Comment on "Čerenkov effect and the Lorentz contraction"

G. Cavalleri* and E. Tonni
Dipartimento di Matematica e Fisica, Università Cattolica del Sacro Cuore, via Trieste 17, I-25121 Brescia, Italy
(Received 23 March 1999; published 4 January 2000)

In a recent paper [Phys. Rev. A **55**, 1647 (1997)], Pardy has proposed an experiment to measure the Lorentz contraction starting from the zeros of the power spectral density of the Čerenkov radiation emitted by two bunches of electrons. It is shown that this is not the case since all the calculations and results are relative to the laboratory system S. The Lorentz contraction would be measured only if the rest length $l_0=\gamma l$ in the system S' comoving with the two bunches of electrons could be performed.

PACS number(s): 03.30.+p, 41.60.Bq

In an interesting and stimulating paper, Pardy [1] has derived the power spectra formula of Čerenkov radiation in the case of a system of two (bunches of) equal charges. The framework used is the source theory devised by Schwinger, Tsai, and Erber [2]. Pardy has also investigated the modification of the spectrum of the two-charge Čerenkov radiation because of the radiation correction in the photon propagator. We disagree only with the claim that the knowledge of the spectral formula can be used to verify the Lorentz contraction of the relativistic length. Actually, the experiment proposed by Pardy implies measurements in a single reference system S, whereas the Lorentz contraction implies measurements in two reference systems, one in the laboratory system S and the other in S' at rest with the two (bunches of) charges moving with velocity $\mathbf{v}$ with respect to S. Let us clarify this point.

Pardy [1] proposes to use either a linear or a circular accelerator to accelerate two bunches of electrons to a relativistic speed v. The distance l between the two bunches measured in the laboratory system S does not depend on v since, if the motion for the two bunches is the same (in steady-state conditions), the position $x_2(t)$ reached at time t by bunch 2 is simply related to that $x_1(t)$ of bunch 1 by $x_2(t)=x_1(t)+l$. The situation is completely different from the case of an accelerated rigid body (according to Born [3]), where the Lorentz contraction acts. For example, a rigid rod accelerated longitudinally contracts toward the point P of application of the external force and P moves as a pointlike particle having the same mass m as that of the rod and acted upon by the same force F [4]. Consequently, a rod pushed by F accelerates, on an average over the velocities of its points, less than the same rod pulled by the same F [4]. On the contrary, free particles keep the same distance l if accelerated by the same field. The rest distance l_0 measured by S' moving with the final speed v of the two bunches of electron is

$$l_0=(1-v^2/c^2)^{-1/2}l=\gamma l. \tag{1}$$

Thus, Lorentz' contraction is real, as supported by Pardy [1] and Rindler [3], in the sense that if l and l_0 are the lengths measured in the laboratory S and in the comoving system S', respectively, Eq. (1) always remains valid. The point is that the rest length l_0 remains the same for a rod before and after its acceleration, i.e., if initially measured in S when at rest in S, and then in S' when at rest in S'. For free particles the behavior is different. If they are subjected to the same acceleration $\mathbf{a}(t)$, it is

$$x_2(t)-x_1(t)=l+\int_0^t dt'\int_0^{t'} dt''a(t'') - \left[0+\int_0^t dt'\int_0^{t'} dt''a(t'')\right] = l, \tag{2}$$

as recognized by Pardy himself [Eqs. (48) and (49) of Ref. [1]]. In this case what remains the same is the length l before and after the acceleration if measured in the laboratory system S. Now, since the rest length l_0 (measured in S') satisfies Eq. (1), l_0 is larger than the initial distance between the two particles (or bunches of particles). Consequently, if an inextensible thread were connected with the two particles, the thread would break. This is an old relativity problem, presented in the 1950s to some Nobel Laureates in physics. Since most of them gave the wrong answer, Dewan and Beran [5] gave the right answer considering two identically constructed rockets initially at rest in an inertial frame S and connected with a silk thread. At a prearranged time both rockets are simultaneously (with respect to S) fired up, so that their velocities with respect to S are always equal (even though they are functions of time). This means that with *respect to S* the distance between the two rockets does not change [see Eq. (2)] even when they speed up to relativistic velocities. Consequently, the thread (assumed to be taut at the start) cannot contract and breaks. This correct prediction was not understood by Nawrocki [6] and was further clarified by Dewan [7] and definitely by Romain [8] on the basis of the nonconservation of simultaneity between separate events. This last relativistic property has been shown by Mansouri and Sexl [9] to be a consequence of the internal synchronization of the clocks (belonging to the *same* reference system). If one uses the external synchronization, there is conservation of the simultaneity in special relativity and the relevant transformations are those of Tangherlini [10]. But in this formulation, the bodies in motion with respect to S (which is the single system that has performed an *internal*

*Electronic address: g.cavalleri@dmf.bs.unicatt.it

synchronization while all the other systems perform an *external* synchronization by means of local coincidences of their clocks with those of S) undergo the Lorentz contraction. Consequently, the thread tends to contract and breaks because its ends are kept at the same constant distance l_0.

The calculations performed by Pardy [1] of the power spectral radiation emitted by the Cerenkov mechanism of the two-charge system present some zeros for angular frequencies ω_n given by

$$\omega_0, \quad \frac{\omega_n a}{2v} = \frac{2n-1}{2}\pi, \quad n=1,2,3,\ldots, \tag{3}$$

where the length $a=|\mathbf{a}|$ is defined in Eqs. (11) and (12) of Ref. [1] as the distance between the two bunches in the laboratory system S. In fact, the current density $\mathbf{j}$ is defined as

$$\mathbf{j}=e\mathbf{v}[\delta(\mathbf{x}-\mathbf{v}t)+\delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t)]. \tag{4}$$

It follows that the a value derivable from Eq. (3), i.e.,

$$a=\frac{(m-n)2\pi v}{\omega_m-\omega_n}, \tag{5}$$

must be equal to l and not, as written by Pardy in his Eq. (28), to $\gamma^{-1}l$.

We conclude that Pardy's proposal to use the Čerenkov power spectral density relevant to two bunches of electrons to measure the Lorentz contraction would be correct only if the rest length l_0 in the system S' comoving with the bunches could be measured.

[1] M. Pardy, Phys. Rev. A **55**, 1647 (1997).

[2] J. Schwinger, W.Y. Tsai, and T. Erber, Ann. Phys. (N.Y.) **96**, 303 (1976).

[3] See W. Rindler, *Essential Relativity* (Springer-Verlag, New York, 1977).

[4] G. Cavalleri and G. Spinelli, Nuovo Cimento B **66**, 11 (1970).

[5] E. Dewan and M. Beran, Am. J. Phys. **27**, 517 (1959).

[6] P.J. Nawrocki, Am. J. Phys. **30**, 771 (1962).

[7] E.M. Dewan, Am. J. Phys. **31**, 383 (1963).

[8] J.E. Romain, Am. J. Phys. **31**, 576 (1963).

[9] R. Mansouri and R.V. Sexl, Gen. Relativ. Gravit. **8**, 497 (1977); **8**, 515 (1977); **8**, 809 (1977). For this new formulation of special relativity, see also G. Cavalleri and C. Bernasconi, Nuovo Cimento B **104**, 545 (1989); C.M. Will, Phys. Rev. D **45**, 403 (1992).

[10] F.R. Tangherlini, Nuovo Cimento Suppl. **20**, 1 (1961). For an application of the Tangherlini formulation, see G. Cavalleri and E. Tonni, Phys. Rev. E **57**, 3478 (1998); E. Tonni, Nuovo Cimento B **114**, 561 (1999).