Accurate determination of Compton backscattering in germanium at 86.5 keV on an absolute scale

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The double-differential cross section $d^2\sigma/d\Omega\, dE$ for Compton backscattering in germanium was measured at the photon energy of 86.5 keV. The experimental setup with two high-purity germanium detectors operating in the coincidence mode was applied. A multiline radioactive source of $^{155}_{63}$Eu was used as the source of photons. The Compton spectrum with a very small background was obtained. A fast cascade was found to little influence the data due to the 86.5 keV crossover transition. The contribution of a group of processes, which is described as double cross talk between the two detectors, was also studied and results of calculations show their weak intensity. The calculations of other processes were made as in our earlier papers. The values of $d^2\sigma/d\Omega dE$ were determined to an accuracy of better than 4% on an absolute scale in a broad energy region. The calculated values using the impulse approximation with hydrogenlike wave functions do not reproduce well the cross sections in some energy regions.

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I. INTRODUCTION

The investigation of Compton scattering on bound electrons still raises much interest [1]. Accurate experimental determination of double-differential cross section, $d^2\sigma/d\Omega\, dE$, is an excellent source of data for the study of atomic structure [2]. One of the techniques for measuring the Compton-scattering spectrum, recently introduced by the present authors [3–5], is based on the application of two germanium detectors that operate in the coincidence mode. In that type of measurement, a photon of incident energy E_0 is scattered in the target detector and a scattered photon is absorbed in another (second) detector. Coincidence detection of the total recoil-electron energy (E_1) and the total energy of the Compton-scattered photon (E_2) is observed. Such events are seen in the, ''events line'' $E_1+E_2=E_0$ in the E_1-E_2 two-dimensional spectrum. The analysis of data along this line yields a clean spectrum in a relatively much wider energy range than that obtained in a singles-mode measurement [6,7]. The obtained spectrum is corrected for other (single) cross-talk (detector-to-detector) processes, which also cause the splitting of energy of incident photons among the detectors. Since these corrections are the most important ones, but on the other hand, weak compared to the single Compton scattering, one is led to the conclusion that the coincidence method can give very reliable Compton spectra on absolute scale.

The accurate experimental determination of the Compton double-differential cross section $d^2\sigma/d\Omega\, dE$ is our main goal in the present work. For this purpose, it is necessary to investigate other cross-talk processes, which have not been studied previously, but also contribute to the events line. There is a group of such processes, which can be described as double cross talk between the two detectors. In these processes, an incident photon is scattered one or more times in the target detector, a scattered photon escapes it, and reaches the second detector where it is also scattered one or more times, and finally a scattered photon escapes the second detector and is absorbed in the target detector.

An important advantage of the coincidence measurement of the Compton spectrum over the measurement in the singles mode is the possibility of using of multiline photon sources. For the observation of scattering at the energy E_0, there are no contributions from photons of lower energies, while the contributions of incident photons of higher energy are minor. In addition, a multiline source with several prominent lines of similar intensity is suitable for a simultaneous measurement of Compton spectra at these energies. That was demonstrated in our first experiment [3], where a source of x rays was used. In the present experiment, a complex radioactive multiline γ source of $^{155}_{63}$Eu decaying to $^{155}_{64}$Gd was used. In the decay, the 86.54 keV state decays either by a direct transition to the ground state, or by the 26.53–60.01 keV cascade. The two cascade-transition photons are scattered in the target detector independently of each other and could reach the second detector. Since these cascades are much faster than the response time of the detectors and the coincidence setup, the summation of pulses takes place in each single detector resulting in a coincidence spectrum in the events line $E_1+E_2=E_0$. The energy profile of that spectrum is also examined.

The experimental setup

Two high-purity germanium detectors in head-on geometry were applied in the experiment (Fig. 1). The active volume of the detectors is of a cylidrical shape of nominal size of 200 mm$^2\times$13 mm thick. The energy resolution of the detectors was about 360 eV at 59.4 keV. The radioactive $^{155}_{63}$Eu

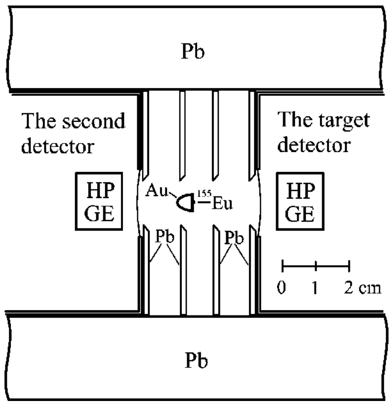

FIG. 1. A part of the experimental setup for the coincidence measurement of the Compton spectrum.

was used as the source of photons, giving a complex multiline spectrum shown in Fig. 2. The most intensive line, of the energy of 86.54 keV, was used in the investigation. A tiny source, obtained by the evaporation of $^{155}EuCl_2$ solution, was mounted on the source shield made of pure gold. The source shield was placed between the two detectors, and was used to expose essentially only the target detector to photons emitted by the source. A satisfactory asymmetry ratio of 72, defined as the ratio of numbers of incident photons which reached the target detector and the second detector, was achieved in the experiment for 86.54 keV photons. Four lead plates were placed between the detectors to reduce detection of photons scattered by the lead plates and other materials surrounding the detectors. The average angle between the directions of the incident and detected scattered photons was about $\vartheta_p \approx 174°$. The average value of the solid angle of the second detector for photons scattered in the target detector was $\Delta\Omega_1 \approx 0.1$ sr, and for the reverse processes $\Delta\Omega_2 \approx 0.095$ sr.

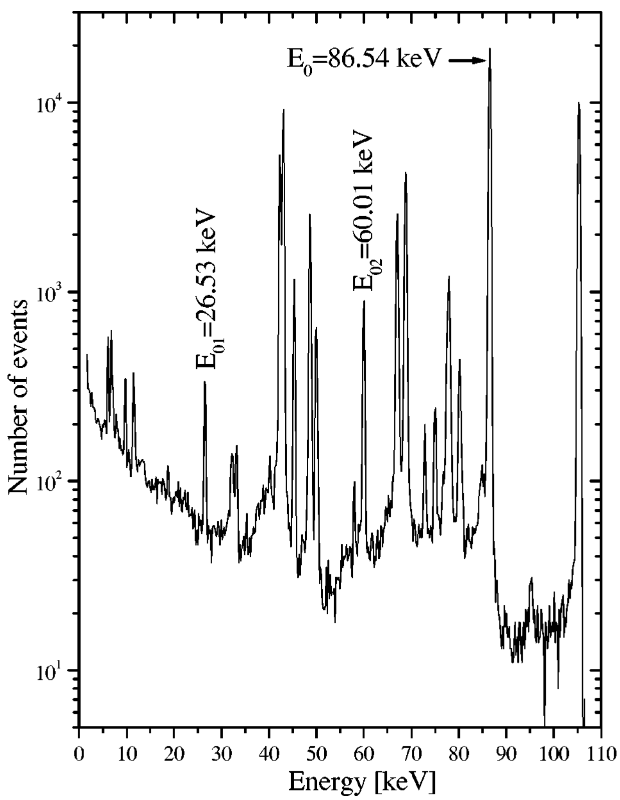

FIG. 2. The spectrum of $^{155}_{63}Eu$ measured in the target detector. The Compton spectrum was determined for the 86.54-keV γ rays.

The same fast-slow coincidence system with a three-parameter $128\times512\times512$ channel pulse-height analyzer was used as in the two previous experiments [3,4]. A triad of channel numbers k_0, k_1, k_2 was recorded for each coincidence event, where k_0 is the time difference of the pulses, k_1 the amplitude of pulse from the target detector, and k_2 the amplitude of pulse from the second detector. The coincidence unit was set to 200 ns. The full width and half maximum time resolution was about 20 ns.

II. PROCESSING OF DATA AND RESULTS

The pulse-height spectrum of events in the $E_1+E_2=E_0$ line, without any subtractions, versus energy deposited in the second detector is shown in Fig. 3. To obtain the single-Compton-scattering data, all other processes which contribute to the events line must be calculated and subtracted. The calculations are very laborious because for each scattering process (Compton, Rayleigh, and photoelectric effect), each electron subshell has been separately taken into account. The results for various combinations of subshells and processes were summed to obtain the calculated results shown in Fig. 3. The calculations were performed on absolute scale, except for the peaks due to the escape of characteristic Ge K x rays, from which the product $N_0\Delta\Omega_1$ was determined (N_0 is the number of photons of incident energy which reached the target detector during the experiment). In the calculations, the impulse approximation with hydrogenlike wave functions [8,9] was used for the Compton-scattering differential cross section, the theory of Heitler [10–12] was used for bremsstrahlung of secondary photoelectrons, and tabulated values for the modified relativistic form factor [13,14] were used for the elastic differential cross section. All processes were classified into single- and double-cross-talk processes.

A. Single-cross-talk processes

Here, we review the single-cross-talk (detector-to-detector scattering) processes following our Refs. [3–5], where they are explained in more detail. These types of processes are described as single- and double-scattering single-cross-talk processes.

The number of events per channel due to the single scattering, in which energy E is deposited in the second detector, is given by the following expression:

$$n(E_0,E)=N_0\frac{d^2\sigma/d\Omega\, dE}{\mu(E_0)+\mu(E)/\cos(\vartheta_p)}\Delta D, \qquad (1)$$

where E_0 is the incident-photon energy, $d^2\sigma/d\Omega\, dE$ is the differential cross section, and $\mu(E_0)$ and $\mu(E)$ are the attenuation coefficients in germanium [15,16] for the incident energy and energy E of the scattered photon. $\Delta D = \epsilon_c\epsilon_2\epsilon(E)N_{Ge}A_2\exp[-\mu_{air}(E)d]\Delta\Omega_1$, where $\epsilon_c=0.98$ is

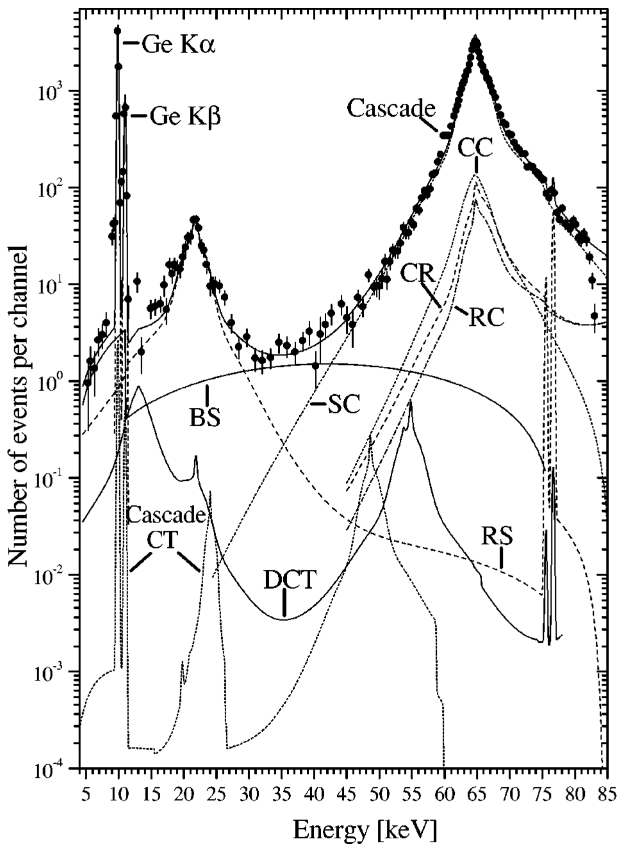

FIG. 3. The pulse-height distribution in the event line $E_1+E_2=E_0$ and calculated contributions of processes: single cross talk (SC—single Compton scattering in the target detector; BS—bremsstrahlung of photoelectrons ejected by incident radiation; Ge $K\alpha$ and Ge $K\beta$—peaks due to characteristic K x rays of germanium, RS—reverse scattering; CC, RC, and CR—Compton-Compton, Rayleigh-Compton, and Compton-Rayleigh double scattering); DCT—double-cross-talk processes; Cascade—a small peak due to absorbtion of 26.53 keV photons in the target detector and of 60.01 keV photons (after passing source shield) in the second detector; Cascade CT—both cascade photons enter the target detector and secondary photons from either one or both photons are absorbed in the second detector.

the efficiency of the coincidence, $\epsilon_2=0.95$ is the estimated efficiency (not including the escape of characteristic Ge K x rays) of the second detector, $\epsilon(E)$ gives the efficiency of the second detector involving only the escape of characteristic Ge K x rays, N_{Ge} is atomic density in germanium, $A_2=0.205$ keV/channel is the channel width, μ_{air} is the attenuation coefficient in air, and d is an average path length of scattered photons in air.

The single Compton scattering, the most interesting process at present, predominantly produces photons in the middle- and high-energy range in the events line. The cross talk via photoelectron bremsstrahlung radiation in the target detector dominates at low energies. In Compton-scattering coincidence measurements, the bremsstrahlung must be calculated in order to estimate the low-energy end of the Compton spectrum. The escape of characteristic Ge $K\alpha$ and $K\beta$ x rays following the photoabsorbtion of the incident photon is also a single-scattering process. It is suitable for the determination of the absolute scale. The ''reverse'' single scattering has been also included in which considered photons pass through the source shield or are elastically scattered in the surrounding materials and first enter the second detector to be scattered into the target detector, where they are absorbed. The numbers of counts for reverse scattering are also calculated by Eq. (1), but the result obtained is interpreted as the number of counts at energy E_0-E.

The double scattering in single-cross-talk processes are low-intensity scattering processes. They contribute to the event line in the same energy region as the single Compton scattering. In Ref. [4], we proved the approximate proportionality of the double- and the single-backscattering Compton spectrum. Rayleigh-Compton and Compton-Rayleigh double scattering produce relatively more high-energy photons. The general expression for the number of events per channel produced by double scattering, in which energy E is deposited in the second detector [17,18], is given by

$$n_D(E_0,E)=2N_0\Delta D\sum_i \sum_j \int_{E-B_i}^{E_0-B_j} dE_1 \int_0^{\pi} d\varphi' \times\left(\int_0^1 SF_1 dx_1+\int_{-1}^0 SF_2 dx_1\right), \quad (2)$$

where

$$S=\frac{d^2\sigma_i(E_0,E_1,x_1)}{d\Omega_1' dE_1}\,\frac{d^2\sigma_j(E_1,E,x_2)}{d\Omega_2' dE},$$

$$F_1=\frac{1}{\mu(E_0)-\mu(E)/x_p}\,\frac{1}{\mu(E_1)-\mu(E)x_1/x_p},$$

and

$$F_2=\left(\frac{1}{\mu(E_0)-\mu(E)/x_p}-\frac{1}{\mu(E_0)-\mu(E_1)/x_1}\right)\times\frac{1}{\mu(E_1)-\mu(E)x_1/x_p},$$

$$x_2=\sqrt{1-x_p^2}\sqrt{1-x_1^2}\cos\varphi'+x_p x_1 .$$

σ_i and σ_j are the cross sections for Compton or Rayleigh scattering by electrons in the subshells i and j in germanium atoms $(1s,2s,2p,\ldots)$ of the target detector, respectively. E_1 and E are energies after the first and second scattering, respectively, $x_1=\cos\vartheta_1'$, $x_2=\cos\vartheta_2'$, and $x_p=\cos\vartheta_p$, where ϑ_1' and ϑ_2' are angles of the first and second scattering, respectively. B_i is the binding energy of shell i.

In deriving Eq. (2), normal incidence, infinite thickness, and infinite radius of the cylinder of the detector active volume were assumed. These approximations are well satisfied since $[1/\mu(E_0)\ll D$, where D is the thickness of the target, and $\mu R_f\geq 4$, where R_f is the radius of the target [19]]. The polarization effects, which occur in multiple scattering [20], were not taken into account.

B. Double-cross-talk processes

There are many scattering modes involved in the double cross talk among the two detectors. Each of the mentioned single- or double-scattering processes in one detector can be combined with the same or other scattering processes in the other detector. An exception is a combination of escapes of Ge K x rays from both detectors. Since we expect a very low intensity rate of all double-cross-talk processes, only the combinations of single-scattering processes are considered (Compton scattering, bremsstrahlung, and the escape of the Ge characteristic x ray), including a combination of elastic scattering of incident photons in the target detector with other single-scattering processes in the second detector.

We consider the single scattering processes of incident photons in the target detector in which $n_2(E_1)dE_1$ photons are produced, which enter the second detector, where they are also singly scattered. The number of produced tertiary photons of energy E_2 per unit energy interval, which reach the target detector and are absorbed therein, is

$$dn_1(E_2)=n_2(E_1)dE_1N_{Ge}\frac{\sum_j \dfrac{d^2\sigma_j(E_1,E_2,\cos\vartheta_p)}{d\Omega\, dE_2}}{\mu(E_1)+\mu(E_2)/\cos\vartheta_p}\exp[-\mu_{air}(E_2)d]\Delta\Omega_2\varepsilon_c\varepsilon_1\varepsilon(E_2), \tag{3}$$

where

$$n_2(E_1)=N_0N_{Ge}\frac{\sum_i \dfrac{d^2\sigma_i(E_0,E_1,\cos\vartheta_p)}{d\Omega dE_1}}{\mu(E_0)+\mu(E_1)/\cos\vartheta_p}\exp[-\mu_{air}(E_1)d]\Delta\Omega_1 .$$

In these relations σ_i and σ_j are the cross sections for bremsstrahlung and single Compton scattering by electrons in the subshells i and j in germanium atoms of the target and second detector, respectively. The energy deposited in the target detector is $E_{D1}=E_0-E_1+E_2$ while the energy deposited in the second detector is $E_{D2}=E_1-E_2$.

The expression in Eq. (3) is essentially the application of Eq. (1) to double cross talk among the two detectors. Because the number of (coincident) counts per channel $N(E_{D2})$, which deposited energy E_{D2} in the second detector, is required, the substitution $E_2=E_1-E_{D2}$ must be done as well as summation over all combinations of energies E_1 and E_2 which give the same value of E_{D2}. Therefore, the following transformation of Eq. (3) gives the required number of counts:

$$N(E_{D2})=\int_{E_{D2}}^{E_0-B_i}n_2(E_1)dE_1N_{Ge}\frac{\sum_j \dfrac{d^2\sigma_j(E_1,E_1-E_{D2},\cos\vartheta_p)}{d\Omega\, dE_1}}{\mu(E_1)+\mu(E_1-E_{D2})/\cos\vartheta_p}\exp[-\mu_{air}(E_1-E_{D2})d]\Delta\Omega_2\varepsilon_c\varepsilon_1\varepsilon(E_1-E_{D2}). \tag{4}$$

Integration of Eq. (4) must be performed using the following conditions for values of E_{D2}: $E_{D2}>E_{tr2}$, $E_{D2}\geqslant B_j$, $E_{D2}<E_0-E_{tr1}$, where E_{tr1} and E_{tr2} are the energies corresponding to the discriminator thresholds of the first and second detector, respectively.

Only bremsstrahlung or Compton photons of energy $E_{D2}+E_{K\alpha,\beta}$ emitted from the target detector, which induced the escape of the characteristic Ge K x rays in the second detector, can contribute to the events line at energy E_{D2}. Thus, it is necessary to modify Eq. (4) for the calculation of these events

$$N(E_{D2})=n_2(E_{D2}+E_{K\alpha,\beta})N_{Ge}\frac{\dfrac{d\sigma_{K\alpha,\beta}(E_{D2}+E_{K\alpha,\beta},\cos\vartheta_p)}{d\Omega}}{\mu(E_{D2}+E_{K\alpha,\beta})+\mu(E_{K\alpha,\beta})/\cos\vartheta_p}\exp[-\mu_{air}(E_{K\alpha,\beta})d]\Delta\Omega_2\varepsilon_c\varepsilon_1\varepsilon(E_{K\alpha,\beta}),$$

where $E_{K\alpha,\beta}$ is the energy of Ge $K\alpha$ and $K\beta$ x rays, and $\sigma_{K\alpha,\beta}(E,\cos\vartheta_p)$ is the cross section for their production by photons of energy E, which enter germanium at the angle ϑ_p with respect to the normal of its surface.

A combination of elastic scattering of incident photons in the active volume of the target detector and single scattering in the second detector makes a new subgroup of double-cross-talk processes. These processes show the same behavior as the reverse single scattering. They are also calculated by Eq. (1) and the result obtained is interpreted as the number of counts at energy E_0-E.

The results of calculations of all double-cross-talk processes (Fig. 3) show their very low intensity rates compared to the single-cross-talk processes. It should be noted that this ratio is proportional to the solid angle $\Delta\Omega_2$. However, in a Compton experiment in which a good definition of the scattering angle is desired, the solid angle is always small. Therefore, in Compton scattering measurements using two Ge detectors, the double cross talk between them can be neglected. The same conclusion can be expressed in another way [21]: the component of the response function of the second detector due to the escape of secondary radiation has a minor contribution to the Compton-scattering data.

C. Background due to the cascade transition

${}^{155}_{63}$Eu decays by β^- decay to the $5/2^+$, 3/2 states of ${}^{155}_{64}$Gd with a branching ratio of 26%. The states decay to the ground state by emission of γ photons of 86.54 keV or by cascade of emissions via the intermediate $5/2^-$ state in which photons of 26.53 keV and 60.01 keV are emitted. We examined the coincidence counts for events in which both photons enter the target detector and secondary radiation due to the scattering of one, of the other, or of both photons is absorbed in the second detector.

We consider a cascade in which emission of a photon of energy E_{01} is followed by emission of a photon of energy E_{02} with the probability $W_{Emis.}(E_{02})$. The number of coincidences per channel due to two cascade photons, which entered the target detector, recorded in the events line $E_1+E_2=E_0$, in which the second detector registered energy $E_S=E'_{01}+E'_{02}$, is given by

$$N_{Cascade\ CT}(E_S)=N_{01}A_2\int W(E_{01,}E'_1)W_{Emis.}(E_{02})\times W(E_{02,}E'_2)dE'_1, \qquad (5)$$

where N_{01} is the number of photons of energy E_{01} emitted during the experiment, and E'_{01} and E'_{02} are the energies of the scattered first (E_{01}) and second (E_{02}) incident photon after their scattering. $W(E_{0i},E'_i)$ is the probability per unit energy interval and per emitted incident cascade photon of energy E_{0i}, for total absorption, given by

$$W(E_{0i},E'_i)=\epsilon_c\epsilon_2\epsilon(E_{0i})\,\delta(E'_i)$$

or for single scattering, given by

$$W(E_{0i},E'_i)=\frac{\sum_j d^2\sigma_j/d\Omega\,dE}{\mu(E_0)+\mu(E)/\cos(\vartheta_p)}\,\frac{\Delta D}{A_2}.$$

Equation (5) with the numerical values $E_{01}=26.53$ keV, $E_{02}=60.01$ keV, $N_{01}=0.01N_0$, and $W_{Emis.}(E_{02})=1.03\times10^{-2}$ gives the spectrum $N_{Cascade\ CT}(E_S)$ shown in Fig. 3. The very low contribution of the cascade transition is essentially caused by the relatively low branching ratio of the cascade.

The only visible contribution of the cascade transition in the experimental data is the peak (denoted Cascade in Fig. 3) due to the coincidence absorption of the first cascade photon (E_{01}) in the target detector and the second cascade photon (E_{02}) in the second detector. This peak could be eliminated by a more efficient source shield.

D. The differential cross sections

The pulse-height distribution n_{exp} in the events line $E_1+E_2=E_0$ has been corrected for nonsingle Compton scattering in the target detectors, and the obtained spectrum has subsequently been converted into the values of experimental cross sections (Fig. 4) using the following relation:

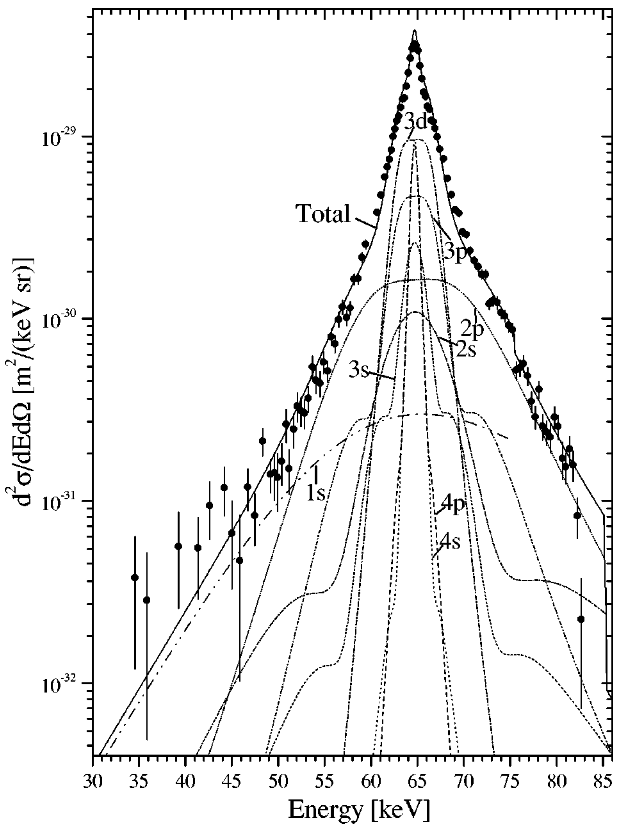

FIG. 4. The experimental values of the differential cross section $d^2\sigma/d\Omega\,dE$ for Compton scattering in germanium and the theoretical values calculated using the impulse approximation with the hydrogenlike wave functions.

$$\left(\frac{d^2\sigma}{d\Omega\,dE}\right)_{expt}=\frac{n_{expt}[\mu(E_0)+\mu(E)/\cos(\vartheta_p)]}{N_0\Delta D}.$$

The obtained experimental Compton spectrum is in the energy range from 33 keV to 82 keV, which is 58% of the complete energy range. The low-energy end is limited by the bremsstrahlung of photoelectrons, while the upper end is limited by the threshold of the discriminator of the target detector. The range of values of the differential cross section is about 6×10^3. The uncertainties of the data shown in Fig. 4 are statistical only. The uncertainty of the absolute scale is about 4% and the main reason for it is the not quite precise knowledge of efficiency of the Ge detectors. The results of theoretical calculation of the cross section based on the impulse approximation using the hydrogenlike wave functions are also shown in Fig. 4. The theoretical curve obtained is convoluted with the function of the detector resolution, represented by the Gaussian function. Fair agreement between the theoretical and experimental cross section values is obtained. The disagreement appears in the same energy regions as in the last two experiments [3,4], but it is more pronounced. That is due to the higher energy of incident pho-

TABLE I. Total Compton cross sections from each subshell and their sum calculated using the impulse approximation with hydrogenlike functions.

Subshell	$1s$	$2s$	$2p$	$3s$	$3p$	$3d$	$4s$	$4p$	Sum
$\frac{d\sigma_i}{d\Omega}\left[10^{-30}\frac{m^2}{sr}\right]$	6.38	8.93	27.31	9.17	27.62	46.05	9.21	9.21	143.89

tons. The main reason for the disagreement arises from the application of the hydrogenlike wave functions in the impulse approximation.

The values of $d\sigma_i/d\Omega$ at 174°, calculated in the impulse approximation for each subshell in the energy range 33–82 keV (for $1s$ shell to 75.44 keV to take into account the binding energy), are shown in Table I. These values are almost equal to the integrals over the whole energy regions because the differential cross sections outside the region are very small. The sum of the values is also shown in Table I. It compares well with the experimental value for the same energy region which amounts to $(138\pm 7)\times 10^{-30}$ m^2/sr (the error is mainly due to the uncertainly of the absolute scale).

III. CONCLUSIONS

The coincidence measurements give very clean and reliable results for the differential cross sections of Compton scattering. The measured pulse-height distribution contains very little background. To obtain the single-scattering Compton spectrum, only small corrections for other cross-talk processes were needed. The conversion to the differential cross sections is accurate to better than 4% on absolute scale. The differential cross sections have been obtained in a relatively much wider energy interval than what could be expected in a singles-mode measurement. The low-energy end is limited by the bremsstrahlung process, while the upper-energy end is limited by the threshold of the target detector discriminator. Almost any radioactive x and γ source could be applied in the coincidence Compton experiments, independently of its complexity. Even crossover transitions with relatively weak fast cascades can be used. In many cases, the multiline sources will be convenient for a simultaneous measurement at several incident energies. The contribution of the double-cross-talk processes to the Compton spectrum is very weak in coincidence measurements.

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