Quantum teleportation using quantum nondemolition technique

D. B. Horoshko* and S. Ya. Kilin
Institute of Physics, Belarus National Academy of Sciences, Minsk 220072, Belarus

(Received 31 August 1999; published 10 February 2000)

We propose a scheme and a protocol for quantum teleportation of a single-mode optical field state, based on the entanglement produced by the quantum nondemolition (QND) interaction. We show that the recently attained results in the QND technique allow one to perform the teleportation in the quantum regime. We also show that an application of QND coupling to squeezed fields will significantly improve the quality of teleportation for a given degree of squeezing.

PACS number(s): 03.67.−a, 03.65.−w, 42.50.Dv

Quantum teleportation is a transport of a quantum state of a system to another similar system via a classical channel [1]. The principles of quantum mechanics demand that for realization of such a transport, the sender and the receiver must have two ancillary quantum systems, which are strongly quantum-mechanically correlated (entangled). Quantum teleportation is a fundamental phenomenon of quantum world and also one of the key procedures in the rapidly growing area of quantum information [2]. Recently several teleportation schemes have been proposed [3,4] and the successful realization of some of them has been reported: teleportation of the polarization state of a single photon [5,6], of the state of a single-mode optical field [7,8], and of nuclear spin state [9]. In this paper we propose a new scheme for quantum teleportation of the state of single-mode optical field. In contrast to the existing schemes [7,8], where a superposition of two squeezed fields has been used as a source of entanglement, we propose to use the entanglement produced by the quantum nondemolition (QND) interaction [10,11], realized in the last years for optical fields in various media [12], the best results being obtained for coupling two optical beams by parametric interaction in a $\chi^{(2)}$ crystal [13] or using cold atoms in a trap [14].

Our scheme is depicted in Fig. 1. The nonlinear QND interaction produces the following coupling between two bright coherent fields E_a and E_b [11]:

$$\tilde{E}_a = E_a + \frac{g}{2}(E_b^+ + E_b), \tag{1}$$

$$\tilde{E}_b = E_b + \frac{g}{2}(E_a^+ - E_a), \tag{2}$$

where $\tilde{E}_a$ and $\tilde{E}_b$ are the outgoing fields and g is the QND gain. It is easy to find that the quadratures $X = E^+ + E$ and $Y = i(E^+ - E)$ of both fields are transformed as

$$\tilde{X}_a = X_a + gX_b, \quad \tilde{Y}_a = Y_a,$$
$$\tilde{X}_b = X_b, \quad \tilde{Y}_b = Y_b - gY_a.$$

The quadratures X_b and Y_a are the so-called QND variables, which are not affected by coupling. The conjugated quadratures X_a and Y_b become correlated with X_b and Y_a, respectively, and for infinitely large QND gain, or for infinitely squeezed Y_b (X_a), the measurement of Y_b (X_a) gives a result proportional to that of measurement of Y_a (X_b). This is the main idea of a QND measurement. In our approach this measurement is not performed, but the entanglement produced by QND coupling is used for quantum teleportation, as described below.

One of the beams, resulting from the QND interaction ($\tilde{E}_a$) passes to the sender Alice, and another one ($\tilde{E}_b$) passes to the receiver Bob for further reconstruction of the teleported state. The protocol of quantum teleportation is as follows. Alice superimposes the field $E_{\rm in}$, which is to be teleported, with the field $\tilde{E}_a$ at a beam splitter with the transmittance ϵ, and, using homodyne photodetection, measures the X quadrature of the reflected (for $E_{\rm in}$) field and the

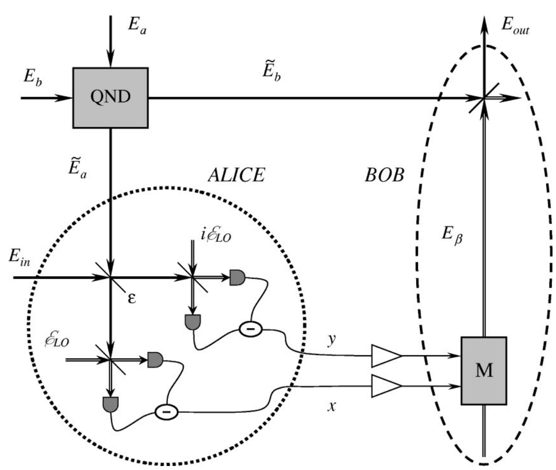

FIG. 1. Schematic of quantum teleportation arrangement. Two entangled beams $\tilde{E}_a$ and $\tilde{E}_b$ are produced by the QND device. The incoming field $E_{\rm in}$ is mixed by Alice with one of the entangled beams and the quadratures of the resulting fields are measured by two homodyne detectors, using a purely real local oscillator field $\mathcal{E}_{\rm LO}$ and a purely imaginary one $i\mathcal{E}_{\rm LO}$. The measurement results are sent via a classical channel to Bob who prepares a laser beam in a proper coherent state by means of the amplitude and phase modulator *M*. This coherent field is superimposed with the second entangled beam, resulting in the output field. The state of the latter reproduces closely the state of the incoming field.

*Electronic address: horoshko@ifanbel.bas-net.by

Y quadrature of the transmitted field, obtaining the values

$$x=\epsilon^{1/2}(X_a+gX_b)+(1-\epsilon)^{1/2}X_{\rm in}, \tag{3}$$

$$y=\epsilon^{1/2}Y_{\rm in}-(1-\epsilon)^{1/2}Y_a, \tag{4}$$

where the field variables are written in the Wigner representation. The measured values are sent to Bob via a classical channel, and upon receiving them Bob prepares the bright laser beam E_β by means of phase and amplitude modulation in a coherent state with the amplitude $\beta=\Gamma G(x+igy)/2$, where $\Gamma\gg 1$ and G are some real gains. After that Bob superimposes the field E_β with the beam $\tilde{E}_b$ at a beam splitter with very low transmittance $\epsilon'=\Gamma^{-2}\ll 1$, so that the reflected field $\tilde{E}_b$ is just shifted by $G(x+igy)/2$, but no noise is added. The result of interference of the reflected field $\tilde{E}_b$ and the transmitted field E_β is the output $E_{\rm out}$:

$$\begin{aligned}2E_{\rm out}&=2\Gamma^{-1}E_\beta-2\tilde{E}_b=G(x+igy)-X_b-iY_b+igY_a\\ &=G\epsilon^{1/2}X_a+(Gg\epsilon^{1/2}-1)X_b+G(1-\epsilon)^{1/2}X_{\rm in}\\ &\quad+ig[1-G(1-\epsilon)^{1/2}]Y_a-iY_b+iGg\epsilon^{1/2}Y_{\rm in}.\end{aligned} \tag{5}$$

Now, if the electronic gain G and the transmittance ϵ satisfy for a given QND gain g the relations $G=(1-\epsilon)^{-1/2}$, $g=(1-\epsilon)^{1/2}\epsilon^{-1/2}$, the output field reads

$$2E_{\rm out}=X_{\rm in}+iY_{\rm in}+g^{-1}X_a-iY_b, \tag{6}$$

that is, for $\langle X_a\rangle=\langle Y_b\rangle=0$, it reproduces the incoming field with additional noise caused by quadratures X_a and Y_b. For infinite QND gain g and infinitely squeezed quadrature Y_b the teleportation is ideal. However, even for finite QND gain and not squeezed Y_b the proposed teleportation scheme works in an essentially quantum regime.

This can be seen from comparing this scheme to classical teleportation. In the latter case Alice and Bob do not have correlated ancillary systems and all they can do in order to accomplish a true reproduction of the average values of $X_{\rm in}$ and $Y_{\rm in}$, is to measure these quadratures with the uncertainty imposed by the Heisenberg's principle and create a coherent field with the average values of quadratures equal to the measured ones. So the protocol of classical teleportation is as follows. Alice divides the incoming field $E_{\rm in}$ in two parts by a 50:50 beam splitter and measures the X quadrature of the reflected field and the Y quadrature of the transmitted field, obtaining the values

$$x=\frac{1}{\sqrt{2}}X_{\rm in}+\frac{1}{\sqrt{2}}X_v, \tag{7}$$

$$y=\frac{1}{\sqrt{2}}Y_{\rm in}-\frac{1}{\sqrt{2}}Y_v, \tag{8}$$

where $E_v=(X_v+iY_v)/2$ is the vacuum field at the unused port of the beam splitter. The measured values are sent to Bob who creates a coherent state

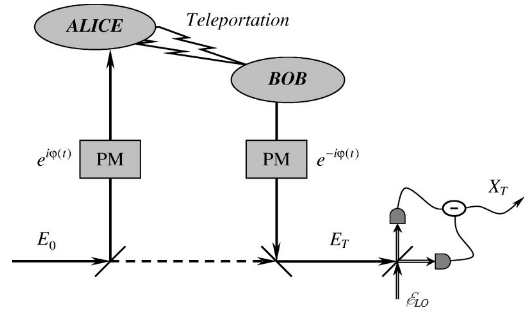

FIG. 2. The verification procedure. The verifier, completely independent of Alice and Bob, prepares an ensemble of states for field E_0. This field can be directly sent to the homodyne detector (dashed line) for measuring the distribution of its real quadrature. Alternatively it can be sent to Alice for teleportation with a subsequent measurement of the field received from Bob (solid line). The difference in the noise level between two paths is a measure of the teleportation success. The phase shifts, introduced by the phase modulators (PM) allow one to control the noise added to the corresponding quadrature of the teleported field.

$$\begin{aligned}2E_{\rm out}&=\sqrt{2}(x+iy)+2E_w\\ &=X_{\rm in}+iY_{\rm in}+X_v-iY_v+X_w+iY_w,\end{aligned} \tag{9}$$

where $E_w=(X_w+iY_w)/2$ is the vacuum fluctuation inevitably added at the reconstruction stage [e.g., as in the previous protocol, Bob can superimpose a strong coherent field $2E_\beta=\Gamma\sqrt{2}(x+iy)$ with vacuum field E_w at a low transmittance $\epsilon'=\Gamma^{-2}$ beam splitter and use the reflected (for E_w) field as the output]. Now, to compare quantum [Eq. (6)] and classical [Eq. (9)] teleportation, we must calculate a quantitative measure of teleportation success, connected with some verification procedure.

Let us consider the verification scheme shown in Fig. 2, similar to that of Ref. [7]. The verifier Victor, independent of Alice and Bob, prepares an ensemble of states of field E_0. Victor can directly measure the average value and the variance of the field quadrature X_0. Alternatively he can shift the phase of the field by a random phase $\varphi(t)$ and give it to Alice who teleports the state to Bob. After receiving the state $E_{\rm out}$ from Bob, Victor again shifts the phase, this time by $-\varphi(t)$ and measures the X quadrature of the obtained field E_T. If the teleportation is ideal, the average value and variance of X_T will be the same as for X_0. The teleportation scheme in this paper gives the following expression for X_T:

$$X_T=X_0+g^{-1}X_a\cos\varphi-Y_b\sin\varphi. \tag{10}$$

So for $\langle X_a\rangle=\langle Y_b\rangle=0$ the average value of X_T is equal to that of X_0. The variance of X_T is given by

$$\langle(\delta X_T)^2\rangle=\langle(\delta X_0)^2\rangle+g^{-2}\langle(\delta X_a)^2\rangle\overline{\cos^2\varphi}+\langle(\delta Y_b)^2\rangle\overline{\sin^2\varphi}, \tag{11}$$

where $\delta X=X-\langle X\rangle$ and the bar denotes averaging over the random phase shift. If the phase is distributed homogeneously in $[0,2\pi]$, then $\overline{\cos^2\varphi}=\overline{\sin^2\varphi}=\frac{1}{2}$, and

$$N_{\text{add}} \equiv \langle (\delta X_T)^2 \rangle - \langle (\delta X_0)^2 \rangle$$

$$= \frac{1}{2g^2} \langle (\delta X_a)^2 \rangle + \frac{1}{2} \langle (\delta Y_b)^2 \rangle, \tag{12}$$

where N_{add} is the noise added to the teleported light. We use the added noise as a measure of teleportation success instead of commonly used fidelity $F = \langle \psi_{\text{in}} | \rho_{\text{out}} | \psi_{\text{in}} \rangle$ (where $|\psi_{\text{in}}\rangle$ is the incoming state and ρ_{out} is the density matrix of the output), for the two following reasons. First, fidelity generally depends on the incoming state, while for the teleportation of field state, which is a linear in field transformation, the output field is always the incoming one plus some noise; so it is natural to describe the teleportation by this added noise, independent of the incoming state. Second, the output state can differ from the input one in two ways: in averages (displacement) and in variances of quadratures (distortion). Fidelity (as well as the parameters introduced in Ref. [8]) takes into account both these effects. However, classical teleportation always can be arranged so that the average values (of quadratures) are transported correctly. So only a measure of state distortion is necessary for characterizing the quantum nature of teleportation, and the added noise just provides such a measure. The added noise is related closely to other measures of quantum teleportation: it coincides with one half of the sum of conditional variances $V_{cv}^{+} + V_{cv}^{-}$ of Refs. [8,15] in a particular case of unity gain, and with $\sigma_W - 1$ of Ref. [7] in a particular case of coherent input state (where σ_W is the variance of the Wigner function of the output state).

It is easy to find from Eq. (9) that for classical teleportation

$$N_{\text{add}}^{\text{cl}} = \frac{1}{2} \langle (\delta X_v)^2 \rangle + \frac{1}{2} \langle (\delta Y_v)^2 \rangle + \frac{1}{2} \langle (\delta X_w)^2 \rangle + \frac{1}{2} \langle (\delta Y_w)^2 \rangle \geqslant 2 \tag{13}$$

for any states of fields E_v and E_w, which can be seen from the inequality $a^2 + b^2 \geqslant 2ab$ and the Heisenberg's uncertainty principle. So, to reach the quantum regime of teleportation, one must reach added noise $0 \leqslant N_{\text{add}} < 2$, under condition that $\langle X_T \rangle = \langle X_0 \rangle$ for any phase φ. In the proposed scheme it can be obtained by using coherent fields E_a and E_b (with imaginary and real amplitudes, respectively) and QND gain $g > 1/\sqrt{3}$.

The entanglement produced by a QND device (or its quantum state preparation regime) is usually characterized by the conditional variance of signal given a measured value of meter, $V_{s|m}$ [16]. For the transformation described by Eqs. (1), (2) and coherent input fields E_a and E_b this variance is given by

$$V_{s|m} \equiv \langle (\delta \tilde{X}_b)^2 \rangle - \frac{|\langle \delta \tilde{X}_a \delta \tilde{X}_b \rangle|^2}{\langle (\delta \tilde{X}_a)^2 \rangle} = \frac{1}{1+g^2} \tag{14}$$

and the condition $g > 1/\sqrt{3}$ corresponds to the condition $V_{s|m} < \frac{3}{4}$. In a recent QND experiment using cold atoms in a trap as a nonlinear medium [14] the value $V_{s|m} = 0.45$ has been measured ($V_{s|m} = 0.37$ with corrections for optical losses). At the same time, our requirements for field phases are exactly that needed for the QND interaction based on the cross-phase modulation, implemented in this experiment. For QND schemes based on $\chi^{(2)}$ crystals, a value $V_{s|m} = 0.65 - 0.7$ has been achieved [13], without restrictions on the phases of interacting beams. These results speak about a very good possibility to reach the quantum regime of teleportation using the existing experimental techniques and the protocol presented in this paper.

Let us briefly compare our scheme to that using squeezed states [7,8]. In classical teleportation the output field is a sum of the input one and four additional noises (quantum duties [4]), as shown by Eq. (9), the noise added to a random quadrature being always more than 2. Quantum teleportation can be realized by using correlated ancillary fields E_v and E_w, produced by mixing two squeezed fields E_1 and E_2 at a 50:50 beam splitter, so that in Eq. (9) $X_v + X_w = \sqrt{2} X_1$ and $Y_v - Y_w = \sqrt{2} Y_2$, and the added noise is given by

$$N_{\text{add}}^{\text{sq}} = V_1 + V_2, \tag{15}$$

where V_1 and V_2 are the variances of the quadratures X_1 and Y_2, respectively. When the quadratures X_1 and Y_2 are squeezed, the added noise becomes less than the classical value of 2. In our approach the ancillary fields are correlated via a QND interaction. Two QND variables (X_b and Y_a) are automatically cancelled in the output field due to QND correlations. One of the remaining noises (X_a) can be made as small as desired by increasing the QND gain and/or by squeezing this quadrature. The noise of Y_b can be reduced by squeezing this quadrature only. The added noise in this case reads ($V_{s|m} < 1$)

$$N_{\text{add}}^{\text{QND}} = \frac{1}{2} \frac{V_{s|m}}{1 - V_{s|m}} V_a + \frac{1}{2} V_b, \tag{16}$$

where V_a and V_b are the variances of the quadratures X_a and Y_b, respectively. Comparing Eq. (16) with Eq. (15) we see that with decreasing conditional variance the QND coupling decreases significantly the requirements for the degree of squeezing of ancillary beams (so that for $V_{s|m} < \frac{3}{4}$ they can be not squeezed at all).

It should be noted that a principal possibility to use QND coupling for quantum teleportation has been pointed out in Ref. [17]. However this paper considers only a QND gain $g = 1$ and two ideally squeezed ancillary beams, in which case no benefits from QND coupling can be seen, compared to mixing these beams at a beam splitter. On the other hand, as a QND device can be approximated by mixing the signal beam with a squeezed one on a beam splitter [16], the teleportation protocols relying on two squeezed states can be considered as a particular case of our approach.

In summary, we have described a protocol for quantum teleportation of a single mode optical field using QND coupling as a source of entanglement of ancillary laser beams. In our scheme the QND device is used in the quantum state preparation regime, and the recent achievements for realizing this regime satisfy the requirements for quantum teleportation very well. On the other hand, the existing schemes for quantum teleportation of single-mode field state are limited by the restricted degree of squeezing, available at present, and using QND coupling for squeezed ancillary beams can provide further improvement of quantum teleportation technique.

We thank T. Ralph and B. Buchler for valuable comments. This work was supported by INTAS, Grant No. 96-0167.

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