the optical potential inside the cavity. However, the situation is more complex here than for the case of laser cooling for two reasons. First, the light intensity itself depends on the condensate wave function and hence on time. Second, the condensate wave function obeys the *nonlinear* Schrödinger equation. Hence, for any given *z* position of the condensate (any given time) the lowest energy state has to be found by self-consistently solving Eq. (7) and the GPE (6).

In Fig. 2 we show the time evolution of the intracavity field intensity I and the overlap P_0 of the condensate wave function with the lowest energy band, as the condensate is falling through the cavity without atomic collisions $(g_{coll}=0)$. For simplicity we assume that the initial condensate wave function is the free momentum state of zero momentum. The conservation of the Bloch momentum q then ensures that at any time only the Bloch states with $q=0$ are populated, and the overlap with the lowest energy band is given by

$$P_0=|\langle\psi|\phi_{n=0,q=0}\rangle|^2. \tag{11}$$

For a condensate falling with a velocity $v_z=w/\tau_R$, the transfer of the free wave function into the optical potential is not adiabatic (especially not on entering and leaving the cavity),

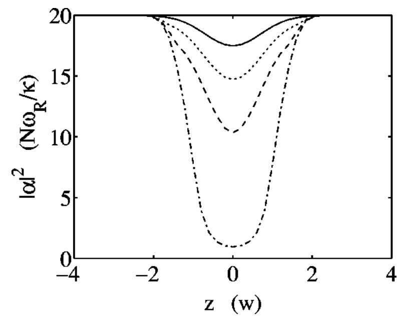

FIG. 3. Photon number corresponding to the self-consistent ground state vs the position of condensate in the cavity mode. $Ng_{coll}=0\omega_R$, $5\omega_R$, $10\omega_R$, and $20\omega_R$ (from top to bottom), $\Delta_c=0$, and $NU_0=10\kappa$. [For Rb and Na experiments, $Ng_{coll}=\omega_R$ corresponds to condensate densities of the order of 30 atoms per potential well, i.e., per $(\lambda/2)^3$.]

and the occupation of the ground state drops to about 90%. Hence there is a significant occupation of excited modes and the corresponding spatial oscillation of the condensate is reflected in the oscillation of the cavity output. For $v_z=w/(3\tau_R)$, however, nearly all of the population is transferred, into the lowest bound state, and accordingly the oscillations are suppressed.

For the parameters of Fig. 2, $U_0>0$, and the condensate is attracted to the *nodes* of the light field. Hence the lowest bound state is localized at these positions which leads to a much reduced coupling of the condensate to the cavity and correspondingly to a much smaller frequency shift of the cavity resonance. This can easily be seen by comparing the results for the cavity field with the solid line of Fig. 1, which is taken for the same parameters, but for a condensate falling so fast that the wave function does not have the time to change due to the presence of the optical potential wells.

Figure 3 shows the effect of collisions on the intracavity photon number in the adiabatic regime $(v_z\ll w/\tau_R)$. The curves are derived from the self-consistent solution of Eq. (7) and the GPE for different values of g_{coll}. For increasing g_{coll} the atoms in the condensate increasingly repell each other, counteracting the confining effect of the optical potential. The wave function becomes broader and the coupling of the condensate to the cavity stronger, which in turn leads to larger shifts in the cavity resonance frequency and a reduced cavity field intensity. Hence the decrease in the cavity output provides a direct measure of the atom-atom interaction within the condensate. Note that this could be used for *in situ* measurements of Feshbach resonances, if one manipulates the *s*-wave scattering length by applying a magnetic field [25–28]. Finally, in Fig. 4 we plot the energy of the self-consistent ground state and of the two lowest collective excitations of the condensate. These excitations are obtained from the investigation of the behavior of small deviations of the condensate wave function $\psi(x,t)$ from the stationary, self-consistent ground state [which we denote by $\phi(x)$ in the following]. We thus write $\psi(x,t)=\phi(x)+\delta\psi(x,t)$ and insert this ansatz into the GPE (6). After linearization in the small deviation $\delta\psi(x,t)$ for the time evolution we obtain

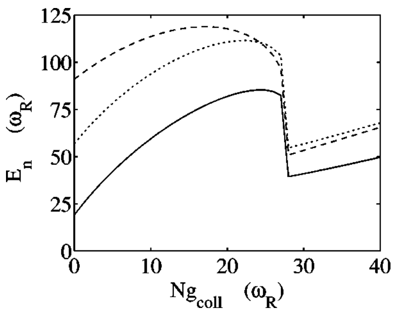

FIG. 4. Eigenenergies E_n of the self-consistent ground state ($n=0$, solid curve), of the first antisymmetric collective condensate excitation ($n=1$, dotted curve), and of the first symmetric excitation ($n=2$, dashed curve) vs collision parameter g_{coll} for $\Delta_c=0$, $NU_0=10\kappa$, $\eta^2=40N\kappa\omega_R$.

$$i\frac{d}{dt}\delta\psi=\left\{\frac{\hat{p}^2}{2m}+|\alpha_0|^2U+2Ng_{coll}|\phi|^2-E_0\right\}\delta\psi$$
$$+Ng_{coll}\phi^2\delta\psi^*-NU\phi\frac{\partial|\alpha_0|^2}{\partial\Delta_c}(\langle\phi|U|\delta\psi\rangle+\text{c.c.}),\tag{12}$$

where E_0 is the eigenenergy of the ground state $\phi(x)$ and α_0 the corresponding cavity field given by Eq. (7). The first two terms on the right-hand side of Eq. (12) describe the well-known collective excitations of a condensate in a trap with a fixed depth [17], while the additional last term results from the variation of the optical potential according to the changing cavity field. Equation (12) together with its complex conjugate form a linear set of equations for $\delta\psi$ and $\delta\psi^*$ which we solve numerically to obtain the eigenfrequencies plotted in Fig. 4. The difference between these frequencies is responsible for the oscillation of the cavity output in the case of nonperfect adiabatic transfer of the condensate wave function into the lowest bound state, as shown previously in Fig. 2.

For $g_{coll}=0$ the ground state is strongly localized at the antinodes of the cavity, leading to a large intracavity field intensity. Hence the optical potential is well approximated by a harmonic oscillator, and the ground state is the harmonic oscillator ground state and thus an even function of the position x. Since the first excited state is an odd function with respect to x, the last term in Eq. (12) vanishes, and this excited state is also a harmonic-oscillator state. For the next higher excitation, however, this term is nonzero and leads to a modification of the state and the eigenenergy.

For increasing g_{coll} the internal (collisional) energy of the condensate and hence the ground-state energy E_0 grow. Simultaneously the collisional term in Eq. (12) increases, and the excitation spectrum changes accordingly. However, the excitations obey the same symmetry as in the case of $g_{coll}=0$, and therefore all odd states are found to be the same as for a trap of constant depth, whereas the dynamical cavity field changes the energies of the even states. This gives rise to the crossing of the lowest two excitation energies at $Ng_{coll}\approx 24\omega_R$ in Fig. 4.

As the wave function broadens with increasing g_{coll}, the cavity field intensity decreases and above a certain value ($Ng_{coll}\approx 28\omega_R$ for the parameters in Fig. 4) no bound state exists in the optical potential. Hence the eigenfunctions above this critical value resemble free-space momentum states which leads to the significant change in the behavior of the spectrum of excitation energies.

In conclusion we have shown that a high-quality optical cavity provides a powerful tool to investigate properties of Bose-Einstein condensates in a nondestructive way. For slow initial velocities the condensate is adiabatically transfered into the self-consistent ground state of the optical potential, which contains ample information on condensate properties.

In addition, the system suggests many interesting applications. Using cavities with a decay rate κ of the order of the condensate vibrational frequencies, the finite response time of the cavity implies a damping (or amplification) of the condensate oscillations, as has been shown for cooling and trapping of a single atom [13,14,16]. By changing the intensity and/or the frequency of the driving laser depending on the cavity output, one gets a handle on the controlled nondestructive *in situ* manipulation of the condensate wave function. Similarly, changing the magnitic field during the condensate passage should allow a direct measurement of the effective scattering length and the relaxation dynamics.

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