Rigorous calculation of heating in alkali-metal traps by background gas collisions

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The finite depth $\mathcal{E}$ of an atom trap results in an upper bound for the energy transfer in collisions with the background gas that will result in heating but not in loss of an atom. The energy transfer rate is accurately predicted as function of the well depth by applying a versatile semiempirical model function for the small-angle differential cross section, covering the full range from pure diffractive scattering to classical scattering. Simple scaling laws for the energy transfer rate are presented that can be readily applied. For the diffraction dominated regime we find an energy transfer rate proportional to $(\mathcal{E}/\mathcal{E}_{\mathrm{ref}})^2$ with $\mathcal{E}_{\mathrm{ref}}=(k_B T_b \ \theta_0^2/4)$, a system-dependent energy determined by the ambient temperature T_b and the diffraction angle θ_0. In the classical regime we find the usual result of an energy transfer rate proportional to $(\mathcal{E}/\mathcal{E}_{\mathrm{ref}})^{5/6}$.

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I. INTRODUCTION

In 1995, Bose-Einstein Condensation (BEC) was first observed in a cold dilute sample of trapped alkali-metal atoms, by three groups using Rb [1], Na [2], and Li [3], respectively. Recently, BEC was also achieved in a dilute gas of H atoms [4,5]. Since the first observation of BEC in a dilute gas, a whole new field of atom optics and coherent matter has emerged. At this moment, most efforts in the field are directed to Bose-Einstein condensation of the alkali atoms. Emphasis has been laid on Rb, the workhorse in this new field, with Na as the runner-up. For Cs it is not clear if BEC can be reached, due to the recent experimental evidence on large spin-spin relaxation rates for the $|F,m_F\rangle=|4,4\rangle$ and $|3,3\rangle$ states. Progress toward achieving BEC in Cs without magnetic trapping is reported by DePue *et al.* [6], using a crossed far off-resonance optical trap.

The road to BEC is well known: cold atoms are first trapped in a magneto-optical trap and then transferred to a magnetic trap where evaporative cooling is applied to achieve the ultralow temperatures in the 1 μK to 10 nK range where the transition to BEC takes place. At magnetic trap densities in the range $n \leqslant 10^{14}$ cm^{-3}, collisions with the background gas at ambient temperature are the dominant mechanism of trap loss: the energy transferred in these collisions is much larger than the trap depth. However, for collisions with a scattering angle of a few mrad, the energy transfer is of the same order of magnitude as the trap depth. These atoms do not leave the trap, but dissipate the energy gained in secondary collisions with other trapped atoms. Of course, this effect is larger for deeper traps. For atoms in a magnetic trap the resulting energy transfer rate is usually counteracted by using an rf shield which limits the trap depth to a small value, i.e., which pumps the hotter atoms to non-trapped states. This is the same as keeping the evaporative cooling switched on at a certain cutoff [7,8]. Quantitative data on heating in magnetic traps is still lacking.

For atoms trapped in a far off-resonance trap (FORT), a very powerful tool for trapping atoms without the limitations of trapping dc magnetic fields, there is no simple means for counteracting the collisional heating. Recently, there has been a strong interest in trying to achieve BEC in such an optical trap because of its superior confinement and rapid control of trapping parameters. In this case, the trap depth is an important tool for minimizing the heating by the background gas collisions. The depth of a FORT covers a wide range from 4 μK to 4 mK. In this case it is essential to have detailed knowledge on the process of collisional heating.

In this paper we calculate the energy transfer rate by background gas collisions very accurately. We treat both the small-angle collisions in the quantum mechanical diffraction-dominated range within the diffraction angle, as well as the range of slightly larger angles which is dominated by classical scattering. Both ranges are connected smoothly and accurately by using a well-tested and versatile semi-empirical model function for the small angle differential cross section. All results are given as simple analytical formulas. The effect of averaging over the thermal Maxwell Boltzmann distribution is included as an additional convolution factor. Typical results are given for the alkali-metal gases Li through Cs.

Recently, this approach has been applied in a study to investigate the prospects for achieving BEC in a trapped sample of metastable neon and metastable helium atoms [9]. In this study, the collisional energy transfer rate is due to secondary collisions of the ionic and ground state products of residual ionizing collisions in a spin-polarized gas. For ground-state atom–metastable-atom collisions the diffractive contribution is dominant; for (dimer)ion–metastable-atom collisons the range of classical scattering is most important. The scaling of the collisonal energy-transfer rate with the trap depth and trap geometry helps to define the trapping configuration that is to be preferred for pursueing BEC in these two metastable gases. However, in this study it was not necessary to investigate the full transition region between the diffractive and the classical regime.

In a recent paper by Bali *et al.* [10], the contribution of the quantum-mechanical diffraction-dominated range of scattering angles to the heating of trapped alkali atoms by background collisions is discussed. For the case of a shallow trap with a depth *much smaller* than a system dependent reference value, e.g., 0.98 mK for Rb and 0.51 mK for Cs (in our notation of Table I), their results are in agreement with our calculations. These conditions are typically met in a FORT.

TABLE I. System parameters for the alkali gases, together with collision parameters k, Q, $\sigma(0)$, and θ_0 at a velocity $v_m=(2k_BT_b/m)^{1/2}$ corresponding to the maximum of the Maxwell Boltzmann distribution of the background gas atoms at temperature $T_b=300$ K.

alkali	Li	Na	K	Rb	Cs
m (a.u.)	7	23	39	87	133
C_6 (a.u.)[a]	1393	1556	3897	4691	6851
v_m (m/s)	1034	570	438	293	237
k (Å^{-1})	5.72	10.4	13.5	20.2	24.9
Q (Å^2)	877	1163	1865	2359	2988
$\sigma(0)$ (Å^2 sr)	2.44×10^{5}	1.41×10^{6}	6.14×10^{6}	2.19×10^{7}	5.37×10^{7}
θ_0 (mrad)	20.9	10.0	6.1	3.6	2.6
$n_b\tau_1(1-\Delta Q/Q)$ (m^{-3}s)	1.10×10^{14}	1.51×10^{14}	1.22×10^{14}	1.45×10^{14}	1.41×10^{14}
$\mathcal{E}_{\text{ref}}$ (mK)	32.8	7.54	2.77	0.982	0.507
$v_mQ\mathcal{E}_{\text{ref}}$ ($\text{m}^3\text{K/s}$)	2.98×10^{-16}	5.00×10^{-17}	2.26×10^{-17}	6.80×10^{-18}	3.60×10^{-18}

[a]Ref. [19]

When the trap depth is equal to this reference value, the approach of Bali *et al.* [10] results in heating rates that are approximately 70% larger than our calculations, which take into account a fully correct description of the small-angle differential cross section. For deeper traps, as is the case for magnetic traps, it is essential to cover the whole range of scattering angles in full accuracy. Only then accurate heating rates of trapped alkali atoms by collisions with the background gas are obtained, as discussed in this work.

In Sec. II we discuss small-angle collisions, including the semiempirical model function for the small-angle differential cross section. The calculation of the effective product of cross section and energy transfer is presented in Sec. III, as a function of universal scaling parameters for all alkali atoms. In Sec. IV we discuss the energy transfer rate for the trapped alkali gases as a function of the trap depth. A comparison to experimental data for Na, Rb, and Cs in magnetic traps and optical dipole traps is given in Sec. V. In Sec. VI we argue that the glory oscillations will only have a negligible effect on the calculated energy transfer rates for binary collisions of alkali atoms. Finally, in Sec. VII we present some concluding remarks.

II. SMALL-ANGLE COLLISIONS

The finite depth $\mathcal{E}$ of the trap results in an upper bound for the energy transfer that will still result in heating but not in the loss of an atom. For a collision with scattering angle θ in the reduced system or center-of-mass system, the velocity transferred to the cold atom is equal to $\Delta v=v\,\theta\mu/m$, with μ the reduced mass, v the velocity of the impinging background gas atom with mass m_b, and m the mass of the atoms in the trap. The condition for collisional heating then is $\frac{1}{2}m(\Delta v)^2\leqslant\mathcal{E}$. This in turn limits the range of scattering angles that we have to take into account,

$$\theta\leqslant(m_b/m)^{1/2}(m/\mu)(\mathcal{E}/E)^{1/2}=\theta_{\max}, \tag{1}$$

with $E=\frac{1}{2}m_bv^2$ the kinetic energy of the background gas atom in the laboratory system. Of course, for binary collisions of an alkali-metal background gas with the same species in the trap, the mass factor reduces to $(m_b/m)^{1/2}(m/\mu)=2$. For collisions with thermal sodium atoms at $T_b=300$ K and a trap depth of $\mathcal{E}=1$ mK, the maximum scattering angle is $\theta_{\max}=3.7$ mrad. This determines the angular range where we need a correct description of the differential cross section $\sigma(\theta)$.

For an inverse power-law potential $V(R)=C_s/R^s$ (with $s=6$ for the induced dipole-dipole interaction of atom-atom scattering), simple analytical formulas for $\sigma(\theta)$ are available. The small-angle differential cross section $\sigma(\theta)$ scales with the characteristic diffraction angle [11–15],

$$\theta_0=(4\pi/k^2Q)^{1/2}, \tag{2}$$

with $k=\mu v/\hbar$ the wave number. This can be directly understood by considering the equivalent situation in physical optics. An orifice (or equivalent black disc) with area A and radius $b=(A/\pi)^{1/2}$ causes a small-angle diffraction pattern that scales as $J_1^2(kb\,\theta/2)/(kb\,\theta/2)^3$, with J_1 the spherical Bessel function and k the optical wave number. The characteristic angle for diffraction in optics is thus $\theta=2/kb$, which differs only by a factor $2^{1/2}$ with the angle defined in Eq. (2) for atomic diffraction.

For angles $\theta\ll\theta_0$ we obtain a *diffraction* peak, which can be approximated by [15,16]

$$\sigma(\theta)/\sigma(0)=1-\alpha_1(s)(\theta/\theta_0)^2, \tag{3}$$

with $\alpha_1(6)=0.9975$ a numerical constant. For angles $\theta\gg\theta_0$ we find the classical result [13–15]

$$\sigma(\theta)/\sigma(0)=\alpha_2(s)(\theta/\theta_0)^{-2(s+1)/s}, \tag{4}$$

with the constant $\alpha_2(6)=0.2846$. This contribution corresponds to *refraction*, in analogy to geometrical optics when light is scattered by an inhomogeneous medium. The absolute value of the differential cross section $\sigma(0)$ in the forward direction and the total elastic cross section Q are given by [17,18]

$$\sigma(0)=k^2Q^2/[16\pi^2\cos^2(\pi/(s-1))]$$

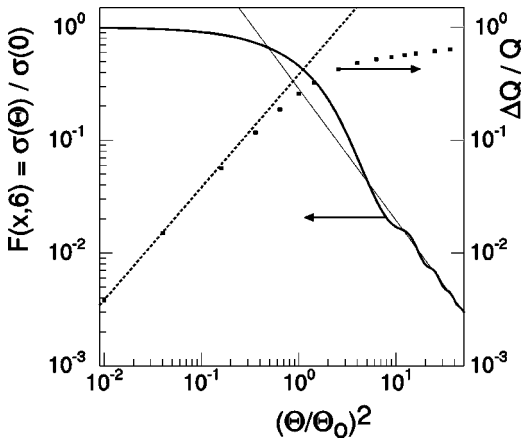

FIG. 1. Small-angle differential cross section $\sigma(\theta)/\sigma(0)$ for the case of a C_6/R^6 potential for atom-atom thermal collisions. Full curve: quantum-mechanical calculations; full line: classical mechanics. Filled squares: partial cross section ΔQ for heating collisions with a scattering angle less than $\theta=\theta_{\max}$, normalized to the elastic total cross section Q; dashed line: diffractive limit $\Delta Q = 0.382\ (\theta_{\max}/\theta_0)^2$.

$$Q=\alpha_3(s)(C_s/\hbar v)^{2/(s-1)}, \tag{5}$$

with $\alpha_3(6)=8.083$. The scattering parameters $\sigma(0)$, Q and θ_0 are not independent, but related by $\pi\theta_0^2\ \sigma(0)/Q = 4\cos^2[\pi/(s-1)]^{-1}=\alpha_4(s)$ with $\alpha_4(6)=0.382$.

Various semiempirical formulas connecting the two regions have been given by several authors [13,15]. A very accurate semiempirical representation is given by [15]

$$\mathcal{F}(x;s)=\sigma(\theta)/\sigma(0)$$

$$=\{1-c_1(s)\sin[c_2(s)x^2]+c_3(s)x^2\}^{-(s+1)/s},$$

$$\mathcal{F}(x;s)\approx\begin{cases}1-\{(s+1)/s\}[c_3(s)-c_1(s)c_2(s)]x^2, & x\ll 1\\ c_3(s)^{-(s+1)/s}x^{-2(s+1)/s}, & x\gg 1\end{cases} \tag{6}$$

with $x=\theta/\theta_0$ and the constants equal to $c_1(6)=3.75$, $c_2(6)=0.556$ and $c_3(6)=2.94$. The function $\mathcal{F}(x;6)$ is shown in Fig. 1, together with the refractive limit corresponding to classical mechanics [Eq. (4)]. In the angular range $0\leqslant x\leqslant 3.9$ the maximum relative deviation of the model function $\mathcal{F}(x,6)$ from the quantum-mechanical cross section is only 4.5%. The asymptotic behavior of this model function is in excellent agreement with the predictions of the semiclassical approximation at small angles $x\ll 1$ [Eq. (3)] and the classical or high energy approximation at large angles $x\gg 1$ [Eq. (4)]. The latter is valid for large impact parameters where the collision energy is much larger than the intermolecular potential, due to its rapid fall-off, scaling with R^{-6}. The great advantage of $\mathcal{F}(x;s)$ is the accurate description obtained in the transition region, smoothly connecting the two asymptotic regimes. The integral of the model function $\mathcal{F}(x;6)$ over the range $x=0$ to $\theta_{\max}$ determines the cross section ΔQ related to the heating of trapped atoms, as given by

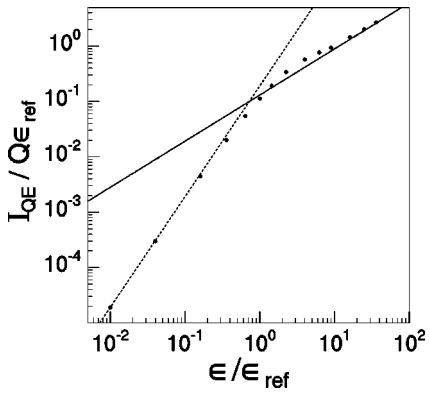

FIG. 2. The product $\mathcal{I}_{QE}$ of cross section and energy transfer as a function of the scaled trap depth $\mathcal{E}/\mathcal{E}_{\rm ref}=(\theta_{\max}/\theta_0)^2$. Data points: numerical result [Eq. (9)]; dashed line: diffractive approximation [Eq. (11)]; full line: classical or refractive approximation [Eq. (12)].

$$\Delta Q=\int_0^{\theta_{\max}}\sigma(\theta)2\pi\sin\theta\ \ \mathrm{d}\theta$$

$$\approx\frac{Q}{2\cos^2[\pi/(s-1)]}\int_0^{x_{\max}}\mathcal{F}(x;s)x\ \ \mathrm{d}x, \tag{7}$$

with $x_{\max}=(\theta_{\max}/\theta_0)$. For small-angle scattering the approximation $\sin\theta=\theta$ is fully justified. For $s=6$ we find $\Delta Q(x_{\max}\to\infty)=1.002Q$, illustrating the quality of the description based on $\mathcal{F}(x;6)$. The calculated values of $\Delta Q/Q$ are also shown in Fig. 1. We see that 26% of the total cross section is determined by small angle scattering for $x<1$, i.e., $\theta<\theta_0$.

III. EFFECTIVE PRODUCT OF CROSS SECTION AND ENERGY TRANSFER

The essential input for the calculation of the energy transfer rate of trapped atoms by collisions with the background gas is the energy-transfer integral $\mathcal{I}_{QE}$, i.e., the integral of the product of the differential cross section $\sigma(\theta)$ and the associated energy transfer $\Delta E(\theta)=\frac{1}{2}m\Delta v(\theta)^2$ in the laboratory system. This integral is given by

$$\mathcal{I}_{QE}=\int_0^{\theta_{\max}}\sigma(\theta)\Delta E(\theta)2\pi\sin\theta\ \ \mathrm{d}\theta, \tag{8}$$

where the integration ranges from $\theta=0$ to the trap-depth limited value $\theta_{\max}$. By using the approximation $\sin\theta\approx\theta$ and the semiclassical approximation of Eq. (6), the collision integral of Eq. (8) can be written as

$$\mathcal{I}_{QE}=\frac{\mathcal{E}_{\rm ref}Q}{2\cos^2[\pi/(s-1)]}\int_0^{x_{\max}}\mathcal{F}(x;s)x^3\ \ \mathrm{d}x, \tag{9}$$

with $\{2\cos^2[\pi/(s-1)]\}^{-1}=0.764$ for $s=6$. The parameter Q is the total elastic cross section as given by Eq. (5). The

parameter $\mathcal{E}_{\rm ref}$ is a system-dependent scaling value for the energy transfer, i.e., a reference value for the trap depth as given by

$$\mathcal{E}_{\rm ref} = E\,\theta_0^2(\mu/m)^2(m/m_b). \tag{10}$$

The scaling factor of the integral can also be written as $\mathcal{E}_{\rm ref}Q = 2\pi\hbar^2/m$, a surprisingly simple result that only depends on the mass of the trapped atoms. Please note that $x_{\rm max}^2 = (\theta_{\rm max}/\theta_0)^2 = \mathcal{E}/\mathcal{E}_{\rm ref}$.

The calculation of the collision integral of Eq. (8) is thus reduced to calculating a nondimensional integral with a scaled trap depth $\mathcal{E}/\mathcal{E}_{\rm ref}$ determining its range. The results of Eq. (9) are plotted in Fig. 2 as a function of the scaled trap depth $\mathcal{E}/\mathcal{E}_{\rm ref}$.

For the two limiting cases of pure diffractive scattering for $x \ll 1$ and refractive scattering for $x \gg 1$, the nondimensional integral can be evaluated analytically. We then find the asymptotic results

$$\mathcal{I}_{QE} \approx \begin{cases} 0.191\mathcal{E}_{ref}Q(\mathcal{E}/\mathcal{E}_{ref})^2 = 0.191(2\pi\hbar^2/m)(\mathcal{E}/\mathcal{E}_{ref})^2 & x \ll 1 \quad (11) \\ 0.131\mathcal{E}_{ref}Q(\mathcal{E}/\mathcal{E}_{ref})^{5/6} = 0.131(2\pi\hbar^2/m)(\mathcal{E}/\mathcal{E}_{ref})^{5/6} & x \gg 1, \quad (12) \end{cases}$$

where we have used $s=6$ to calculate the numerical factors. For comparison to the numerical results for $\mathcal{I}_{QE}$ we have plotted the two asymptotic expressions in Fig. 2. It is clear that the two approximations do not connect smoothly. The discrepancy between the numerical result and the diffractive approximation increases from -3% at $\mathcal{E}/\mathcal{E}_{\rm ref}=0.04$ to -41% at $\mathcal{E}/\mathcal{E}_{\rm ref}=1$. For the refractive contribution the discrepancy with the refractive approximation first rises from -14% at $\mathcal{E}/\mathcal{E}_{\rm ref}=1$ to $+39\%$ at $\mathcal{E}/\mathcal{E}_{\rm ref}=4$, with a decrease to $+4\%$ at $\mathcal{E}/\mathcal{E}_{\rm ref}=36$. For a correct prediction of $\mathcal{I}_{QE}$ within 10%, the range of validity of these approximations is restricted to $\mathcal{E}/\mathcal{E}_{\rm ref} \leqslant 0.16$ or $\mathcal{E}/\mathcal{E}_{\rm ref} \geqslant 4$. The approximation of Eq. (11) is equal to the expression used by Bali *et al.* [10] for calculating the energy transfer rate.

IV. ENERGY TRANSFER RATE

The energy input $\dot{U}$ per second to the total number N of trapped atoms is given by the usual expression

$$\dot{U} = Nn_b\langle v\mathcal{I}_{QE}\rangle_{T_b} = Nn_b v_m \mathcal{I}_{QE_m} f_{T_b} = \dot{U}_m f_{T_b}, \tag{13}$$

with n_b the density of the background gas. The notation $\langle\ \ \rangle_{T_b}$ indicates an average over the Maxwell-Boltzmann distribution of the relative velocity v of the atoms in the background gas with temperature T_b. The last part of Eq. (13) descibes the effect of the convolution over the Maxwell-Boltzmann distribution in terms of the ''velocity $\times$ cross section $\times$ energy transfer'' factor at velocity $v_m = (2k_BT_b/m)^{1/2}$ multiplied by a convolution factor f_{T_b} which is on the order of unity. The velocity v_m corresponds to the maximum of the Maxwell-Boltzmann distribution. A subscript m has been used to indicate that the energy transfer integral $\mathcal{I}_{QE_m}$ has been evaluated at velocity v_m.

The results for the energy transfer rate $\dot{U}_m/N$ of the alkali-metal atoms Li through Cs are given in Fig. 3 as a function of the trap depth $\mathcal{E}$ in the range 0.1 to 50 mK. The temperature of the background gas is $T_b = 300$ K; the density is $n_b = 2.42\ \ 10^{13}\ \ {\rm m}^{-3}$, corresponding to a pressure of $1\ \ 10^{-9}$ mbar. The figure gives the analytical result $\dot{U}_m/N$ of Eq. (13) without the effects of convolution over the Maxwell Boltzmann distribution. The convolution factor f_{T_b} decreases from 1.03 at $\mathcal{E}/\mathcal{E}_{\rm ref}=0.04$ to 0.96 at $\mathcal{E}/\mathcal{E}_{\rm ref}=2$, followed by an increase to 1.06 at $\mathcal{E}/\mathcal{E}_{\rm ref}=9$ and a fall-off to 1.03 at $\mathcal{E}/\mathcal{E}_{\rm ref}=36$: in all cases the correction for the thermal convolution is only a minor effect. The C_6 values [19] used as input are given in Table I, together with the scaling energy $\mathcal{E}_{\rm ref}$ and the characteristic collision parameters Q, $\sigma(0)$, and θ_0. At low values of the trap depth we observe an increase of the energy transfer rate when going from Li to Cs, due to the dominant influence of the rapidly increasing value of $\mathcal{E}/\mathcal{E}_{\rm ref}$.

For large values of $\mathcal{E}$ we see a complete inversion of this order in Fig. 3: the light alkali gases suffer the largest energy transfer rate. This can be directly understood if we consider the scaling of the energy transfer rate with the trap depth in the limits $\mathcal{E} \ll \mathcal{E}_{\rm ref}$ and $\mathcal{E} \gg \mathcal{E}_{\rm ref}$, by combining Eqs. (11) and (12) with Eq. (13). For the former case of diffractive scattering, the energy transfer rate for the different alkali gases

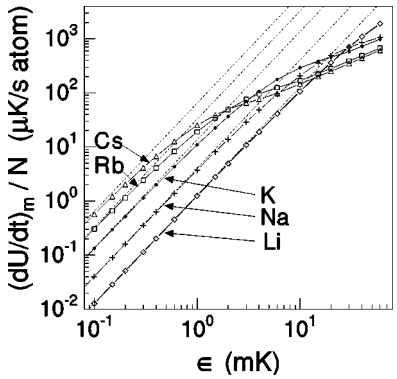

FIG. 3. Numerical results for the energy transfer rate $\dot{U}_m/N$ of trapped alkali atoms at $T_b = 300$ K and a background gas density $n_b = 2.42\times10^{13}$ m^{-3} corresponding to a pressure of 1.0×10^{-9} mbar, given as a function of the trap depth $\mathcal{E}$. The dashed lines (with a slope equal to two) represent the asymptotic behavior of the heat input in the limit of diffractive scattering for $\mathcal{E} \ll \mathcal{E}_{\rm ref}$ [Eq. (11)].

scales as $v_m Q \mathcal{E}_{\rm ref}^{-1}$; for the latter case of refractive scattering the scaling is given by $v_m Q \mathcal{E}_{\rm ref}^{1/6}$. The additional factor $\mathcal{E}_{\rm ref}^{7/6}$ in the refractive case is sufficient to reverse the order of energy transfer rates, with the largest rates for the light alkali atoms. Actually, the order of K and Na is not reversed: the energy transfer rate for K is slightly larger than for Na at even larger values of $\mathcal{E}$ than shown in Fig. 3.

Usually, an accurate value of the density of the background gas in the trapping chamber is not available directly from the experimental setup. This would severly limit the application of our model to predict absolute values of the energy transfer rate. However, we can derive the background density from a simple decay measurement of the trap population. We use the characteristic time constant τ_1 for the density-*independent* decay of the trap population. The loss of atoms from the trap by these collisions is given by

$$\dot{N} = -N n_b \langle v(Q-\Delta Q)\rangle_{T_b} = -N/\tau_1, \tag{14}$$

with $Q-\Delta Q$ the partial cross section for scattering over an angle *larger* than $\theta_{\rm max}$, i.e., the cross section corresponding to collisions leading to trap loss. The value of ΔQ can be calculated using the approximation of Eq. (6) and the integral expression of Eq. (7). In Fig. 1 we show the numerical results for $\Delta Q/Q$ as a function of the scaled energy depth $(\theta_{\rm max}/\theta_0)^2 = (\mathcal{E}/\mathcal{E}_{\rm ref})$. These results can be used to relate $\tau_1^{-1} \approx n_b v_m Q(1-\Delta Q/Q)$ to the density of the background gas.

Finally, for comparison with experimental data on trap temperatures, we have to convert the energy transfer rate $\dot{U}$ to an increase $\dot{T}$ of the temparature of the trapped atoms. For an internal energy $U = N\eta k_B T$ this results in

$$\dot{T} = \dot{U}/(N\eta k_B), \tag{15}$$

with $\eta = 3/2$ for a square-well potential and $\eta = 3$ for a harmonic trapping potential.

V. COMPARISON TO EXPERIMENTS

In a recent experiment on Raman cooling in a far-off resonance optical trap (FORT) for Cs atoms a residual energy transfer rate of $\approx 4\,\mu$K/s has been observed [20]. The well depth is $\mathcal{E} = 0.16$ mK, leading to a value of $\mathcal{E}/\mathcal{E}_{\rm ref} = 0.32$ and $\theta_{\rm max}/\theta_0 = 0.56$. The cross section for collisional loss is determined by $\Delta Q/Q = 0.105$; the lifetime $\tau_1 = 2$ s then leads to a background density $n_b = 7.9\times 10^{13}$ m^{-3} in the vapor cell. The energy transfer rate is then equal to $\dot{U}_m/(Nk_B) = 4.5$ μK/s. This result is in excellent agreement with the experimental data.

We can also apply our model to predict the energy transfer rate of Rb atoms in a circularly polarized FORT, using the experimental conditions in the recent paper of Corwin *et al.* [21]. Typical conditions are a trap depth of $\mathcal{E} = 1.6$ mK and a loss time due to background collisions equal to $\tau_1 = 10$ s. Using Table I we find $\mathcal{E}/\mathcal{E}_{\rm ref} = 1.63$, i.e., $\theta_{\rm max}/\theta_0 = 1.28$. The cross section for collisonal loss by collisions with the background gas is determined by $\Delta Q/Q = 0.35$; the lifetime $\tau_1 \approx 10$ s leads to $n_b = 2.2\times 10^{13}$ m^{-3}. The predicted energy transfer rate is $\dot{U}_m/(Nk_B) = 35$ μK/s; the increase in temperature in the approximation of a harmonic potential then is $\dot{T} = 11.5$ μK/s. These are the boundary conditions for all cooling efforts in this trap.

For the BEC experiment in Na at MIT, the lifetime in the magnetic trap is very short unless the trap depth is lowered by rf shielding, allowing collisionally heated atoms to escape [7,8]. If we assume a depth of the magnetic trap equal to $\mathcal{E} \approx 10$ mK, i.e., a depth of 150 G for Na with a magnetic moment of $1\,\mu_B$, we can calculate the energy transfer rate to support their observation. Using a number density $n_b = 5\times 10^{12}$ m^{-3} we find $\dot{U}_m/(Nk_B) = 43$ μK/s. An rf shield at 20 μK directly results in a decrease of the energy transfer rate to $\dot{U}_m/(Nk_B) = 3.4\times 10^{-4}$ μK/s as follows from the scaling with $\mathcal{E}^2$.

Finally, we can apply our model to a condensate of Na atoms, loaded in a FORT after having achieved Bose-Einstein condensation by the usual method of evaporative cooling in a magnetic trap. For this comparison we use the data of the group of Ketterle at MIT [22]. For the ''strong'' optical trap with a depth $\mathcal{E} = 2$ μK they find a loss time equal to $\tau_1 \approx 30$ s. Using Table I we find $\mathcal{E}/\mathcal{E}_{\rm ref} = 2.7\times 10^{-4}$ and $\theta_{\rm max}/\theta_0 = 1.6\times 10^{-2}$. The cross section for collisional loss can be calculated in the small angle limit, i.e., $\Delta Q/Q \approx 0.382\; x_{\rm max}^2 = 1.03\times 10^{-4}$; the lifetime then leads to a density $n_b = 5\times 10^{12}$ m^{-3}. The predicted energy transfer rate then is $\dot{U}_m/(Nk_B) = 3.4\times 10^{-6}$ μK/s, fully determined by the diffractive contribution. This low energy transfer rate is completely due to the shallowness of the FORT.

VI. GLORY OSCILLATIONS

The semiclassical model function used to calculate the energy transfer rate of alkali atoms in a trap fully describes the contribution of the long range interaction to the total cross section and the differential cross section for elastic scattering. Not included is the contribution of the glory oscillations, i.e., the extra contribution due to the interference of the classical trajectory that also has a scattering angle $\theta = 0$ by the compensating effects of the attractive and repulsive interatomic forces. These glory oscillations as a function of the velocity can be fully described in semiclassical terms [14], using the depth $\epsilon_{\rm min}$ and position $R_{\rm min}$ of the potential well as input. The amplitude δQ_{gl} of the glory oscillations in the total cross section scales as $\delta Q_{gl} \approx 7 R_{\rm min}^{3/2} k^{-1/2}$ and does not depend on the magnitude of the long-range interaction C_6. For the alkali-metal–alkali-metal systems, with their large C_6 value, the relative amplitude $\delta Q_{gl}/Q$ is rather small. For example, for the Na+Na system at thermal velocity v_m, we find $\delta Q_{gl}/Q = 0.013$ and 0.021 for the singlet and triplet potential, respectively; the relative amplitude of the glory oscillations in $\sigma(0)$ is larger by a factor $2\cos[\pi/(s-1)] = 1.6$.

The phase of the glory oscillations scales as $\Phi_{gl} \approx 0.9(2\epsilon_{\rm min} R_{\rm min}/\hbar v)$, resulting in a full glory period $\delta v_{gl} \approx 7(\hbar v^2/2\epsilon_{\rm min} R_{\rm min})$. For example, for the Na+Na system we

find $\delta v_{gl}=3$ m/s and 76 m/s for the singlet and the triplet potential, respectively, as compared to $v_m=570$ m/s. Because the collisions with the background gas have a random orientation of the spin vector of the background gas atoms, the glory oscillations of both the singlet and the triplet potentials will contribute. In general, these glory oscillations will not be in phase, adding to an effective damping.

Taking the average over the Maxwell Boltzmann distribution will effectively wash out the net contribution of the glory oscillations in Q and $\sigma(0)$ if we consider the small period of the oscillations as a function of the velocity. The decreasing relative amplitude and oscillation period of the glory oscillations with increasing well depth, well position and C_6, as is the case when going from Li to Cs, will emphasize this effect for the heavier alkali atoms. We conclude that the net effect of the glory oscillations on the calculated energy transfer rates for the alkali background collisions is indeed very small. The approximation of only treating the contribution of the long-range attractive forces is fully justified.

VII. CONCLUDING REMARKS

The results of this paper describe the boundary conditions for cooling experiments of the alkali-metal gases in a FORT and a magnetic trap in a very tractable fashion. The analytical results for the two limiting cases show where the experiment can be optimized for obtaining low temperatures. The numerical results for the energy transfer rate for a well depth in the transition region $0.1<\mathcal{E}/\mathcal{E}_{\rm ref}<20$ can be directly applied using Figs. 1 and 2 or by calculating the nondimensional integrals via MAPLE or MATHEMATICA.

The scaling of the energy transfer rate with $(\mathcal{E}/\mathcal{E}_{\rm ref})^2$ for values of the well depth that are small compared to the reference value $\mathcal{E}_{\rm ref}$, i.e., in the diffraction-dominated quantum regime of small-angle collisions, is useful for minimizing the energy transfer rate by relaxing the well depth during the cooling phase. For a FORT, where different methods are used for cooling down the trap temperature, it is good to keep this scaling rule in mind. For a magnetic trap, the usual process of evaporative cooling takes care of this.

A final question that has to be answered: is the heating rate by diffractive scattering the ultimate limiting factor in reaching high phase-space densities in traps? In our opinion, this is not the case. With increasing density (or column density) of the trap, a *new* process of heating will start to become increasingly effective. Primary collisions of the background gas with trapped alkali-metal atoms, resulting in an energy transfer that is *larger* than the trap depth, will also start to contribute to the heating process. The scattered low-energy alkali-metal atom, with an energy in the range of 1 mK to 10 K or more, has a finite chance to collide with a trapped alkali-metal atom *before* leaving the trap. This transfer of energy by low-energy secondary collisions is very efficient. The heating rate due to these secondary collisions is proportional to the column density of the trapped atoms, in agreement with the experimental observations in many magnetic traps [23]. Work is in progress to treat these problems within the same framework as presented in this paper.

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