

# Scattering of light and atoms in a Fermi-Dirac gas with Bardeen-Cooper-Schrieffer pairing

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We theoretically study the optical properties of a Fermi-Dirac gas in the presence of a superfluid state. We calculate the leading quantum-statistical corrections to the standard column density result of the electric susceptibility. We also consider the Bragg diffraction of atoms by means of light-stimulated transitions of photons between two intersecting laser beams. Bardeen-Cooper-Schrieffer pairing between atoms in different internal levels magnifies incoherent scattering processes. The absorption linewidth of a Fermi-Dirac gas is broadened and shifted. Bardeen-Cooper-Schrieffer pairing introduces a collisional local-field shift that may dramatically dominate the Lorentz-Lorenz shift. For the case of the Bragg spectroscopy the static structure function may be significantly increased due to superfluidity in the near-forward scattering.

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## I. INTRODUCTION

As a result of dramatic progress in cooling and trapping of alkali-metal atomic gases the quantum statistics of atoms has observable consequences. Perhaps, the most striking effect is the Bose-Einstein (BE) condensation of bosonic atoms [1–3]. Nevertheless, Fermi-Dirac (FD) gases are also expected to exhibit a rich and complex behavior. One especially fascinating property of FD gases is that with effectively attractive interaction between different particles the ground state of the system may become unstable with respect to the formation of bound pairs of quasiparticles or Cooper pairs [4,5]. This effect is analogous to the Bardeen-Cooper-Schrieffer (BCS) transition in superconductors. FD gases have been an active subject of research already quite some time [6–22].

We recently showed [17] that the coherent quasiparticle pairing between atoms in different internal levels may enhance the optical interactions. In particular, the resonance line of a FD gas (the extinction of light from coherent laser beam) is broadened and shifted as a result of the BCS pairing. In this paper we present a more detailed study of the propagation of light in a dilute FD gas in the presence of a superfluid state. We also investigate the signatures of the BCS pairing in light-stimulated transitions of photons between two intersecting nonparallel laser beams. We demonstrate the dramatic dependence of the Bragg diffraction rate on the BCS order parameter. The Bragg diffraction of atoms has already been experimentally used as a method to split a BE condensate [23] and to perform spectroscopic measurements on the condensates [24,25].

In this paper we theoretically study the optical response of a superfluid state in a zero-temperature FD gas. First, we consider the propagation of low-intensity light and calculate the leading low-density correction to the standard century-old column density result of the electric susceptibility, also known as the Clausius-Mossotti relation [26]. This correction is a direct consequence of the quantum-statistical position correlations between different atoms that modify the optical interactions at small interatomic separations.

FD statistics exhibit a short-range ordering of atoms in the gas. In the absence of a superfluid state the FD statistics

force a regular spacing between the atoms in the same internal state within the characteristic correlation length  $\xi_{\uparrow\uparrow} \sim 1/k_F$ . Here  $k_F$  denotes the Fermi wave number. Fermionic atoms repel each other and short interatomic separations are suppressed by the Pauli exclusion principle. The pair correlation function between two atoms displays antibunching.

The resonant dipole-dipole interactions between different atoms, which behave as  $1/r^3$ , are dominant at small interatomic separations. The Fermi repulsion suppresses these optical interactions and therefore the incoherent scattering of light in the atomic sample is reduced. As a result a FD gas exhibits a dramatic narrowing of the absorption line for the coherently scattered light [15,16].

In the  $s$ -wave BCS transition the particles near the Fermi surface having opposite momenta and different internal quantum numbers tend to appear in pairs. This leads, e.g., to a finite energy gap  $\Delta$  in the excitation spectrum of the system and to a nonvanishing expectation value of the anomalous correlation function for the matter-field annihilation operators  $\langle \psi_{\uparrow}(\mathbf{r})\psi_{\downarrow}(\mathbf{r}) \rangle$ . Here the two internal states are referred to as  $\uparrow$  and  $\downarrow$ .

The BCS  $s$ -wave pairing also introduces short-range ordering of atoms within the characteristic correlation length  $\xi_{\uparrow\downarrow} \sim \epsilon_F/(\Delta k_F)$  between atoms in different internal states [4]. Here  $\epsilon_F$  denotes the Fermi energy. Due to the macroscopic two-particle coherence the atoms in different internal levels attract each other and the BCS pairing enhances small interatomic separations. The pair correlation function between two atoms with different quantum numbers displays bunching. As a result the optical interactions and the incoherent scattering of light in the atomic gas are *enhanced*. This broadens the absorption line for the coherently scattered light.

As a second topic we study the Bragg spectroscopy of a FD gas with BCS pairing. In the Bragg spectroscopy an optical potential couples to the local number density of atoms and connects two external atomic states of different momenta. The scattering rate of atoms is proportional to the dynamical structure function of the atomic gas [6,24,25]. The structure function has been extensively studied for the case of incoherently scattered light in optically thin BE [6,27–29] and FD [6,10,14] atomic gases.

The structure function contains distinct quantum-statistical features in the case of fermionic and bosonic atomic gases [6]. For a FD gas it demonstrates the Fermi *inhibition* of the spontaneous scattering of photons [6,10,14]. The structure function may also be used as a method of determining the relative phase between two BE condensates [28,30].

In this paper we show that a superfluid state may significantly increase the value of the static structure function. This is because due to the BCS pairing atoms and holes near the Fermi surface are mixed; with a given recoil momentum there exist more unoccupied states to which atoms can scatter. The effect of superfluidity is stronger for nearforward scattering.

In Sec. II we introduce the Hamiltonian density for ground-state atoms in the absence of the driving light field. The analysis of the quasiparticles follows the standard BCS theory [4]. We emphasize the effect of the quantum statistics on the pair correlation function. In Sec. III we study the propagation of light in a FD atomic gas. The interaction between light and matter is discussed in general terms in Sec. III A. The atomic polarization is solved for the low-density atomic gas, complete with the dependence on the atomic level scheme and on various light polarizations in Sec. III B. The electric susceptibility representing the damping and the phase velocity of the light beam is obtained in Sec. III C. The effects of the quantum statistics and the *s*-wave interactions manifest themselves in two distinguishable parameters. A few remarks about the light propagation are made in Sec. III D. In Sec. IV we consider the possibilities to observe the BCS pairing via Bragg spectroscopy. The Bragg diffraction of atoms in the laser field probes the structure function of the gas. In Sec. IV B we calculate the structure function for a superfluid state. A few concluding remarks are made in Sec. V.

## II. GROUND-STATE ATOMS

### A. Hamiltonian

We assume a FD gas occupying two different internal sublevels  $|g, \uparrow\rangle$  and  $|g, \downarrow\rangle$  of the same atom with electronically excited levels  $|e, \nu\rangle$ . In the absence of the driving light field, atoms in the electronic ground state are described in second quantization by the Hamiltonian density  $\mathcal{H}_g$  [4,5]:

$$\mathcal{H}_g = \sum_{\nu} \psi_{g\nu}^{\dagger} (H_{\text{c.m.}}^{g\nu} - \mu_{g\nu}) \psi_{g\nu} + \hbar u_g \psi_{g\uparrow}^{\dagger} \psi_{g\downarrow}^{\dagger} \psi_{g\downarrow} \psi_{g\uparrow}, \quad (1)$$

where  $\psi_{g\nu}(\mathbf{r}t)$  denotes the atom-field annihilation operator for level  $|g, \nu\rangle$  in the Heisenberg picture,  $\mu_{g\nu}$  is the corresponding chemical potential, and  $H_{\text{c.m.}}^{g\nu}$  stands for the center-of-mass (c.m.) Hamiltonian. We have approximated the finite-range interparticle potential by a contact interaction with the strength given by  $u_g = 4\pi a_g \hbar/m$ . Here  $a_g$  and  $m$  denote the *s*-wave scattering length and the mass of the atom. The atoms in different internal states can interact via *s*-wave scattering. On the other hand, due to the Pauli exclusion

principle there only is a very weak *p*-wave scattering between two atoms in the same level, which is ignored in Eq. (1).

### B. BCS pairing

Before the light is switched on, the system is described by the Hamiltonian density  $\mathcal{H} = \mathcal{H}_g$  [Eq. (1)]. The assumption that the driving light only weakly disturbs the system allows us to evaluate the electric susceptibility by using the ground-state atom correlations determined by  $\mathcal{H}_g$ , even in the presence of the driving light. We assume a homogeneous atomic sample and introduce a plane-wave basis for the field operators:

$$\psi_{g\nu}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} b_{\mathbf{k}\nu} e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (2)$$

In the Hamiltonian (1) we introduce the standard canonical transformation to the Bogoliubov quasiparticles [4,5]

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}} b_{\mathbf{k}\downarrow} - v_{\mathbf{k}} b_{-\mathbf{k}\uparrow}^{\dagger}, \quad (3a)$$

$$\beta_{-\mathbf{k}} = u_{\mathbf{k}} b_{-\mathbf{k}\uparrow} + v_{\mathbf{k}} b_{\mathbf{k}\downarrow}^{\dagger}, \quad (3b)$$

where  $u_{\mathbf{k}}$  and  $v_{\mathbf{k}}$  are real, depend only on  $|\mathbf{k}|$ , and satisfy  $u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 = 1$ . The requirement that linearized mean field fluctuations of  $\mathcal{H}_g$  in Eq. (1) be diagonal in the quasiparticle representation sets an additional constraint and we obtain

$$u_{\mathbf{k}}^2 = \frac{1}{2} \left( 1 + \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right), \quad v_{\mathbf{k}}^2 = \frac{1}{2} \left( 1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right), \quad (4)$$

where  $E_{\mathbf{k}} = \sqrt{\Delta^2 + \xi_{\mathbf{k}}^2}$ ,  $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \bar{\mu} + \hbar u_g (\rho_{\uparrow} + \rho_{\downarrow})/2$ , and the energy gap

$$\Delta = -\frac{\hbar u_g}{V} \sum_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} (1 - \bar{n}_{\alpha\mathbf{k}}^q - \bar{n}_{\beta\mathbf{k}}^q). \quad (5)$$

In equilibrium, the quasiparticle occupation numbers  $\bar{n}_{\alpha\mathbf{k}}^q \equiv \langle \alpha_{\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}} \rangle$  and  $\bar{n}_{\beta\mathbf{k}}^q \equiv \langle \beta_{\mathbf{k}}^{\dagger} \beta_{\mathbf{k}} \rangle$  satisfy FD statistics with  $\bar{n}_{\alpha\mathbf{k}}^q = \bar{n}_{\beta\mathbf{k}}^q = (e^{E_{\mathbf{k}}/k_B T} + 1)^{-1}$ . The dispersion relation for free particles is given by  $\epsilon_{\mathbf{k}} = \hbar^2 k^2 / (2m)$  and the average of the chemical potentials is  $\bar{\mu} = (\mu_{\uparrow} + \mu_{\downarrow})/2$ . For simplicity, we assume  $\mu_{\uparrow} = \mu_{\downarrow}$ . For the gap parameter at  $T=0$  we use the weak-coupling approximation  $\Delta \approx 1.76 k_B T_c$  [4,9], where

$$k_B T_c \approx \frac{8\epsilon_F}{\pi} e^{\gamma-2} \exp \left[ -\frac{\pi}{2k_F |a_g|} \right]. \quad (6)$$

Here the Fermi wave number  $k_F = (6\pi^2 \rho)^{1/3}$  is defined in terms of the atom density  $\rho$ ,  $T_c$  denotes the critical temperature of the BCS phase transition, and  $\gamma \approx 0.5772$ .

In the superfluid phase transition the atoms in the different internal states  $\uparrow$  and  $\downarrow$  form quasiparticle pairs resulting in a nonvanishing anomalous correlation  $\langle \psi_{\uparrow}(\mathbf{r}_1) \psi_{\downarrow}(\mathbf{r}_2) \rangle$ . The effect of this macroscopic two-particle coherence on the atomic position correlations is particularly transparent in the case of the pair correlation function:

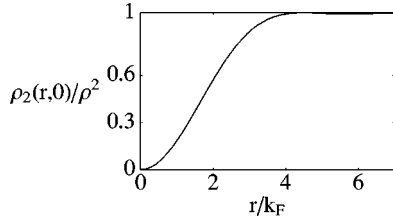


FIG. 1. The pair correlation function for an ideal homogeneous Fermi-Dirac gas at zero temperature. The two atoms have the same internal quantum numbers. The pair correlation function represents the joint probability distribution for the position of the second atom given that the first atom sits at the origin. Two fermionic atoms repel each other and small interatomic separations are suppressed.

$$\rho_2(\mathbf{r}_1\nu\eta, \mathbf{r}_2\sigma\tau) \equiv \langle \psi_{g\nu}^\dagger(\mathbf{r}_1) \psi_{g\sigma}^\dagger(\mathbf{r}_2) \psi_{g\tau}(\mathbf{r}_2) \psi_{g\eta}(\mathbf{r}_1) \rangle. \quad (7)$$

In the ground state of  $\mathcal{H}_g$  [Eq. (1)], determined by the vacuum of the Bogoliubov quasiparticles [Eq. (3)], the pair correlation function reads (for  $\nu \neq \sigma$ )

$$\rho_2(\mathbf{r}_1\nu\nu, \mathbf{r}_2\sigma\sigma) = \rho_\nu \rho_\sigma + |\langle \psi_{g\nu}(\mathbf{r}_1) \psi_{g\sigma}(\mathbf{r}_2) \rangle|^2, \quad (8a)$$

$$\rho_2(\mathbf{r}_1\nu\nu, \mathbf{r}_2\nu\nu) = \rho_\nu^2 - |\langle \psi_{g\nu}^\dagger(\mathbf{r}_1) \psi_{g\nu}(\mathbf{r}_2) \rangle|^2. \quad (8b)$$

The pair correlation function represents the joint probability distribution for the positions of two atoms. We note from Eq. (8b) that the fermionic atoms in the same internal state repel each other analogously to antibunching of photons and short interatomic separations are inhibited. For an ideal homogeneous FD gas, in the absence of a superfluid state, we can analytically evaluate Eq. (8b). We obtain [16]

$$\rho_2(r; \nu\nu, \nu\nu) = \rho^2 \left\{ 1 - \frac{9}{k_F^4 r^4} \left[ \frac{\sin k_F r}{k_F r} - \cos k_F r \right]^2 \right\}, \quad (9)$$

where  $r \equiv |\mathbf{r}_1 - \mathbf{r}_2|$ . In Fig. 1 we have shown the pair correlation function (9). The FD repulsion between different atoms results in suppressed dipole-dipole interactions and in the Fermi *inhibition* of incoherently scattered light [15,16].

As a result of the BCS pairing atoms in different internal states attract each other analogously to bunching of photons according to Eq. (8a). Therefore short interatomic separations are enhanced. In the next section we find that this may lead to *enhanced* optical interactions and incoherent scattering of light.

### III. LIGHT PROPAGATION

#### A. Basic relations

In this section we introduce terms in the Hamiltonian that result from the electromagnetic fields. We briefly recapitulate and generalize our previous quantum field-theoretical analysis of the light-matter interactions [31,32,15–17]. As a basic assumption, atoms are represented as point dipoles and the radiated field in a medium has the familiar expression of the dipolar field [Eq. (15b)].

#### 1. Electromagnetic field

We consider the nonrelativistic Hamiltonian formalism of electrodynamics. It is advantageous to study the propagation of light by introducing the dipole approximation for atoms and the corresponding Hamiltonian in the *length* gauge obtained in the Power-Zienau-Woolley transformation [33–35].

The electromagnetic field introduces additional terms in the system Hamiltonian. The Hamiltonian for the free electromagnetic field-energy is

$$H_F = \int d^3r \mathcal{H}_F = \sum_q \hbar \omega_q a_q^\dagger a_q. \quad (10)$$

Here  $\omega_q$  and  $a_q$  denote the mode frequency and the photon annihilation operator. The mode index  $q$  incorporates both the wave vector  $\mathbf{q}$  and the transverse polarization  $\hat{\mathbf{e}}_q$ . In the length gauge the basic dynamical degree of freedom for the light field is the electric displacement  $\mathbf{D}(\mathbf{r})$  which interacts with the atomic polarization  $\mathbf{P}(\mathbf{r})$ ,

$$\mathcal{H}_D = - \frac{1}{\epsilon_0} \mathbf{P}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r}). \quad (11)$$

In the dipole approximation the polarization is given in terms of the density of atomic dipoles  $\mathbf{P}(\mathbf{r}) = \sum_i \mathbf{d}_i \delta(\mathbf{r}_i - \mathbf{r})$ . Here  $\mathbf{d}_i$  and  $\mathbf{r}_i$  denote the dipole operator and the c.m. position operator for the  $i$ th atom. In second quantization the positive frequency component of the polarization is given by

$$\mathbf{P}^+(\mathbf{r}) = \sum_{\nu, \eta} \mathbf{d}_{g\nu e \eta} \psi_{g\nu}^\dagger(\mathbf{r}) \psi_{e\eta}(\mathbf{r}) \equiv \sum_{\nu, \eta} \mathbf{P}_{\nu\eta}^+(\mathbf{r}), \quad (12)$$

$$\mathbf{P}_{\nu\eta}^+(\mathbf{r}) \equiv \mathbf{d}_{g\nu e \eta} \psi_{g\nu}^\dagger(\mathbf{r}) \psi_{e\eta}(\mathbf{r}), \quad (13)$$

where  $\mathbf{d}_{g\nu e \eta}$  stands for the dipole matrix element for the transition  $|e, \eta\rangle \rightarrow |g, \nu\rangle$

$$\mathbf{d}_{g\nu e \eta} \equiv \mathcal{D} \sum_{\sigma} \hat{\mathbf{e}}_{\sigma} \langle e \eta; 1g | 1\sigma; g\nu \rangle. \quad (14)$$

Here  $\mathcal{D}$  denotes the reduced dipole matrix element, which is chosen to be real, and  $\mathbf{d}_{e\eta g\nu} = \mathbf{d}_{g\nu e \eta}^*$ . The summation in Eq. (14) runs over the unit circular polarization vectors  $\sigma = \pm 1, 0$  weighted by the Clebsch-Gordan coefficients of the corresponding optical transitions. Here light fields with the polarizations  $\hat{\mathbf{e}}_{\pm}$  and  $\hat{\mathbf{e}}_0$  drive the transitions  $|g, \nu\rangle \rightarrow |e, \nu \pm 1\rangle$  and  $|g, \nu\rangle \rightarrow |e, \nu\rangle$ , respectively.

For a weak external magnetic field the nuclear spin  $I$ , the electron spin  $S$ , and the orbital angular momentum  $L$  may be coupled. In that case  $F$  and  $m_F$ , defined by  $\mathbf{F} = \mathbf{I} + \mathbf{S} + \mathbf{L}$ , are good quantum numbers. With stronger magnetic fields the nuclear spin decouples and the optical transitions are between different sublevels of  $J$ , where  $\mathbf{J} = \mathbf{S} + \mathbf{L}$ . In the latter case we may assume that the atoms occupy a single manifold of  $m_I$ . In the following calculation we consider atomic sublevels determined by  $m_F$ .

The positive frequency component of the electric field  $\mathbf{E}^+$  may be expressed in terms of the positive frequency compo-

ment of the driving electric displacement  $\mathbf{D}_F^+$ , with the wave number  $k$ , and the dipole radiation field [31]:

$$\epsilon_0 \mathbf{E}^+(\mathbf{r}) = \mathbf{D}_F^+(\mathbf{r}) + \int d^3 r' \mathbf{G}(\mathbf{r} - \mathbf{r}') \mathbf{P}^+(\mathbf{r}'), \quad (15a)$$

$$\mathbf{G}_{ij}(\mathbf{r}) = \left[ \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} - \delta_{ij} \nabla^2 \right] \frac{e^{ikr}}{4\pi r} - \delta_{ij} \delta(\mathbf{r}). \quad (15b)$$

We also have the familiar relation

$$\mathbf{D}(\mathbf{r}) = \epsilon_0 \mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r}). \quad (16)$$

Equation (15) is the integral representation for the Maxwell's wave equation of the electric field in the presence of dipole atoms and the monochromatic dipole radiation kernel  $\mathbf{G}(\mathbf{r})$  coincides with the corresponding classical expression [26]. The explicit form of the radiated field from a dipole with the amplitude  $\hat{\mathbf{p}}$  reads

$$\begin{aligned} \mathbf{G}(\mathbf{r}) \hat{\mathbf{p}} = & \frac{k^3}{4\pi} \left\{ (\hat{\mathbf{n}} \times \hat{\mathbf{p}}) \times \hat{\mathbf{n}} \frac{e^{ikr}}{kr} + [3\hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \hat{\mathbf{p}}) - \hat{\mathbf{p}}] \right. \\ & \left. \times \left[ \frac{1}{(kr)^3} - \frac{i}{(kr)^2} \right] e^{ikr} \right\} - \frac{\hat{\mathbf{p}} \delta(\mathbf{r})}{3}, \end{aligned} \quad (17)$$

where  $\hat{\mathbf{n}} = \mathbf{r}/r$ . The volume integral over  $1/r^3$  in Eq. (17) is not absolutely convergent in the neighborhood of the origin. The expression (17) should be understood in such a way that the integral of the term inside the curly brackets over an infinitesimal volume enclosing the origin vanishes [31]. We note that Eq. (15a), with the correct delta function contribution from Eq. (17), yields  $\nabla \cdot \mathbf{D}(\mathbf{r}) = 0$  for neutral atoms justifying the use of the electric displacement, instead of the electric field, in Eq. (11).

The nonrelativistic propagator (17) involves an explicit high-frequency cutoff [31]. In situations where integral expressions containing the propagator  $\mathbf{G}(\mathbf{r})$  are not absolutely convergent the integrals are defined in such a way that the integration over spherical angles should be performed first [31].

In the length gauge the Hamiltonian also contains the polarization self-energy term:

$$\mathcal{H}_p = \frac{1}{2\epsilon_0} \mathbf{P}(\mathbf{r}) \cdot \mathbf{P}(\mathbf{r}). \quad (18)$$

However, this is proportional to the overlap of the atomic dipoles and, in the limit of low light-intensity, all contact interaction terms between different dipole atoms were shown in Ref. [32] to be inconsequential for light-matter dynamics. Although the result is valid for an arbitrary number of atoms, the cancelation of the contact interactions is especially transparent in the case of a superradiant decay of only two point dipoles [32].

## 2. *S-wave interactions*

Two cold fermionic atoms in different internal states interact by means of *s*-wave scattering. This introduces inter-

action terms between ground-state atoms in Eq. (1). In the presence of driving light we also have *s*-wave scattering between different electronically excited levels and between electronically excited and ground-state atoms.

We write the contribution to the Hamiltonian density that consists of the excited-level operators as

$$\begin{aligned} \mathcal{H}_e = & \sum_{\nu} \psi_{e\nu}^{\dagger} (H_{\text{c.m.}}^{e\nu} + \hbar \omega_0 - \mu_{e\nu}) \psi_{e\nu} \\ & + \frac{\hbar}{2} \sum_{\nu\eta\sigma\tau} u_{e\nu\eta,e\sigma\tau} \psi_{e\nu}^{\dagger} \psi_{e\nu}^{\dagger} \psi_{e\eta}^{\dagger} \psi_{e\sigma} \psi_{e\tau}. \end{aligned} \quad (19)$$

Here  $u_{e\nu\eta,e\sigma\tau} = 4\pi\hbar a_{e\nu\eta,e\sigma\tau}/m$  describes the two-body *s*-wave scattering between the atoms. For simplicity, the frequency of the optical transition  $\omega_0$  is assumed to be independent of the atomic sublevel. For typical values of the optical linewidth the c.m. motion for the excited atoms may be omitted [6]. In this paper we consider a situation where the intensity of the driving light field is low and therefore the density of the excited atoms is low. Hence, to leading order in the limit of low light-intensity the second term in Eq. (19) makes no contribution to the optical response, and we do not address its explicit form in more detail.

We assume that to leading order all remaining interactions between the ground-state and excited-state atoms, which cannot be accounted for when the atoms are modeled as point dipoles, are governed by the following interactions [35]:

$$\mathcal{H}_{ge} = \hbar \sum_{\nu\eta\sigma\tau} u_{g\nu\eta,e\sigma\tau} \psi_{g\nu}^{\dagger} \psi_{g\nu}^{\dagger} \psi_{e\eta}^{\dagger} \psi_{e\sigma} \psi_{g\tau}. \quad (20)$$

If we assume the conservation of the angular momentum of the colliding particles, the two-body interaction can be written in the following form [36]:

$$\mathcal{H}_{ge} = \hbar \sum_{F=|f_a-f_b|}^{f_a+f_b} \sum_{m_F=-F}^F u_{F,m_F} \mathcal{O}_{Fm_F}^{\dagger} \mathcal{O}_{Fm_F}, \quad (21)$$

where  $u_{F,m_F} = 4\pi\hbar a_{F,m_F}/m$  and

$$\mathcal{O}_{Fm_F} \equiv \sum_{m_a m_b} \langle Fm_F; f_a f_b | f_a, m_a; f_b, m_b \rangle \psi_{f_a m_a} \psi_{f_b m_b}. \quad (22)$$

Here  $\langle Fm_F; f_a f_b | f_a, m_a; f_b, m_b \rangle$  is the Clebsch-Gordan coefficient for forming the state  $|Fm_F\rangle$  from two colliding particles in the angular momentum states  $|f_a m_a\rangle$  and  $|f_b m_b\rangle$ .

As an example we consider the  $f=1/2 \rightarrow 3/2$  transition. By expanding Eq. (21) we obtain  $\mathcal{H}_{ge} = \mathcal{H}_{ge}^A + \mathcal{H}_{ge}^B$ , where  $\mathcal{H}_{ge}^A$  does not mix the population between the different sublevels. The explicit form reads

$$\mathcal{H}_{ge}^A = \hbar \sum_{\nu,\sigma} u_{g\nu,e\sigma} \psi_{e\sigma}^{\dagger} \psi_{g\nu}^{\dagger} \psi_{g\nu} \psi_{e\sigma}. \quad (23)$$

For the  $f=1/2 \rightarrow 3/2$  transition the summation runs over  $\nu = \pm 1/2$  and  $\sigma = \pm 1/2, \pm 3/2$ . The interaction strengths  $u_{g\nu,e\sigma}$

are determined in terms of the interaction strengths  $u_{F,m_F}$  for the scattering channels  $|Fm_F\rangle$ :

$$\begin{aligned} u_{g\pm\frac{1}{2},e\pm\frac{3}{2}} &= u_{2,\pm 2}, \\ u_{g\pm\frac{1}{2},e\mp\frac{1}{2}} &= \frac{1}{2}(u_{1,0} + u_{2,0}), \\ u_{g\pm\frac{1}{2},e\mp\frac{3}{2}} &= \frac{3}{4}u_{1,\mp 1} + \frac{1}{4}u_{2,\mp 1}, \\ u_{g\pm\frac{1}{2},e\pm\frac{1}{2}} &= \frac{1}{4}u_{1,\pm 1} + \frac{3}{4}u_{2,\pm 1}. \end{aligned}$$

On the other hand,  $\mathcal{H}_{ge}^B$  consists of spin-exchange collision terms

$$\begin{aligned} \mathcal{H}_{ge}^B/\hbar &= \frac{1}{2}(u_{2,0} - u_{1,0})\psi_{e\frac{1}{2}}^\dagger\psi_{g-\frac{1}{2}}^\dagger\psi_{g\frac{1}{2}}\psi_{e-\frac{1}{2}} \\ &+ \sqrt{\frac{3}{4}}(u_{2,1} - u_{1,1})\psi_{e\frac{3}{2}}^\dagger\psi_{g-\frac{1}{2}}^\dagger\psi_{g\frac{1}{2}}\psi_{e\frac{1}{2}} \\ &+ \sqrt{\frac{3}{4}}(u_{2,-1} - u_{1,-1})\psi_{e-\frac{3}{2}}^\dagger\psi_{g\frac{1}{2}}^\dagger\psi_{g-\frac{1}{2}}\psi_{e-\frac{1}{2}} + \text{H.c.} \end{aligned} \quad (24)$$

Similar nonlinear wave-mixing terms can have interesting effects on the dynamics of spinor BE condensates [37].

If the  $s$ -wave scattering amplitude  $u_{F,m_F}$  is independent of  $F$  we obtain  $\mathcal{H}_{ge}^B = 0$ . Furthermore, if the scattering is also independent of  $m_F$ ,  $u_{g\nu,e\sigma}$  in Eq. (23) does not depend on sublevels  $\nu$  and  $\sigma$ . For simplicity, in the following we assume that  $\mathcal{H}_{ge}^B = 0$ .

## B. Atomic polarization

The dipole radiation [Eq. (15)] describes the scattered light in a medium. In this section we study the optical response of an atomic gas by solving nonperturbatively the polarization of the matter-field for a low-density atomic gas in the limit of low light-intensity. The main items are the steady-state equation of the atomic polarization [Eq. (29)] and its solution by means of the low-density decorrelation approximation [Eq. (32)].

### 1. Equation of motion

We consider the optical response of the atomic polarization in the limit of low light-intensity. We derive the corresponding Heisenberg equations of motion for the matter-field operators. Alternative approaches have used, e.g., the Schwinger-Keldysh techniques [38]. We assume that the spin-exchanging collisions between the ground-state and the excited-state atoms vanish indicating  $\mathcal{H}_{ge} = \mathcal{H}_{ge}^A$ . Then the Heisenberg equations of motion for atomic field operators are obtained from the Hamiltonian densities  $\mathcal{H}_g$  [Eq. (1)],  $\mathcal{H}_D$  [Eq. (11)],  $\mathcal{H}_e$  [Eq. (19)], and  $\mathcal{H}_{ge}^A$  [Eq. (23)].

In the limit of low light-intensity we derived from the Heisenberg equations of motion a hierarchy of equations for correlation functions involving atomic polarization and atom density [31,32]. In the case of the present system we may proceed similarly. As far as the optical response is concerned it is again assumed that we can concentrate on the dynamics of internal degrees of freedom for the atoms and the light. Hence, in the equation of motion for the atomic polarization the kinetic energy of the atoms is neglected. Nevertheless, the quantum statistics of the different c.m. motional states is still fully included.

The vacuum electromagnetic fields are eliminated by performing the field-theory version of the Born and Markov approximations [31]. We then obtain the equation of motion for the polarization operator component  $\mathbf{P}_{\nu\eta}^+(\mathbf{r})$ ,

$$\begin{aligned} \frac{d}{dt}\mathbf{P}_{\nu\eta}^+ &= (i\delta - \gamma)\mathbf{P}_{\nu\eta}^+ + i\kappa\psi_{g\eta}^\dagger\psi_{g\sigma}\mathbf{P}_{\eta\sigma}^{\nu\eta}\cdot\mathbf{D}_F^+ \\ &+ i\kappa\int d^3r' \mathbf{P}_{\eta\sigma}^{\nu\eta}\cdot\mathbf{G}'(\mathbf{r}-\mathbf{r}')\psi_{g\nu}^\dagger\mathbf{P}^+(\mathbf{r}')\psi_{g\sigma} \\ &- i\kappa\int d^3r' \mathbf{P}_{\sigma\nu}^{\nu\eta}\cdot\mathbf{G}'(\mathbf{r}-\mathbf{r}')\psi_{e\sigma}^\dagger\mathbf{P}^+(\mathbf{r}')\psi_{e\nu} \\ &- i\kappa\psi_{e\sigma}^\dagger\psi_{e\nu}\mathbf{P}_{\sigma\nu}^{\nu\eta}\cdot\mathbf{D}_F^+ + iu_g(1-\delta_{\nu,\sigma})\psi_{g\nu}^\dagger\psi_{g\sigma}^\dagger\psi_{g\sigma}\psi_{e\eta} \\ &+ iu_{g\nu,e\sigma}\psi_{g\nu}^\dagger\psi_{e\sigma}^\dagger\psi_{e\sigma}\psi_{e\eta} - iu_{g\sigma,e\eta}\psi_{g\nu}^\dagger\psi_{g\sigma}^\dagger\psi_{g\sigma}\psi_{e\eta} \\ &+ i(u_{e\xi\eta,e\sigma\tau} - u_{e\eta\xi,e\sigma\tau})\psi_{g\nu}^\dagger\psi_{e\xi}^\dagger\psi_{e\sigma}\psi_{e\tau}. \end{aligned} \quad (25)$$

Here we use the notational convention that the repeated indices  $\sigma$ ,  $\tau$ , and  $\xi$  indicate the summation over the corresponding sublevels. In Eq. (25) we have shown explicitly only the nonlocal position dependence. The atom-light detuning is denoted by  $\delta$  and the Kronecker delta function by  $\delta_{\nu,\sigma}$ . The spontaneous linewidth  $\gamma$  is given by

$$\gamma \equiv \frac{\mathcal{D}^2 k^3}{6\pi\hbar\epsilon_0}. \quad (26)$$

We have also defined  $\kappa \equiv \mathcal{D}^2/(\hbar\epsilon_0)$  in terms of the reduced dipole matrix element  $\mathcal{D}$ . In Eq. (25) we introduced the following projection operator:

$$\begin{aligned} \mathbf{P}_{\xi\tau}^{\nu\eta} &\equiv \frac{\mathbf{d}_{g\nu e\eta}\mathbf{d}_{e\xi g\tau}}{\mathcal{D}^2} \\ &= \sum_{\sigma_1,\sigma_2} \hat{\mathbf{e}}_{\sigma_1}\hat{\mathbf{e}}_{\sigma_2}^* \langle e\eta;1g|1\sigma_1;g\nu\rangle \langle e\xi;1g|1\sigma_2;g\tau\rangle, \end{aligned} \quad (27)$$

to include the dependence of the scattered light on the polarizations and on the atomic level structure.

In the present formalism the atoms are represented by ideal point dipoles which may overlap. Obviously, real atoms with a hard-core interatomic potential cannot overlap. We remove the contact dipole-dipole interactions between different atoms. This is done by introducing in Eq. (25) the propagator

$$\mathbf{G}'_{ij}(\mathbf{r}) \equiv \mathbf{G}_{ij}(\mathbf{r}) + \frac{\delta_{ij}\delta(\mathbf{r})}{3} \quad (28)$$

that explicitly cancels the contact interaction of  $\mathbf{G}(\mathbf{r})$  displayed in Eq. (17). The purpose of this definition is to yield a vanishing integral for  $\mathbf{G}'(\mathbf{r})$  over an infinitesimal volume enclosing the origin. As shown for the case of low-intensity light in Ref. [32], we can make the substitution (28) without changing the outcome of the optical response, and without any additional assumption of the hard-core interatomic potentials, even for ideal point dipoles. This is because due to the divergent dipole-dipole interactions all correlation functions for atomic dipoles vanish whenever two position arguments are the same. As a result the contact interaction terms between different dipole atoms do not have any effect on the light-matter dynamics. The independence of the optical response of the collection of dipole atoms on the substitution (28) is the underlying explanation for the Lorentz-Lorenz local-field correction to the electric susceptibility.

We consider the limit of low intensity of the driving light. Obviously, light has to be present in order to produce population in the electronically excited levels and an excited-state field amplitude is proportional to the light field amplitude. Therefore, we take the low-intensity limit by retaining only those products of operators in Eq. (25) that involve at most one excited state field operator or the electric displacement amplitude [31]. Then, e.g., the third line and the last line in Eq. (25) make no contribution to the equation of motion for  $\mathbf{P}^+(\mathbf{r})$  for low light-intensity.

We also note that in the low-intensity limit the pair correlation function (7) is determined by the Hamiltonian density in the absence of the driving light  $\mathcal{H}_g$  [Eq. (1)]. This is because the effect of the light on the ground-state field amplitudes involves terms that contain at least one excited-state field operator *and* one light-field amplitude:

$$\dot{\psi}_g(\mathbf{r}) \propto \mathbf{d}_{ge} \cdot \mathbf{D}^-(\mathbf{r}) \psi_e(\mathbf{r}).$$

Thus, to leading order in the low-intensity limit the effect of the driving light on the ground-state atom correlation functions vanishes.

For the expectation value of the polarization we use the notation  $\mathbf{P}_{1\nu\eta} \equiv \langle \mathbf{P}_{\nu\eta}^+ \rangle$ , with  $\nu$  and  $\eta$  denoting the atomic sublevel. The steady-state solution of  $\mathbf{P}_{1\nu\eta}$  in the limit of low light-intensity is given by

$$\begin{aligned} \mathbf{P}_{1\nu\eta}(\mathbf{r}_1) &= \alpha \rho_\nu \mathbf{P}_{\eta\nu}^{\nu\eta} \cdot \mathbf{D}_F^+(\mathbf{r}_1) + \sum_{\sigma} \mathcal{F}_{\sigma\nu}^{\sigma\eta} \mathbf{P}_2(\mathbf{r}_1 \sigma \sigma; \mathbf{r}_1 \nu \eta) \\ &+ \alpha \sum_{\sigma\tau\xi} \int d^3 r_2 \mathbf{P}_{\eta\sigma}^{\nu\eta} \cdot \mathbf{G}'(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{P}_2(\mathbf{r}_1 \nu \sigma; \mathbf{r}_2 \tau \xi). \end{aligned} \quad (29)$$

Here  $\alpha = -\mathcal{D}^2 / [\hbar \epsilon_0 (\delta + i\gamma)]$  is the polarizability of an isolated atom and  $\rho_\nu \equiv \langle \psi_{g\nu}^\dagger \psi_{g\nu} \rangle$  denotes the ground-state atom density in level  $\nu$ . We have also defined

$$\mathbf{P}_2(\mathbf{r}_1 \nu \eta; \mathbf{r}_2 \sigma \tau) \equiv \langle \psi_{g\nu}^\dagger(\mathbf{r}_1) \mathbf{P}_{\sigma\tau}^+(\mathbf{r}_2) \psi_{g\eta}(\mathbf{r}_1) \rangle, \quad (30)$$

$$\mathcal{F}_{\tau\nu}^{\sigma\eta} \equiv \frac{1}{\delta + i\gamma} [u_{g\sigma e} \eta - (1 - \delta_{\tau\nu}) u_g]. \quad (31)$$

The normally ordered expectation value  $\mathbf{P}_2(\mathbf{r}_1 \nu \eta; \mathbf{r}_2 \sigma \tau)$  describes correlations between an atomic dipole at  $\mathbf{r}_2$  and a ground-state atom at  $\mathbf{r}_1$ . In the integral of Eq. (29) it represents a process in which an excited atom at  $\mathbf{r}_2$  emits a photon and excites a ground-state atom at  $\mathbf{r}_1$ . The tensor  $\mathcal{F}_{\tau\nu}^{\sigma\eta}$  generates the collisionally induced level shifts.

## 2. Low-density approximation

So far, we have obtained a steady-state solution for the atomic polarization (29) that acts as a source for the secondary radiation in Eq. (15a). Equation (29) involves unknown correlation function  $\mathbf{P}_2$ . Basically, we could continue the derivation and obtain the equations of motion for  $\mathbf{P}_2$  and for the higher-order correlation functions. This would eventually result in an infinite hierarchy of equations analogous to the equations in Ref. [31]. However, even in the case of a simple level structure and in the absence of the *s*-wave interactions the solution for the whole system by stochastic simulations is demanding on computer time [15]. In the studies of the refractive index of a quantum degenerate BE gas Morice *et al.* [39,40] considered a density expansion in terms of the number of atoms repeatedly exchanging a photon by introducing certain approximations to the ground-state atom correlations. Although the lowest-order density correction to the susceptibility of a zero-temperature FD gas may be obtained analytically [16], in the presence of highly nontrivial quantum statistical position correlations a rigorous density expansion is in most cases a very challenging task. In this paper we consider low atom densities (in terms of  $\rho/k^3$ ) and approximate Eq. (29) by the decoupling that is analogous to the lowest-order correction in Ref. [39],

$$\mathbf{P}_2(\mathbf{r}_1 \nu \eta; \mathbf{r}_2 \sigma \tau) \approx \frac{\rho_2(\mathbf{r}_1 \nu \eta, \mathbf{r}_2 \sigma \sigma)}{\rho_\sigma} \mathbf{P}_{1\sigma\tau}(\mathbf{r}_2), \quad (32)$$

where the ground-state pair correlation function  $\rho_2$  is defined by Eq. (7).

The decorrelation approximation (32) introduces the lowest-order correction to the optical response in terms of the number of microscopic optical interaction processes between the atoms by ignoring the repeated scattering of a photon between the same atoms [40]. As shown in Ref. [16] in the absence of a superfluid state it also correctly generates the leading low-density correction. The predictions of the expansion by Morice *et al.* [39] were tested for a zero-temperature FD gas in one dimension [15]. The agreement with the exact solution obtained by the numerical simulations was found to be semiquantitative and in the low-density limit excellent.

The dependence of the light propagation on the density fluctuations may now be observed by inserting Eq. (32) into Eq. (29). If the emitting atom at  $\mathbf{r}_2$  and the absorbing atom at  $\mathbf{r}_1$  have the same internal state, the pair correlation function displays repulsion and is determined by Eq. (8b). In the case

of different internal states the atoms attract each other and the pair correlation function is obtained from Eq. (8a).

It is crucial for the low-density limit that the atom operators in Eq. (25) were arranged to normal order. Otherwise, commutators are generated for higher-order correlation functions  $\mathbf{P}_2, \dots$ , that could be of the same order in atom density as the terms in the equation for  $\mathbf{P}_{1\nu\eta}$ .

### C. Electric susceptibility

In the previous section we obtained the steady-state solution for the atomic polarization (29) by means of the low-density approximation (32). The optical response may now be evaluated by eliminating  $\mathbf{D}_F^+$  and  $\mathbf{P}_2$  from Eqs. (15a), (29), and (32). As an example we calculate the vector components of  $\mathbf{P}_1$  for the  $f=1/2 \rightarrow 3/2$  transition having the electronically excited sublevels  $m_f = \pm 1/2, \pm 3/2$ . The pair correlation function in Eq. (32) is nonvanishing only with  $\nu = \eta$ . Because we are dealing with a linear theory, the electric field and the polarization are related by the susceptibility as  $\mathbf{P}^+ = \epsilon_0 \chi \mathbf{E}^+$ . We consider a situation where FD gas fills the half-infinite space  $z > 0$ . For simplicity, we assume equal and constant atom densities for the spin states  $\rho_{\uparrow} = \rho_{\downarrow} \equiv \rho$ . To simplify further, we assume that scattering length  $a_{g\nu\sigma}$  is independent of  $\nu$  and  $\sigma$  corresponding to the case that the  $s$ -wave interactions in Eq. (23) are independent of the scattering channel  $|Fm_F\rangle$ . We write the incoming free field as a plane wave

$$\mathbf{D}_F(\mathbf{r}) = D_F \hat{\mathbf{e}} e^{ikz}. \quad (33)$$

We assume that it is linearly polarized with  $\hat{\mathbf{e}}$  parallel to  $\mathbf{d}_{g\frac{1}{2}e\frac{1}{2}}$ . By choosing  $\hat{\mathbf{e}} \equiv \hat{\mathbf{e}}_x$  we have the following representation for the circular polarization vectors in terms of the unit Cartesian coordinate vectors:

$$\hat{\mathbf{e}}_{\pm} = \mp \frac{1}{\sqrt{2}} (\hat{\mathbf{e}}_y \pm i \hat{\mathbf{e}}_z), \quad \hat{\mathbf{e}}_0 = \hat{\mathbf{e}}_x. \quad (34)$$

With the ansatz  $\mathbf{P}_{1\nu\nu}(\mathbf{r}) = P \hat{\mathbf{e}} \exp(ik'z)$ , for  $\text{Im}(k') > 0$ , we then immediately see that  $\mathbf{P}_{1\nu\eta} = 0$  for  $\nu \neq \eta$ . Finally, by using Eq. (8), and by ignoring the effects of the surface of the atomic gas [15], we obtain a spatially constant susceptibility for the sample as

$$\chi = \frac{k'^2}{k^2} - 1 = \frac{2C\alpha\rho}{1 - 2C\alpha\rho/3 + \Sigma_1 + \Sigma_2}, \quad (35)$$

with

$$\begin{aligned} \Sigma_1 = & -\frac{C\alpha}{\rho} \int d^3r e^{-ikz} \hat{\mathbf{e}}^* \cdot \mathbf{G}'(\mathbf{r}) \cdot \hat{\mathbf{e}} [|\langle \psi_{g\uparrow}(\mathbf{r}) \psi_{g\downarrow}(0) \rangle|^2 \\ & - |\langle \psi_{g\downarrow}^\dagger(\mathbf{r}) \psi_{g\downarrow}(0) \rangle|^2], \end{aligned} \quad (36)$$

$$\Sigma_2 = -\frac{1}{\rho} \sum_{\sigma} \mathcal{F}_{\uparrow\sigma}^\dagger \rho_2(\mathbf{r}\uparrow, \mathbf{r}\sigma). \quad (37)$$

Here we have used the obvious relation  $\rho_2(\mathbf{r}_1\sigma, \mathbf{r}_2\nu) \equiv \rho_2(\mathbf{r}_1\sigma\sigma, \mathbf{r}_2\nu\nu) = \rho_2(\mathbf{r}_1\nu\nu, \mathbf{r}_2\sigma\sigma)$ . The parameter  $C$  denotes the value of the Clebsch-Gordan coefficient  $C = |\langle e\frac{1}{2}; 1g | 10; g\frac{1}{2} \rangle|^2 = 2/3$  in the case of the  $f=1/2 \rightarrow 3/2$  transition. By writing  $\bar{\mathbf{r}} \equiv k\mathbf{r}$  the propagator in Eq. (36) has the following expression in the spherical coordinates:

$$\begin{aligned} \hat{\mathbf{e}}_x^* \cdot \mathbf{G}'(\bar{\mathbf{r}}) \cdot \hat{\mathbf{e}}_x = & \frac{k^3 e^{i\bar{r}}}{4\pi} \left[ (1 - \sin^2 \theta \cos^2 \phi) \frac{1}{r} \right. \\ & \left. + (3 \sin^2 \theta \cos^2 \phi - 1) \left( \frac{1}{r^3} - \frac{i}{r^2} \right) \right]. \end{aligned} \quad (38)$$

In Eq. (35)  $\Sigma_1$  is solely generated by the quantum-statistical position correlations between different atoms. The effect of  $s$ -wave interactions is encapsulated in  $\Sigma_2$ . In an uncorrelated atomic sample the atomic positions are statistically independent and the pair correlation function (7) satisfies  $\rho_2(\mathbf{r}\nu, \mathbf{r}'\sigma) = \rho_\nu \rho_\sigma$  resulting in  $\Sigma_1 = 0$ . For the case of uncorrelated atoms, and in the absence of the  $s$ -wave scattering, we would obtain Eq. (35) with  $\Sigma_1 = \Sigma_2 = 0$ . This is the standard column density result stating that susceptibility equals polarizability of an atom times atom density. Equation (35) also involves the Lorentz-Lorenz local-field correction in the denominator.

The quantum-statistical corrections to the column density result are introduced by  $\Sigma_1$ . It describes the modifications of the optical interactions between neighboring atoms due to the position correlations. The second term in Eq. (36) represents the quantum-statistical contribution to the scattering process in which a photon emitted by an atom in internal state  $\nu$  at position  $\mathbf{r}$  is reabsorbed by another atom in internal state  $\nu$  and located at the origin. According to FD statistics two fermions with the same quantum numbers repel each other and FD statistics forces a regular spacing between the atoms. The optical interactions are dominantly generated at small interatomic distances and the corrections to the susceptibility due to the second term in Eq. (36) correspond to *inhibited* multiple scattering of light resulting in suppressed diffusive radiation. In the absence of a superfluid state FD gas exhibits a dramatic narrowing of the absorption linewidth for coherently scattered light [15,16].

The first term in Eq. (36) represents the quantum-statistical corrections to the reabsorption process between atoms in different internal states due to the two-particle coherence. This term is nonvanishing only in the presence of a superfluid state. Because the total spin of an interacting atom pair in Eq. (1) is an integer, the pairs behave as bosons [4]. According to the Bose-Einstein statistics two bosons attract each other and the BCS pairing favors small interatomic spacing. This results in *enhanced* optical interactions and incoherent scattering of light.

The electric susceptibility exhibits a Lorentzian line shape. The optical line shift  $\mathcal{S}$  and linewidth  $\Gamma$  for the atomic sample are obtained from Eq. (35)

$$\mathcal{S}/\gamma = \frac{4\pi\rho\mathcal{C}}{k^3} + \mathcal{S}_{\text{col}}/\gamma - \frac{6\pi}{k^3} \operatorname{Re}\left(\frac{\Sigma_1}{\alpha}\right), \quad (39)$$

$$\Gamma/\gamma = 1 - \frac{6\pi}{k^3} \operatorname{Im}\left(\frac{\Sigma_1}{\alpha}\right). \quad (40)$$

The collisional line shift  $\mathcal{S}_{\text{col}}$ , which results from  $\Sigma_2$  [Eq. (37)], is generated by the  $s$ -wave interactions. It depends on the BCS order parameter  $\Delta$ :

$$\begin{aligned} \mathcal{S}_{\text{col}} &\equiv (u_g - u_{ge}) \frac{\rho_2(\mathbf{r}\uparrow, \mathbf{r}\downarrow)}{\rho} \\ &= \rho(u_g - u_{ge}) \left[ 1 + \left( \frac{\Delta}{\hbar u_g \rho} \right)^2 \right]. \end{aligned} \quad (41)$$

The first term in the optical lineshift (39) corresponds to the Lorentz-Lorenz local-field correction. As far as the  $s$ -wave interactions can be considered local on the scale of the optical wavelength in Eqs. (1) and (20) also the line shift  $\mathcal{S}_{\text{col}}$  may be considered as a local-field shift. In that case the local-field shift due to the  $s$ -wave scattering in Eq. (39) is larger than the Lorentz-Lorenz shift for  ${}^6\text{Li}$  if

$$\gamma \gtrsim 210 \left[ 1 + \left( \frac{\Delta}{\hbar u_g \rho} \right)^2 \right] \frac{(a_g - a_{ge})}{a_0 \lambda^3} \mu\text{m}^3 \text{s}^{-1}.$$

Here  $\lambda$  denotes the wavelength of the incoming light and  $a_0$  is the Bohr radius. Because  $(\Delta/\hbar u_g \rho)^2$  is expected to be of the order of one [9], the local-field shift could strikingly depend on the BCS order parameter  $\Delta$ . The collisional line shift was recently observed for a hydrogen BE condensate by using a two-photon  $1S \rightarrow 2S$  spectroscopy [41].

If the effective range  $r_u$  of the triplet  $s$ -wave potential in Eqs. (1) and (20) is very short,  $r_u \ll 1/k$ , the resonant dipole-dipole interactions may suppress the effect of the  $s$ -wave scattering on the line shift just as they cancel the effect of the polarization self-energy [32]. However, for a metastable state,  $\gamma^{-1}$  may be large on the time scale of the atomic interactions. In that case the collisional shift could be observable even for very small  $r_u$ .

To calculate the linewidth (40) and line shift (39) from integral (36) we need to evaluate the spatial correlation functions by using Eqs. (3) and (4). For instance, the expectation value for the anomalous correlation function reads

$$\langle \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(0) \rangle = \frac{1}{V} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{\Delta}{2E_{\mathbf{k}}} (1 - \bar{n}_{\alpha\mathbf{k}}^q - \bar{n}_{\beta\mathbf{k}}^q). \quad (42)$$

The chemical potential is solved from  $\rho_{\nu} = \rho_{\nu}(\bar{\mu})$ . Here  $\langle \psi_{\downarrow}(0) \psi_{\uparrow}(0) \rangle = -\Delta/(\hbar u_g)$  is ultraviolet divergent, resulting from the assumption of the contact two-body interaction in Eq. (1). This interaction is momentum independent and it is not valid at high energies. To estimate the pairing a standard procedure is to remove the high-energy divergence by introducing a high-momentum cutoff  $k_c$ . Nevertheless, we find that the optical linewidth of a FD gas [Eq. (40)] is finite even without any high-momentum cutoff. This is because the di-

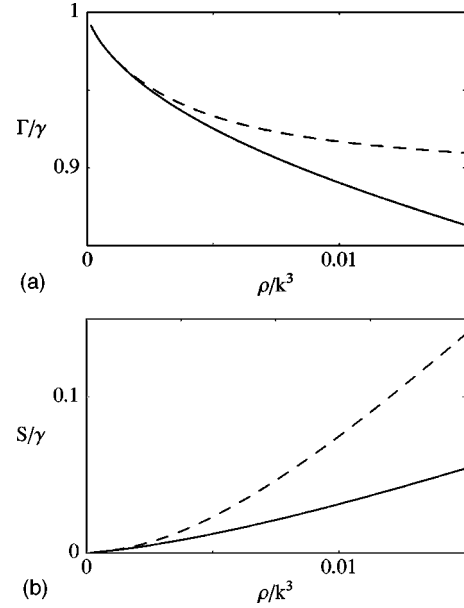


FIG. 2. The optical (a) linewidth and (b) the line shift for the electric susceptibility, in the absence of the collisional line shift, as a function of the atom density per cubic optical wave number of the driving light. The scaling of all the variables is linear. The solid line represents the optical response in the absence of a superfluid state ( $\Delta=0$ ). The BCS pairing broadens the resonance line and increases the line shift already at low atom densities.

pole radiation already involves a high-frequency cutoff [31] that regularizes small  $r$  behavior. The contribution to the optical line shift [Eq. (39)] from the integral (36) is not finite. The lower limit of the integral diverges logarithmically. Although the radiation kernel (15b) involves a cutoff [31], the Lamb shift is not treated rigorously. The small-distance singularities of the dipole radiation kernel may be regularized by introducing explicit regularization factors [42]. However, for the present purposes we may at least obtain an estimate for the shift by using the cutoff  $k_c = 1/r_u$  in the anomalous correlation function (42) with the realistic value  $r_u = 100a_0$  of the triplet  $s$ -wave potential [9].

In Fig. 2 we have shown (a) the absorption linewidth and (b) the line shift for coherently scattered light from Eq. (39) without the collisional shift, i.e., by assuming  $u_g = u_{ge}$  in Eq. (39), for  $\lambda = 900$  nm and for the value of the  $s$ -wave scattering length of  ${}^6\text{Li}$ ,  $a_g = -2160a_0$  [9]. In (a) the solid line represents the linewidth in the absence of the superfluid state ( $\Delta=0$ ). The line narrows as a function of the density already at very low densities [16]. The presence of the superfluid state broadens the optical linewidth (the dashed line). For the BCS state, even without the collisional shift, also (b) the optical line shift is increased.

It is interesting to emphasize that the optical linewidth is almost independent of the high-momentum behavior of the anomalous correlation function (42). This can be seen by introducing a cutoff  $k_c$  in Eq. (42). We found [17] that the optical linewidth is almost independent of the cutoff from  $k_c = \infty$  to  $k_c = 1/(500a_0)$  indicating that the exact short-range behavior of the two-body  $s$ -wave potential is not very crucial for the value of the linewidth. Furthermore, the contribution



of integral (36) in the close neighborhood of the origin to the linewidth is vanishingly small, and therefore the effects of the BCS pair correlations do not result from short-range correlations.

#### D. Summary remarks

We calculated the leading quantum-statistical and collisional corrections to the standard century-old column density result for the electric susceptibility. In Eq. (35) the corrections to the susceptibility are encapsulated in the two parameters  $\Sigma_1$  and  $\Sigma_2$ . Here  $\Sigma_2$  represents the collisional line shift due to the  $s$ -wave interactions and  $\Sigma_1$  position correlations between different atoms. The susceptibility was obtained by means of the decorrelation approximation (32) which neglects all the repeated photon exchange between the same atoms. These are the microscopic mechanism for the collective optical linewidths and line shifts [31]. Therefore,  $\Sigma_1$  is a direct consequence of the quantum-statistical correlations; for an uncorrelated atomic sample, with  $\rho_2(\mathbf{r}\nu, \mathbf{r}'\sigma) = \rho_\nu \rho_\sigma$ ,  $\Sigma_1$  vanishes. In the case of uncorrelated atoms the lowest-order correction to the optical linewidth results from the collective light scattering [16,39,40]. This correction is proportional to atom density. The quantum statistics is different because the correlation length itself depends on the density. For instance, in the low-density limit for an ideal FD gas we obtain [16]  $\Sigma_1 \propto (\rho/k^3)(\xi_{\uparrow\uparrow} k) \propto \rho^{2/3}/k^2$ . Hence, at least in the absence of a superfluid state, Eq. (35) not only represents the lowest-order correction to the susceptibility in terms of microscopic optical interaction processes between the atoms, but it also correctly generates the leading low-density correction.

### IV. BRAGG SPECTROSCOPY

#### A. Diffraction of atoms

In this section we consider diffraction of atoms by means of light-stimulated transitions of photons between two non-parallel laser beams. When an atomic beam interacts with a periodic potential formed by a standing light wave, it can Bragg diffract, analogous to the Bragg diffraction of x rays from a crystal [23]. The Bragg diffraction has been experimentally used as a technique to split a BE condensate [23] and as a spectroscopic method to probe the density fluctuations of a BE condensate [24,25].

In an  $n$ th order Bragg scattering process photons are absorbed from one beam and stimulated to emit into the other  $n$  times [23,24]. Two different momentum states are connected by a  $2n$ -photon process. The change of the energy of a photon upon the scattering satisfies  $E = n\hbar\omega$ , where  $\omega$  stands for the frequency difference of the two lasers with wave number  $k$ . The fractional change of frequency upon scattering is assumed to be negligible,  $|\omega| \ll kc$ , so that one finds the familiar relation between the change of the wave vector  $\Delta\boldsymbol{\kappa}$  and the scattering angle  $\theta$ ,

$$|\Delta\boldsymbol{\kappa}| = 2nk \sin(\theta/2). \quad (43)$$

#### B. Dynamical structure function

In Bragg spectroscopy, the two intersecting laser beams create a moving standing wave with a periodic intensity modulation  $I(\mathbf{r}t) = I \cos(\Delta\boldsymbol{\kappa} \cdot \mathbf{r} - \omega t)$  [25]. The intensity modulation creates an optical potential  $V(\mathbf{r}t) \equiv V_0 \cos(\Delta\boldsymbol{\kappa} \cdot \mathbf{r} - \omega t)$  which couples to the local number density of ground-state atoms. The dependence of  $V_0$  on the atomic level scheme and on light polarizations is analyzed in Ref. [6]. We consider a situation where the internal sublevel of the ground-state atom does not change in the scattering process. The corresponding Hamiltonian density reads

$$\mathcal{H}_B = V_0 \cos(\Delta\boldsymbol{\kappa} \cdot \mathbf{r} - \omega t) \psi_{g\nu}^\dagger(\mathbf{r}) \psi_{g\nu}(\mathbf{r}). \quad (44)$$

According to Fermi's golden rule the excitation rate is then  $2\pi/\hbar (V_0/2)^2 S(\Delta\boldsymbol{\kappa}, \omega)$  [25], where  $S(\Delta\boldsymbol{\kappa}, \omega)$  is the dynamical structure function [43]

$$S(\Delta\boldsymbol{\kappa}, \omega) \equiv \frac{1}{Z} \sum_{i,f} e^{-E_i/\hbar k_B T} |\langle i | \hat{\rho}(\Delta\boldsymbol{\kappa}) | f \rangle|^2 \delta(\hbar\omega + E_f - E_i). \quad (45)$$

Here  $Z$  denotes the grand partition function and the expectation value of the density fluctuation operator

$$\hat{\rho}(\mathbf{q}) = \sum_\nu \int d^3r e^{-i\mathbf{q} \cdot \mathbf{r}} \psi_{g\nu}^\dagger(\mathbf{r}) \psi_{g\nu}(\mathbf{r}) \quad (46)$$

is summed over all possible final states  $|f\rangle$ , with the energy  $E_f$ , and thermally averaged over initial states  $|i\rangle$ , with the energy  $E_i$ . By using the completeness of  $|f\rangle$  and  $\hat{\rho}(-\mathbf{q}) = \hat{\rho}^\dagger(\mathbf{q})$  we may write Eq. (45) as

$$S(\Delta\boldsymbol{\kappa}, \omega) = \frac{1}{2\pi\hbar} \sum_{\nu,\eta} \int dt d^3r_1 d^3r_2 e^{i\omega t} e^{i\Delta\boldsymbol{\kappa} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \times \langle \psi_{g\nu}^\dagger(\mathbf{r}_1 0) \psi_{g\nu}(\mathbf{r}_1 0) \psi_{g\eta}^\dagger(\mathbf{r}_2 t) \psi_{g\eta}(\mathbf{r}_2 t) \rangle. \quad (47)$$

We define the static structure function by

$$\bar{S}(\Delta\boldsymbol{\kappa}) \equiv \hbar \int_{-\infty}^{\infty} d\omega S(\Delta\boldsymbol{\kappa}, \omega). \quad (48)$$

The dynamical structure function mirrors the velocity distribution of atoms and contains qualitative signatures of BE and FD statistics [6]. It displays the modifications of the velocity distribution due to the quantum statistics including the Fermi *inhibition* and the Bose *enhancement* of the scattering process. For the case of two BE condensates it can also exhibit a dramatic dependence of the spectrum on the relative phase between the two condensates [28,30], and the Bragg diffraction could possibly be used as a technique of measuring the relative condensate phase. This is because, due to the macroscopic quantum coherence of the BE condensates, the uncertainty in the initial state of the Bragg diffraction may result in a destructive or constructive interference of the transition amplitudes. The structure function may also provide information about the high-energy quasiparticle

excitations [29]. Here we study the qualitative signatures of a superfluid state in the structure function of a FD gas.

### 1. Ideal Fermi-Dirac gas

First, we consider an ideal FD gas studied in Ref. [6]. We assume a translationally invariant space. In that case the correlation function in Eq. (47) depends only on  $\mathbf{r} \equiv \mathbf{r}_2 - \mathbf{r}_1$ . We are interested in the incoherent scattering processes corresponding to nonforward directions with  $\Delta \boldsymbol{\kappa} \neq \mathbf{0}$ . In the absence of a superfluid state the correlation function in Eq. (47) with  $\nu \neq \eta$  represents only coherent scattering events. With  $\nu = \eta$  we obtain

$$\begin{aligned} & \langle \psi_{g\nu}^\dagger(00) \psi_{g\nu}(00) \psi_{g\eta}^\dagger(\mathbf{r}t) \psi_{g\eta}(\mathbf{r}t) \rangle \\ &= \rho^2 + \langle \psi_{g\nu}^\dagger(00) \psi_{g\eta}(\mathbf{r}t) \rangle \langle \psi_{g\nu}(00) \psi_{g\eta}^\dagger(\mathbf{r}t) \rangle. \end{aligned} \quad (49)$$

For  $\Delta \boldsymbol{\kappa} \neq \mathbf{0}$  we obtain the dynamical structure function from Eq. (47)

$$S = \frac{1}{\hbar} \sum_{\mathbf{k}, \nu} \delta\left(\omega + \omega_R - \frac{\hbar \mathbf{k} \cdot \Delta \boldsymbol{\kappa}}{m}\right) \bar{n}_{\mathbf{k}, \nu} (1 - \bar{n}_{\mathbf{k} - \Delta \boldsymbol{\kappa}, \nu}). \quad (50)$$

Here  $\bar{n}_{\mathbf{k}, \nu} \equiv [\exp(\epsilon_{\mathbf{k}}/k_B T)/z + 1]^{-1}$  denotes the FD occupation numbers and  $z$  fugacity. We have also defined the effective recoil frequency  $\omega_R$  by

$$\omega_R = \frac{\hbar |\Delta \boldsymbol{\kappa}|^2}{2m}. \quad (51)$$

Expression (50) describes a scattering process in which an atom in the ground state  $\nu$  with the c.m. state  $\mathbf{k}$  scatters to the c.m. state  $\mathbf{k} - \Delta \boldsymbol{\kappa}$  still remaining in the state  $\nu$ . The delta function dictates the energy conservation, which coincides with the theory for Doppler velocimetry of atoms [44] shifted by the effective recoil frequency  $\omega_R$  [6].

Classical atoms obey Maxwell-Boltzmann statistics and their velocities are normally distributed resulting in a Gaussian-shaped dynamical structure function [6]. First, FD statistics modifies the velocity distribution; even an ideal FD gas at  $T=0$  exhibits a finite width in Eq. (50). Secondly, the quantum degeneracy affects the scattering processes. The product of the occupation numbers in Eq. (50) indicates the Fermi *inhibition*: The scattering events in which an atom recoils to an already occupied state are forbidden by the Pauli exclusion principle.

It is illustrative to describe the Fermi inhibition in momentum space [16]. At  $T=0$  the fermionic atoms fill the Fermi sphere with  $\bar{n}_{\mathbf{k}, \nu} = \Theta(k_F - |\mathbf{k}|)$ . The scattering satisfies Eq. (43). For the first-order Bragg diffraction, with  $n=1$ , all atoms are scattered out of the Fermi sea, if  $|\Delta \boldsymbol{\kappa}| > 2k_F$ . Moreover, for small scattering angles,

$$\sin(\theta/2) < \frac{k_F}{|\Delta \boldsymbol{\kappa}|} = \left( \frac{6\pi^2 \rho}{|\Delta \boldsymbol{\kappa}|^3} \right)^{1/3}, \quad (52)$$

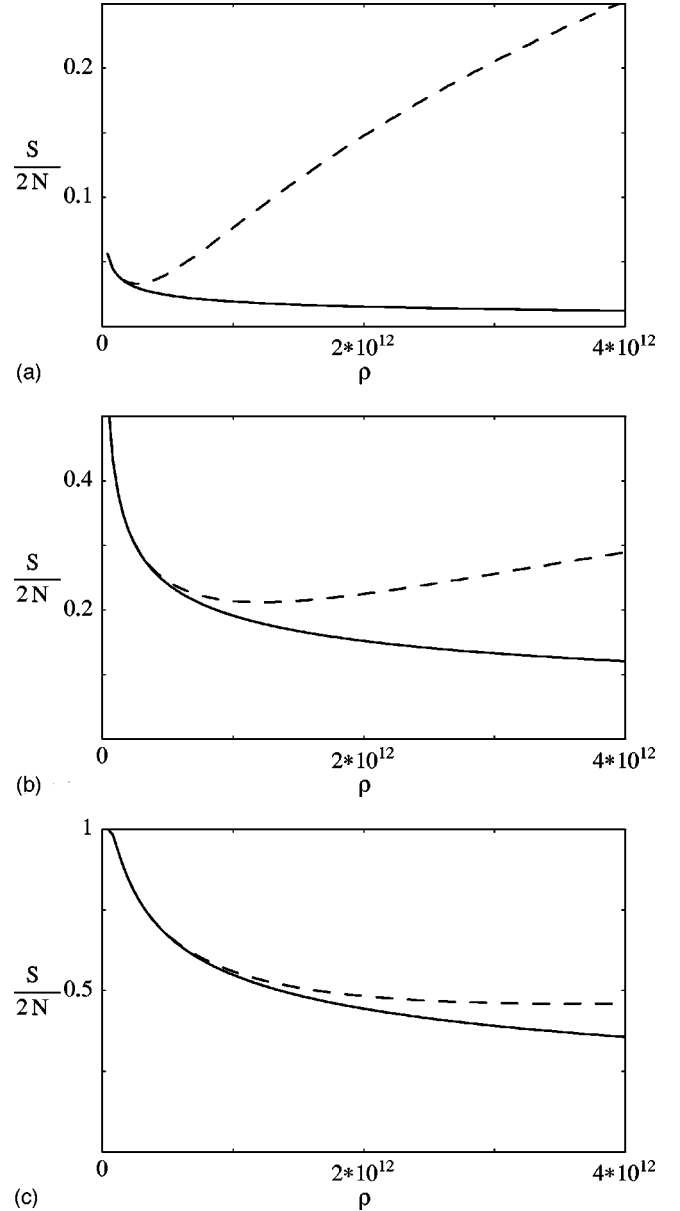


FIG. 3. The static structure function per the total number of atoms  $\bar{S}(\Delta \boldsymbol{\kappa})/(2N)$  as a function of the atom density  $\rho$  in units of  $\text{cm}^{-3}$ . The change of the atomic recoil wave number upon scattering is (a)  $10^3 \text{ cm}^{-1}$ , (b)  $10^4 \text{ cm}^{-1}$ , and (c)  $3 \times 10^4 \text{ cm}^{-1}$ . The solid line represents the diffraction in the absence of a superfluid state ( $\Delta=0$ ). The BCS pairing dramatically increases the incoherent nearforward scattering already at low atom densities.

the incoherent atomic recoil events are forbidden. When the density is increased, at  $k_F \gg |\Delta \boldsymbol{\kappa}|$  the scattering is at least partially suppressed to all nonforward directions.

We consider a situation where the densities of the internal sublevels are equal  $\rho \equiv \rho_\uparrow = \rho_\downarrow \equiv N/V$ . Here  $N$  denotes the number of atoms. The dynamical structure function for an ideal FD gas was displayed in Ref. [6]. At  $T=0$ , and in the degenerate regime with  $|\Delta \boldsymbol{\kappa}| < 2k_F$ , the result exhibits a characteristic shape of a wedge consisting of a linear and a quadratic part. The static structure function per total number of atoms may be evaluated from Eq. (50). For  $|\Delta \boldsymbol{\kappa}| > 2k_F$  we

obtain  $\bar{S}(\Delta\boldsymbol{\kappa})/(2N)=1$  and for  $0<|\Delta\boldsymbol{\kappa}|<2k_F$

$$\frac{\bar{S}(\Delta\boldsymbol{\kappa})}{2N} = \frac{1}{16k_F|\Delta\boldsymbol{\kappa}|} \left( 12|\Delta\boldsymbol{\kappa}|^2 - \frac{|\Delta\boldsymbol{\kappa}|^4}{k_F^2} \right). \quad (53)$$

For free atoms  $\bar{S}(\Delta\boldsymbol{\kappa})/(2N)=1$  and Eq. (53) describes the inhibited scattering.

In the previous discussion we ignored the Rabi oscillation dynamics and considered only the transition rates. This is a good approximation when only a small fraction of atoms is scattered, i.e., when the coupling time  $t_0$  is much shorter than the oscillation period  $t_0 \ll 1/\Omega$  [28]. Here  $\Omega$  denotes the two-photon Rabi frequency. To observe the qualitative features in the spectrum the coupling time should be at least of the order of the characteristic time scale in the spectrum, which according to Ref. [6] is  $(\omega_R \epsilon_F / \hbar)^{-1/2}$ .

### 2. BCS pairing

In the presence of a superfluid state the correlation function in Eq. (47) at  $T=0$  reads (for  $\nu \neq \eta$ ):

$$\begin{aligned} & \langle \psi_{g\nu}^\dagger(00) \psi_{g\nu}(00) \psi_{g\eta}^\dagger(\mathbf{r}t) \psi_{g\eta}(\mathbf{r}t) \rangle \\ &= \rho^2 + |\langle \psi_{g\nu}(00) \psi_{g\eta}(\mathbf{r}t) \rangle|^2. \end{aligned} \quad (54)$$

For the case  $\nu=\eta$  we obtain Eq. (49). Analogously to Eqs. (8) we need to evaluate Eqs. (49) and (54) by means of the Bogoliubov quasiparticles [Eq. (3)]. The dynamical structure function for incoherent scattering ( $\Delta\boldsymbol{\kappa} \neq \mathbf{0}$ ) in the quasiparticle vacuum is

$$\begin{aligned} S(\Delta\boldsymbol{\kappa}, \omega) &= \frac{1}{4} \sum_{\mathbf{k}} \delta(\hbar\omega + E_{\mathbf{k}} + E_{\mathbf{k}-\Delta\boldsymbol{\kappa}}) \left[ \frac{\Delta^2}{E_{\mathbf{k}} E_{\mathbf{k}-\Delta\boldsymbol{\kappa}}} \right. \\ & \left. + \left( 1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right) \left( 1 + \frac{\xi_{\mathbf{k}-\Delta\boldsymbol{\kappa}}}{E_{\mathbf{k}-\Delta\boldsymbol{\kappa}}} \right) \right], \end{aligned} \quad (55)$$

where the quasiparticle energies  $E_{\mathbf{k}}$  are defined in Sec. II B. Expression (55) describes creations of pairs of quasiparticles separated by the wave vector  $\Delta\boldsymbol{\kappa}$ . We note that for a superfluid state in the homogeneous space there exists a finite energy gap in the excitation spectrum  $-\hbar\omega = E_{\mathbf{k}} + E_{\mathbf{k}-\Delta\boldsymbol{\kappa}} \geq 2|\Delta|$ . The corresponding expression for the static structure function  $\bar{S}(\Delta\boldsymbol{\kappa})$  may be obtained from Eqs. (55) and (48).

In Fig. 3 we show the static structure function  $\bar{S}(\Delta\boldsymbol{\kappa})/(2N)$  for a FD gas as a function of the density  $\rho$  for three characteristic values of  $|\Delta\boldsymbol{\kappa}|$ . The  $s$ -wave scattering length  $a_g = -2160a_0$ . The solid line represents an ideal FD gas in the absence of a superfluid state. The superfluid state dramatically increases the structure function (the dashed line) for nearforward scattering. The BCS pairing mixes par-

ticles and holes near the Fermi surface increasing the number of available scattering channels. This effect is particularly striking for the case of small recoil momentum corresponding to nearforward scattering.

We may also consider situations where the internal state of atoms is changed in the scattering process. In this case the two ground states  $|g, \uparrow\rangle$  and  $|g, \downarrow\rangle$  are coupled through a common excited state by the intersecting laser beams. For instance, the scattering rate for the transition  $|g, \downarrow\rangle \rightarrow |g, \uparrow\rangle$  is proportional to  $\langle \psi_{g\downarrow}^\dagger(00) \psi_{g\uparrow}(00) \psi_{g\uparrow}^\dagger(\mathbf{r}t) \psi_{g\downarrow}(\mathbf{r}t) \rangle$  and depends on the quasiparticle pairing.

## V. CONCLUSIONS

We studied the interaction of light with a two-species atomic superfluid gas. First, we considered the propagation of light and evaluated the quantum-statistical corrections to the standard column density result for the electric susceptibility. Secondly, we analyzed the Bragg diffraction of atoms by means of light-stimulated transitions of photons between two laser beams. The effects of BCS pairing may be understood in terms of enhanced incoherent scattering processes resulting in the increased optical linewidth, line shift, and static structure function. These optical properties could possibly signal the presence of the superfluid state and determine the value of the BCS order parameter in dilute atomic FD gases.

One particularly promising candidate to undergo the BCS transition and to become a superfluid is spin-polarized atomic  ${}^6\text{Li}$ . Atoms in two different internal levels can interact via  $s$ -wave scattering and the  ${}^6\text{Li}$  atom has an anomalously large and negative  $s$ -wave scattering length  $a \approx -2160a_0$ . The hyperfine states  $|m_s=1/2, m_i=1\rangle$  and  $|m_s=1/2, m_i=0\rangle$  of  ${}^6\text{Li}$  have been predicted to undergo a superfluid transition at  $10^{-8}$  K with a density of  $10^{12}$  cm $^{-3}$  [8,9]. Here  $m_s$  and  $m_i$  denote the electron and the nuclear spin components.

We assumed a translationally invariant system. A FD gas in a harmonic trap may be considered locally homogeneous [9], provided that the trap length scale  $l = (\hbar/m\omega)^{1/2}$  is much larger than the correlation lengths,  $\xi_{\uparrow\uparrow}$  and  $\xi_{\uparrow\downarrow}$ . The spatial confinement introduces an uncertainty in the recoil momentum. In the case of Bragg spectroscopy, the coherent scattering is negligible, if the change of the wave number of the atoms upon scattering is larger than the inverse size scale of the atomic sample  $1/l \lesssim |\Delta\boldsymbol{\kappa}|$ .

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