Cavity electromagnetically induced transparency of driven-three-level atoms: A transparent window narrowing below a natural width

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Steady-state dynamics of a Λ atom in a ring cavity driven by two coherent fields are studied for arbitrary detunings, arbitrary incoherent pumping, and coherent driving intensities. Effects of both cavity and effective atom number on electromagnetically induced transparency (EIT) are pointed out. New physical pictures for cavity EIT are given in terms of collective cooperative coefficients and dispersion experienced by the probe. In the regime of smaller collective cooperative coefficients, an absorption-gain profile is reduced to that of a general EIT estimated by the imaginary part of a corresponding dipole moment, and its transparency window is directly proportional to power broadening, if the total Rabi frequency is large enough. But in the region of larger collective cooperative coefficients which means a dense atomic medium, longer optical path, or high-Q cavity, EIT is determined not only by the imaginary part but also by the real part of the corresponding dipole moment, which results in the possibility of observing an EIT central peak with a subnatural width, while there may be nearly no power broadening.

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I. INTRODUCTION

In recent years, there has been considerable interest in the quantum interference and coherence effects in a multilevel atom system induced by coherent electromagnetic field(s). Many related phenomena such as electromagnetically induced transparency (EIT) [1], lasing without inversion (LWI) [2], refractive index enhancement without absorption [3], giant nonlinearity [4], and spontaneous emission cancellation [5] have been predicted and subsequently experimentally demonstrated. Specifically, in almost all work done for EIT, an optically thin ensemble of atoms or molecules is generally involved with a configuration of a stronger driving field and a weaker probe field. The key feature is one central transparent peak with a width of about $2(\sqrt{\Omega^2+\gamma^2}-\gamma)$ induced by quantum coherence and interference. For example, in Λ three-level atoms, the width of the transparent peak can be further approximated as 2Ω because the total Rabi frequency $\Omega \gg \gamma$ in a typical EIT. In that sense, power broadening determines the width of the transparent window and is always larger than the characteristic parameter of spontaneous emission γ.

Can we obtain a transparent peak with a subnatural and power-broadened free width? The answer, at least theoretically, is yes. A possible direct way of doing so is by having the width such that $(\Omega/\gamma)\Omega \ll \Omega \ll \gamma$ approximately if both $\Omega \ll \gamma$ and the Rabi frequency of the drive is kept much larger than that of the probe. That procedure is not favorable experimentally, because of poor EIT [1(b)] and poor signal-to-noise ratio. A possible indirect way of obtaining a transparent peak is studied in this paper, and uses a cavity and/or an optically dense medium, which may result in a perfect EIT due to $\Omega \gg \gamma$ and the narrowing of its central peak. In this case, the width of a central transparent peak is dramatically modified as $2[\sqrt{\Omega^2+\gamma^2(1+C)^2}-\gamma(1+C)]$, where is referred to as the collective cooperative coefficient determined by an optically dense medium, a longer optical path and/or a high-Q cavity. If $\gamma \ll \Omega \ll (1+C)\gamma$, the transparent peak width is approximated as $\{\Omega/[1+C)\gamma\}\Omega \ll \Omega$. Accordingly, power broadening can be substantially reduced while a subnatural transparent peak may be observed. As will be seen, the essential physics of an indirect approach is twofold: (1) Absorption cancellation due to quantum interference. (2) The importance of dispersion enhanced by a cavity and/or a coherently prepared dense medium, even though dispersion does not play any important role in the direct approach discussed above. We note that there are some publications about spectroscopy in optically thick samples [6], as well as the importance of dispersion [3]. This paper, however, emphasizes aspects of dispersion enhanced by a cavity and/or a dense medium. Moreover, a different physical picture from recent work [7] is given on the basis of quantum interference.

II. MODEL

For maximizing the utility of the theory given here, we start from one general model of a driven Λ atom (see Fig. 1),

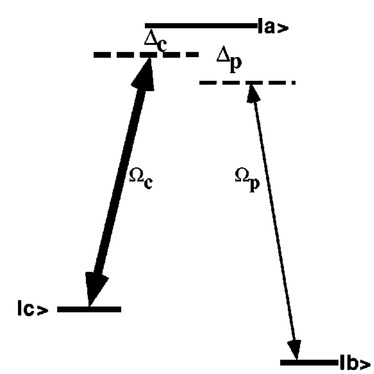

FIG. 1. The atom level scheme under consideration for EIT.

with an arbitrary incoherent pump scheme and any detuning. The equations for the density-matrix elements [8] for the atomic system are

$$\frac{\partial\rho_{bb}}{\partial t}=R_{bb}\rho_{bb}+R_{ba}\rho_{aa}+R_{bc}\rho_{cc}+i\frac{g_p}{2}(\rho_{ab}a_p^*e^{i\phi_p}-\rho_{ba}a_pe^{-i\phi_p}),$$

$$\frac{\partial\rho_{cc}}{\partial t}=R_{cb}\rho_{bb}+R_{ca}\rho_{aa}+R_{cc}\rho_{cc}-i\frac{g_c}{2}(\rho_{ca}a_ce^{-i\phi_c}-\rho_{ac}a_c^*e^{i\phi_c}),$$

$$\rho_{bb}+\rho_{aa}+\rho_{cc}=n_0V, \tag{1}$$

$$\frac{\partial\rho_{ba}}{\partial t}=-\Gamma_{ba}\rho_{ba}+i\frac{g_p}{2}a_p^*e^{i\phi_p}(\rho_{aa}-\rho_{bb})-i\frac{g_c}{2}a_c^*e^{i\phi_c}\rho_{bc},$$

$$\frac{\partial\rho_{bc}}{\partial t}=-\Gamma_{bc}\rho_{bc}+i\frac{g_p}{2}a_p^*e^{i\phi_p}\rho_{ac}-i\frac{g_c}{2}a_ce^{-i\phi_c}\rho_{ba},$$

$$\frac{\partial\rho_{ac}}{\partial t}=-\Gamma_{ac}\rho_{ac}+i\frac{g_c}{2}a_ce^{-i\phi_c}(\rho_{cc}-\rho_{aa})+i\frac{g_p}{2}a_pe^{-i\phi_p}\rho_{bc}.$$

If the ensemble averaging of $A_i\sigma_{jk}$ can be decoupled, the coherent interacting Hamiltonian under the rotating wave approximation is

$$\frac{H}{\hbar}=\Delta_c\sigma_{cc}+\Delta_p\sigma_{bb}-\left[\frac{g_c}{2}A_c\sigma_{ac}e^{-i\phi_c}+\frac{g_p}{2}A_p\sigma_{ab}e^{-i\phi_p}+\text{H.c.}\right], \tag{2}$$

and the dipole moment decay rates are

$$\Gamma_{ba}=\gamma_{ba}+i\Delta_p,$$
$$\Gamma_{bc}=\gamma_{bc}+i(\Delta_p-\Delta_c), \tag{3}$$
$$\Gamma_{ac}=\gamma_{ac}-i\Delta_c,$$

where R_{ij} represents a repopulating rate due to spontaneous emission or an incoherent pump from $|j\rangle$ to $|i\rangle$, and Γ_{ij} is a complex relaxation rate of the corresponding dipole moment ρ_{ij}. g_c,g_p and ϕ_c,ϕ_p are referred to as Rabi frequencies and phases of the atom interacting with two laser fields at frequencies ω_c and ω_p, respectively. Notice that the detunings are defined in a usual way as $\Delta_c=\omega_c-\omega_{ac}$ and $\Delta_p=\omega_p-\omega_{ab}$, respectively and the laser fields have $a_i=\langle A_i\rangle$. n_0 is the atomic density, and V is the effective volume of atoms interacting with the laser fields. For the laser and cavity system, we have

$$\frac{\partial a_p}{\partial t}=-K_pa_p+i\frac{g_p}{2}e^{i\phi_p}\rho_{ab}+\sqrt{2\kappa_p}a_p^{\text{in}},$$

$$\frac{\partial a_c}{\partial t}=-K_ca_c+i\frac{g_c}{2}e^{i\phi_c}\rho_{ac}+\sqrt{2\kappa_c}a_c^{\text{in}}, \tag{4}$$

$$a_i^{\text{out}}=-a_i^{\text{in}}+\sqrt{2\kappa_i}a_i,$$

where a_i^{in}, a_i, and a_i^{out}, $(i=p,c)$ are the field amplitudes of lasers at the front of the input mirror, inside the cavity, and outside the output mirror respectively, and the cavity decay rates are

$$K_p=\kappa_p+i\Delta_{cp},$$
$$K_c=\kappa_c+i\Delta_{cc} \tag{5}$$

Steady-state analysis with the master equation

By setting all time derivatives equal to zero, we solve Eqs. (1) and (4) in the steady state as follows:

$$0=R_{bb}\rho_{bb}+R_{ba}\rho_{aa}+R_{bc}\rho_{cc}+i\frac{g_p}{2}(\rho_{ab}a_p^*e^{i\phi_p}-\rho_{ba}a_pe^{-i\phi_p}),$$

$$0=R_{cb}\rho_{bb}+R_{ca}\rho_{aa}+R_{cc}\rho_{cc}-i\frac{g_c}{2}(\rho_{ca}a_ce^{-i\phi_c}-\rho_{ac}a_c^*e^{i\phi_c}),$$

$$\rho_{bb}+\rho_{aa}+\rho_{cc}=n_0V, \tag{6a}$$

$$0=-\Gamma_{ba}\rho_{ba}+i\frac{g_p}{2}a_p^*e^{i\phi_p}(\rho_{aa}-\rho_{bb})-i\frac{g_c}{2}a_c^*e^{i\phi_c}\rho_{bc},$$

$$0=-\Gamma_{bc}\rho_{bc}+i\frac{g_p}{2}a_p^*e^{i\phi_p}\rho_{ac}-i\frac{g_c}{2}a_ce^{-i\phi_c}\rho_{ba},$$

$$0=-\Gamma_{ac}\rho_{ac}+i\frac{g_c}{2}a_ce^{-i\phi_c}(\rho_{cc}-\rho_{aa})+i\frac{g_p}{2}a_pe^{-i\phi_p}\rho_{bc},$$

$$0=-K_pa_p+i\frac{g_p}{2}e^{i\phi_p}\rho_{ab}+\sqrt{2\kappa_p}a_p^{\text{in}},$$

$$0=-K_ca_c+i\frac{g_c}{2}e^{i\phi_c}\rho_{ac}+\sqrt{2\kappa_c}a_c^{\text{in}}, \tag{6b}$$

$$a_i^{\text{out}}=-a_i^{\text{in}}+\sqrt{2\kappa_i}a_i.$$

After defining

$$D=\Gamma_{ba}\Gamma_{ac}\Gamma_{bc}+\left(\frac{g_p}{2}\right)^2I_p\Gamma_{ba}+\left(\frac{g_c}{2}\right)^2I_c\Gamma_{ac}, \tag{7a}$$

$$M=\Gamma_{bc}\Gamma_{ac}+\left(\frac{g_p}{2}\right)^2I_p, \tag{7b}$$

$$N=\Gamma_{ba}\Gamma_{bc}+\left(\frac{g_c}{2}\right)^2 I_c, \tag{7c}$$

where $I_i=|a_i|^2$ is the laser intensity in the cavity and $(g_i/2)\sqrt{I_i}$ is the corresponding Rabi frequency. We have from the latter three equations in Eqs. (6a):

$$G_{ba}=\frac{\gamma_{ba}}{n_0V}\frac{\rho_{ba}e^{-i\phi_p}}{i(g_p/2)a_p^*}=\frac{\gamma_{ba}}{n_0V}\frac{(\rho_{aa}-\rho_{bb})M+(\rho_{cc}-\rho_{aa})(g_c/2)^2I_c}{D}, \tag{8}$$

$$G_{ac}=\frac{\gamma_{ac}}{n_0V}\frac{\rho_{ac}e^{i\phi_c}}{i(g_c/2)a_c}=\frac{\gamma_{ac}}{n_0V}\frac{(\rho_{cc}-\rho_{aa})N+(\rho_{aa}-\rho_{bb})(g_p/2)^2I_p}{D}, \tag{9}$$

where G_{ij} represents the dimensionless and complex dipole moment of a single atom which is referred to as a complex laser gain. $\rho_{cc}-\rho_{bb}$ is the Raman inversion for the probe laser field. The intracavity amplitude of the probe laser and the driving laser are directly solved from Eqs. (6b) as

$$\begin{aligned}0&=-K_pa_p+\kappa_pC_pG_{ab}a_p+\sqrt{2\kappa_p}a_p^{\rm in},\\ 0&=-K_ca_c-\kappa_cC_cG_{ac}a_c+\sqrt{2\kappa_c}a_c^{\rm in},\\ a_i^{\rm out}&=-a_i^{\rm in}+\sqrt{2\kappa_i}a_i,\end{aligned} \tag{10}$$

where the collective cooperative coefficients between the atom, cavity, and laser field are $C_p=n_0V(g_p/2)^2/(\gamma_{ba}\kappa_p)$ and $C_c=n_0V(g_c/2)^2/(\gamma_{\rm ac}\kappa_c)$. The larger the effective interacting atom number and/or the higher the cavity Q factors, the larger the collective cooperative coefficients: Notice that if the atomic medium in a cavity is 100 ($G_{ba}=0$ and $G_{ac}=0$), one may conclude that $a_i^{\rm out}=a_i^{\rm in}$, which means transparency of the output fields. Now, we calculate steady-state atomic system quantities, such as population inversions. From Eq. (4), we obtain

$$\begin{aligned}0&=R_{bb}\rho_{bb}+R_{ba}\rho_{aa}+R_{bc}\rho_{cc}+P_{b-a}(\rho_{aa}-\rho_{bb})\\&\quad+P_r(\rho_{cc}-\rho_{aa}),\\ 0&=R_{cb}\rho_{bb}+R_{ca}\rho_{aa}+R_{cc}\rho_{cc}-P_{a-c}(\rho_{cc}-\rho_{aa})\\&\quad-P_r(\rho_{aa}-\rho_{bb}),\\ n_0V&=\rho_{bb}+\rho_{aa}+\rho_{cc},\end{aligned} \tag{11}$$

where the efficient pumping rates by the coherent fields are

$$\begin{aligned}P_{b-a}&=\left(\frac{g_p}{2}\right)^2I_p2\,\mathrm{Re}\left(\frac{M}{D}\right),\\ P_{a-c}&=\left(\frac{g_c}{2}\right)^2I_c2\,\mathrm{Re}\left(\frac{N}{D}\right),\\ P_r&=\left(\frac{g_p}{2}\right)^2\left(\frac{g_c}{2}\right)^2I_pI_c2\,\mathrm{Re}\left(\frac{1}{D}\right).\end{aligned} \tag{12}$$

Solving Eqs. (11), we then obtain

$$\rho_{bb}=\frac{n_0V}{D_0}\{R_{ca}R_{bc}-R_{cc}R_{ba}-P_{b-a}R_{cc}+P_{a-c}(R_b-R_{bb})+P_r[R_c-(R_{bc}+R_{cb})]+(P_{b-a}P_{a-c}-P_r^2)\}, \tag{13}$$

$$\rho_{aa}=\frac{n_0V}{D_0}[R_{cc}R_{bb}-R_{cb}R_{bc}-P_{b-a}R_{cc}-P_{a-c}R_{bb}-P_r(R_{bc}+R_{cb})+(P_{b-a}P_{a-c}-P_r^2)], \tag{14}$$

$$\rho_{cc}=\frac{n_0V}{D_0}\{R_{cb}R_{ba}-R_{ca}R_{bb}+P_{b-a}(R_c-R_{cc})-P_{a-c}R_{bb}+P_r[R_b-(R_{bc}+R_{cb})]+(P_{b-a}P_{a-c}-P_r^2)\}, \tag{15}$$

$$D_0=D_c+P_{b-a}(R_c-3R_{cc})+P_{a-c}(R_b-3R_{bb})+P_r\times[-3(R_{bc}+R_{cb})+(R_b+R_c)]+3(P_{b-a}P_{a-c}-P_r^2), \tag{16}$$

where

$$D_c=R_{ca}(R_{bc}-R_{bb})+R_{cb}(R_{ba}-R_{bc})+R_{cc}(R_{bb}-R_{ba}), \tag{17a}$$

$$\begin{aligned}R_b&=R_{bb}+R_{ba}+R_{bc},\\ R_c&=R_{cb}+R_{ca}+R_{cc}.\end{aligned} \tag{17b}$$

The population inversions for the transitions *a-b*, *a-c*, and *b-c* are

$$\rho_{aa}-\rho_{bb}=\frac{n_0V}{D_0}(T_{a-b}-P_{a-c}R_b-P_rR_c), \tag{18}$$

$$\rho_{aa}-\rho_{cc}=\frac{n_0V}{D_0}(-T_{a-c}-P_{b-a}R_c-P_rR_b), \tag{19}$$

and

$$\rho_{cc}-\rho_{bb}=\frac{n_0V}{D_0}[T_{c-b}+P_{b-a}R_c-P_{a-c}R_b+P_r(R_b-R_c)], \tag{20}$$

respectively, where

$$\begin{aligned}T_{a-b}&=R_{cc}R_b-R_cR_{bc},\\ T_{a-c}&=-R_cR_{bb}+R_{cb}R_b,\end{aligned} \tag{21}$$

$T_{c-b}=T_{a-b}+T_{a-c}$, and T_{i-j} represents the effects of incoherent processes in the population inversion between the states $|i\rangle$ and $|j\rangle$. Substituting Eqs. (18)–(20) into Eqs. (8) and (9), the probe and complex gain coefficients of the probe and drive lasers for the transition *a-b* and *a-c* become

$$G_{ba}=\frac{\gamma_{ba}}{G_0}D*\left[(T_{a-b}-P_{a-c}R_b-P_rR_c)M+(T_{a-c}+P_{b-a}R_c+P_rR_b)\left(\frac{g_c}{2}\right)^2 I_c\right]=\frac{\gamma_{ba}}{G_0}\{[T_{a-b}M+(T_{a-c}-2\gamma_{bc}R_b)x_c]D*+x_c(M*-M)[R_bN*+R_cx_p]\}, \tag{22a}$$

$$G_{ac}=\frac{\gamma_{ac}}{G_0}D*\left[(T_{a-c}+P_{b-a}R_c+P_rR_b)N+(T_{a-b}-P_{a-c}R_b-P_rR_c)\left(\frac{g_p}{2}\right)^2 I_p\right]=\frac{\gamma_{ac}}{G_0}\{[T_{a-c}N+(T_{a-b}+2\gamma_{bc}R_c)x_p]D*+x_p(N-N*)\times[R_cM*+R_bx_c], \tag{22b}$$

$$G_0=DD*D_c+2x_p(R_c-3R_{cc})\mathrm{Re}(MD*)+2x_c(R_b-3R_{bb})\mathrm{Re}(ND*)+2x_px_c[R_b+R_c-3(R_{bc}+R_{cb})+6\gamma_{bc}]\mathrm{Re}(D)+3x_px_c(M*-M)(N-N*), \tag{22c}$$

where $G_0=D_0DD*$, $x_p=(g_p/2)^2I_p$, and $x_c=(g_c/2)^2I_c$. In principle, we can substitute Eqs. (22) into Eqs. (10), so that the coupling equations set of the intracavity laser intensity are determined as

$$\frac{x_p}{x_p^{\mathrm{in}}}=\frac{2\kappa_p}{|K_p-\kappa_pC_pG_{ab}|^2},\qquad \frac{x_c}{x_c^{\mathrm{in}}}=\frac{2\kappa_c}{|K_c-\kappa_cC_cG_{ac}|^2}. \tag{23a}$$

If there are no cavity detunings, that is, $\Delta_{cp}=\Delta_{cc}=0$, we have

$$(x_p-x_p^0)/x_p=2C_p\ \mathrm{Re}(G_{ba})-C_p^2\{[\mathrm{Re}(G_{ba})]^2+[\mathrm{Im}(G_{ba})]^2\}$$
$$(x_c-x_c^0)/x_c=-2C_c\ \mathrm{Re}(G_{ac})-C_c^2\{[\mathrm{Re}(G_{ac})]^2+[\mathrm{Im}(G_{ac})]^2\}, \tag{23b}$$

where $x_i^0=2x_i^{\mathrm{in}}/\kappa_i$. Equation (23b) shows that the intensity amplification of the intracavity fields is determined not only by the linear gains Re(G_{ba}) or Re(G_{ac}), but also by the nonlinear gains $[\mathrm{Re}(G_{ba})]^2$ or $[\mathrm{Re}(G_{ac})]^2$, and the corresponding dispersions such as Im(G_{ba}) or Im(G_{ac}). In the smaller C_i regime ($C_i\ll 1$), the contribution of dispersion and nonlinear gain can be neglected so that absorption spectroscopy is determined by the linear gain. But if $C_i\gg 1$, the contributions of the nonlinear gain and dispersion may be larger than that of the linear gain, which represents the effects of the combination of dense medium, longer optical path, and cavity. In this case, the dispersion property will dramatically modify the absorption profile, so that a substantial narrowing of a central transparency peak may be observed. On the other hand, because Eq. (23) is a two-element high-order equation set, in principle, x_p and x_c can be exactly solved. If this step is reached, we can substitute x_p and x_c into Eqs. (10) to determine the ratio of the output laser intensity to the input intensity as

$$\frac{x_p^{\mathrm{out}}}{x_p^{\mathrm{in}}}=\left|\frac{2}{K_p/\kappa_p-C_pG_{ab}}-1\right|^2,\qquad \frac{x_c^{\mathrm{out}}}{x_c^{\mathrm{in}}}=\left|\frac{2}{K_c/\kappa_c+C_cG_{ac}}-1\right|^2 \tag{24a}$$

or

$$\frac{x_p^{\mathrm{out}}}{x_p^{\mathrm{in}}}=\frac{[1+C_p\ \mathrm{Re}(G_{ba})]^2+[\Delta_{cp}/\kappa_p+C_p\ \mathrm{Im}(G_{ba})]^2}{[1-C_p\ \mathrm{Re}(G_{ba})]^2+[\Delta_{cp}/\kappa_p+C_p\mathrm{Im}(G_{ba})]^2},\qquad \frac{x_c^{\mathrm{out}}}{x_c^{\mathrm{in}}}=\frac{[1-C_c\ \mathrm{Re}(G_{ac})]^2+[\Delta_{cc}/\kappa_c+C_c\ \mathrm{Im}(G_{ac})]^2}{[1+C_c\ \mathrm{Re}(G_{ac})]^2+[\Delta_{cc}/\kappa_c+C_c\ \mathrm{Im}(G_{ac})]^2}. \tag{24b}$$

If $\Delta_{cp}=\Delta_{cc}=0$, the above will be further simplified as

$$\frac{x_p^{\mathrm{out}}}{x_p^{\mathrm{in}}}=1+\frac{4C_p\ \mathrm{Re}(G_{ba})}{[1-C_p\ \mathrm{Re}(G_{ba})]^2+[C_p\ \mathrm{Im}(G_{ba})]^2}=1+4C_p\ \mathrm{Re}(G_{ba})\frac{x_p}{x_p^0},$$
$$\frac{x_c^{\mathrm{out}}}{x_c^{\mathrm{in}}}=1-\frac{4C_c\ \mathrm{Re}(G_{ac})}{[1+C_c\ \mathrm{Re}(G_{ac})]^2+[C_c\ \mathrm{Im}(G_{ac})]^2}=1-4C_c\ \mathrm{Re}(G_{ac})\frac{x_c}{x_c^0}. \tag{25a}$$

From Eqs. (23b), optical bistability, multistability and other nonlinearities can be studied. From Eqs. (25a), the effects of atomic density, interacting volume and cavity can be shown, and EIT or LWI can be studied. Because the real or imaginary part of a complex laser gain is always less than 1.0, Eqs. (25) can be approximated in the following way, if the collective cooperative coefficient C_i is much smaller than 1:

$$x_p^{\mathrm{out}}/x_p^{\mathrm{in}}\cong 1+4C_p\ \mathrm{Re}(G_{ba}),\qquad x_c^{\mathrm{out}}/x_c^{\mathrm{in}}\cong 1-4C_c\ \mathrm{Re}(G_{ac}), \tag{25b}$$

which is referred to as a general EIT discussed in Sec. III. When C_i is nearly 1 or above, Eqs. (25b) is no longer valid and we must use Eqs. (25a), which are addressed in Sec. IV and named cavity EIT.

III. GENERAL EIT

In this section, we discuss the case of C_i, which is much smaller than 1, and which is referred to as ‘‘general EIT.’’ First we suppose that

$$M=M_1+iM_2+x_p,$$
$$N=N_1+iN_2+x_c, \tag{26}$$
$$D=D_1+\gamma_{ba}x_p+\gamma_{ac}x_c+i(D_2+\Delta_p x_p-\Delta_c x_c),$$

where the parameters are

$$\begin{aligned} M_1&=\gamma_{ac}\gamma_{bc}+\Delta_c(\Delta_p-\Delta_c),\\ M_2&=\gamma_{ac}(\Delta_p-\Delta_c)-\gamma_{bc}\Delta_c),\\ N_1&=\gamma_{ba}\gamma_{bc}-\Delta_p(\Delta_p-\Delta_c),\\ N_2&=\gamma_{ba}(\Delta_p-\Delta_c)+\gamma_{bc}\Delta_p),\end{aligned} \tag{27a}$$

$$\begin{aligned} D_1&=\gamma_{ba}\gamma_{ac}\gamma_{bc}+\gamma_{ba}\Delta_c(\Delta_p-\Delta_c)-\gamma_{ac}\Delta_p(\Delta_p-\Delta_c)\\ &\quad+\gamma_{bc}\Delta_c\Delta_p,\\ D_2&=\gamma_{ba}\gamma_{ac}(\Delta_p-\Delta_c)-\gamma_{ba}\gamma_{bc}\Delta_c+\gamma_{ac}\gamma_{bc}\Delta_p+\Delta_c\Delta_p\\ &\quad\times(\Delta_p-\Delta_c),\end{aligned} \tag{27b}$$

so that

$$\begin{aligned} G_{ba}&=\frac{\gamma_{ba}}{G_0}\{\{[T_{a-b}(M_1+x_p)+(T_{a-c}-2\gamma_{bc}R_b)x_c](D_1\\ &\quad+\gamma_{ba}x_p+\gamma_{ac}x_c)-2M_2N_2R_bx_c+T_{a-b}M_2(D_2+\Delta_p x_p\\ &\quad-\Delta_c x_c)\}+i\{T_{a-b}M_2(D_1+\gamma_{ba}x_p+\gamma_{ac}x_c)\\ &\quad-[T_{a-b}(M_1+x_p)+(T_{a-c}-2\gamma_{bc}R_b)x_c](D_2+\Delta_p x_p\\ &\quad-\Delta_c x_c)-2M_2x_c[R_b(N_1+x_c)+R_cx_p]\},\end{aligned} \tag{28a}$$

$$\begin{aligned} G_{ac}&=\frac{\gamma_{ac}}{G_0}\{\{[T_{a-c}(N_1+x_c)+(T_{a-b}+2\gamma_{bc}R_c)x_p](D_1\\ &\quad+\gamma_{ba}x_p+\gamma_{ac}x_c)+2M_2N_2R_cx_p+T_{a-c}N_2(D_2+\Delta_p x_p\\ &\quad-\Delta_c x_c)\}+i\{T_{a-c}N_2(D_1+\gamma_{ba}x_p+\gamma_{ac}x_c)\\ &\quad-[T_{a-c}(N_1+x_c)+(T_{a-b}+2\gamma_{bc}R_c)x_p](D_2+\Delta_p x_p\\ &\quad-\Delta_c x_c)+2N_2x_p[R_c(M_1+x_p)+R_bx_c]\},\end{aligned} \tag{28b}$$

$$\begin{aligned} G_0&=\{D_c(D_1+\gamma_{ba}x_p+\gamma_{ac}x_c)+2x_p(R_c-3R_{cc})(M_1+x_p)\\ &\quad+2x_c(R_b-3R_{bb})(N_1+x_c)+2x_px_c[R_b+R_c-3(R_{bc}\\ &\quad+R_{cb})+6\gamma_{bc}]\}(D_1+\gamma_{ba}x_p+\gamma_{ac}x_c)+12x_px_cM_2N_2\\ &\quad+[D_c(D_2+\Delta_p x_p-\Delta_c x_c)+2x_p(R_c-3R_{cc})M_2\\ &\quad+2x_c(R_b-3R_{bb})N_2](D_2+\Delta_p x_p-\Delta_c x_c).\end{aligned} \tag{28c}$$

Some special cases are condidered in the following discussion. If $\Delta_c=0$, and when $(R_{bb},R_{ba},R_{bc})\to(0,\gamma_b,0)$ and $(R_{cb},R_{ca},R_{cc})\to(0,\gamma_c,0)$, we have $D_c=T_{a-b}=T_{a-c}=0$, $\gamma_{bc}=0$, $\Delta_c=\Delta_{cc}=0$, and $\gamma_{ba}=\gamma_{ac}=\gamma$, if atomic collision is neglected. In this case, we have

$$G_{ba}=\frac{1}{G_0}(-2\gamma^2\Delta_p x_c)\{\gamma_b\gamma\Delta_p+i[\gamma_b(x_c-\Delta_p^2)+\gamma_c x_p]\}, \tag{29a}$$

$$G_{ac}=\frac{1}{G_0}2\gamma^2\Delta_p x_p[\gamma_c\gamma\Delta_p+i(\gamma_c x_p+\gamma_b x_c)], \tag{29b}$$

$$\begin{aligned} G_0&=2\gamma\{[x_p\gamma_c x_p+x_c\gamma_b(x_c-\Delta_p^2)+x_px_c(\gamma_b+\gamma_c)](-\Delta_p^2\\ &\quad+x_p+x_c)+6x_px_c\gamma\Delta_p^2+\Delta_p^2(x_p\gamma_c+x_c\gamma_b)(\gamma^2+x_p)\}.\end{aligned} \tag{29c}$$

Under the condition of $C_i\ll 1$ and from Eq. (23b), we have $x_p\approx x_p^0$ and $x_c\approx x_c^0$. Furthermore, from Eqs. (25b), the absorption spectrum of the probe laser is only determined by the linear gain G_{b-a}. It is clear that G_{b-a} has only its maximum value at $\Delta_p=0$. This represents 100 two-photon resonances, and, at infinite time, the atom gradually falls down in the dark state so that there are no interactions among the atoms, laser fields, and vacuum fields. As a consequence, 100 two photon resonances are observed. From Eqs. (29), EIT which is calculated only by the real part of G_{b-a}, can be immediately derived as

$$\frac{I_p^{\text{out}}}{I_p^{\text{in}}}\approx 1-4C_p\frac{\gamma^2\Delta_p^2}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2(2W_{\text{dip}})^2}, \tag{30a}$$

$$\begin{aligned}(2W_{\text{dip}})^2&=\gamma^2(1+\alpha)+x_p2(1+\sqrt{1+\alpha})+x_c2(\sqrt{1+\alpha}-1\\ &\quad+\alpha),\end{aligned} \tag{30b}$$

where $\alpha=(x_p\gamma_c)/(x_c\gamma_b)$. We then have the following results: (1) Because $G_{b-a}=0$ at both $\Delta_p=\pm\infty$ and $\Delta_p=0$, one may define the half-value width of the transparency window and the half-value width of the absorption doublet. The former is exactly calculated as

$$W_{T\text{-window}}=2[\sqrt{(x_p+x_c)\sqrt{1+\alpha}+(W_{\text{dip}})^2}-W_{\text{dip}}]. \tag{31}$$

If the probe intensity α is large enough, the transparency window will increase substantially. Naturally, we can also select a suitable atom species so that γ_c is much smaller than γ_b, which results in the suppression of the window broadening due to the probe. Nevertheless, the width of the transparency widow is dependent on the power broadening and the natural linewidth in the probe channel. (2) The half-value of the absorption dips is exactly equal to $2W_{\text{dip}}$, when $x_c\gg x_p$, and

$$(2W_{\text{dip}})^2\approx(4+3\gamma_c/\gamma_b)x_p+\gamma^2(1+\alpha). \tag{32}$$

Furthermore, when the probe Rabi frequency is much smaller than γ_{ba}, the width of the absorption dip has its minimum value of about $2W_{\text{dip}}\approx\gamma_{ba}$. On the other hand, Eqs. (30) can be rewritten as

$$G_{b-a}(\Delta_p)=\frac{\gamma_{ba}}{4W_{\rm dip}\sqrt{W_{\rm dip}^2-(x_p+x_c)\sqrt{1+\alpha}}}\left[\frac{(L_-)^2}{(\Delta_p)^2+(L_-)^2}-\frac{(L_+)^2}{(\Delta_p)^2+(L_+)^2}\right], \tag{33}$$

where

$$W_{\rm dip}>\sqrt{x_p+x_c}\sqrt[4]{1+\alpha},$$
$$L_\pm=W_{\rm dip}\sqrt{W_{\rm dip}^2-(x_p+x_c)\sqrt{1+\alpha}}. \tag{34}$$

The absorption spectrum is the subtraction of two Lorentzians with different widths but the same height and frequency center. Apparently, this phenomenon is not simply due to Autler-Townes level splitting. If

$$W_{\rm dip}<\sqrt{x_p+x_c}\sqrt[4]{1+\alpha},$$
$$\Omega_0=\sqrt{(x_p+x_c)\sqrt{1+\alpha}-W_{\rm dip}^2}, \tag{35}$$
$$\Omega_0=\sqrt{(x_p+x_c)\sqrt{1+\alpha}-W_{\rm dip}^2},$$

Eqs. (30) can be rewritten as

$$G_{b-a}(\Delta_p)=\frac{\gamma_{ba}}{4\sqrt{(x_p+x_c)\sqrt{1+\alpha}-W_{\rm dip}^2}}\times\left[\frac{-\Delta_p}{(\Delta_p-\Omega_0)^2+(W_{\rm dip})^2}+\frac{\Delta_p}{(\Delta_p+\Omega_0)^2+(W_{\rm dip})^2}\right]. \tag{36}$$

Thus EIT in this case is the addition of two mirror-symmetrical Lorentzian-like components with the same width and different central frequencies. This phenomenon is similar to the result of Autler-Townes level splitting. In other words, general EIT is the consequence of quantum coherence and an ac Stark effect. In the weak-field regime [for example, Eq. (33)], quantum interference and coherence is dominant, while in the strong-field regime, the ac Stark effect and quantum coherence combine together to be dominant. The above is the general EIT discussed elsewhere. Our particular contribution to general EIT is its physical explanation. This physical explanation gives a criterion to distinguish the contribution of quantum interference and coherence from that of level splitting to some extent. In this case, however, the dispersion contribution is neglected and the width of the central transparency peak is formulated in Eq. (31). Since generally $\sqrt{x_c}\geqslant\gamma$, $W_{T\text{-window}}\approx 2\sqrt{x_c}$. Power broadening determines the transparency window. Cavity EIT under the condition of $C_i\geqslant 1$ will be discussed in Sec. IV.

IV. CAVITY EIT WITH $\Delta_c=0$

Cavity EIT defined in Eqs. (25a) is dependent not only on the real part of G_{b-a} (amplitude gain or loss) but also on its imaginary part (dispersion), thus effectively interacting atom number with cavity. In this sense, absorption spectroscopy of the probe may be dramatically modified. From Eqs. (29), we have

$$G_{ba}=\frac{-\gamma^2\Delta_p^2-i\gamma\Delta_p[x_c(1+\alpha)-\Delta_p^2]}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2(2W_{\rm dip})^2}, \tag{37a}$$

$$G_{ac}=\frac{\alpha\gamma^2\Delta_p^2+i\gamma\Delta_p x_p(1+\alpha)}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2(2W_{\rm dip})^2}. \tag{37b}$$

If Eqs. (23b) are used, Eq. (37a) can be substituted into Eqs. (25a), so that

$$\frac{x_p^{\rm out}}{x_p^{\rm in}}=1-\frac{4C_p\gamma^2\Delta_p^2}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2\{(2W_{\rm dip})^2+[2C_p+C_p^2+\beta(\Delta_p)C_p^2]\gamma^2\}}, \tag{38}$$

where

$$\beta(\Delta_p)=\frac{(1+\alpha)(\alpha x_c^2-2x_cx_p-x_p^2)-\Delta_p^2(2x_p+4\alpha x_c+\alpha\gamma^2)}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2(2W_{\rm dip})^2}. \tag{39}$$

Equation (38) is plotted in Fig. 2 if the dependence of x_p and x_c on the detuning is supposed to be neglected. As C_p increases up to or above 1, the absorption spectroscopy of the probe is dramatically changed in the following ways: (1) narrowing of the transparency window below the power broadening and/or the natural width, (2) broadening of the absorption doublet, and (3) the possibility to observe two very small sideband dips at about $\pm(x_p+x_c)\sqrt{1+\alpha}$. The physical explanations are given below. First, we suppose that $\beta(\Delta_p)\ll 1$ for arbitrary detuning (this is true if x_p is much smaller than x_c). Equation (38) approximately becomes

$$\frac{x_p^{\rm out}}{x_p^{\rm in}}\approx 1-\frac{4C_p\gamma^2\Delta_p^2}{[(x_p+x_c)\sqrt{1+\alpha}-\Delta_p^2]^2+\Delta_p^2(2W_{\rm dip}^0)^2}, \tag{40}$$

which is the same formula as Eq. (31a), except for the replacement of $W_{\rm dip}$ by the new $W_{\rm dip}^0$. The new width of the absorption doublet is obtained from

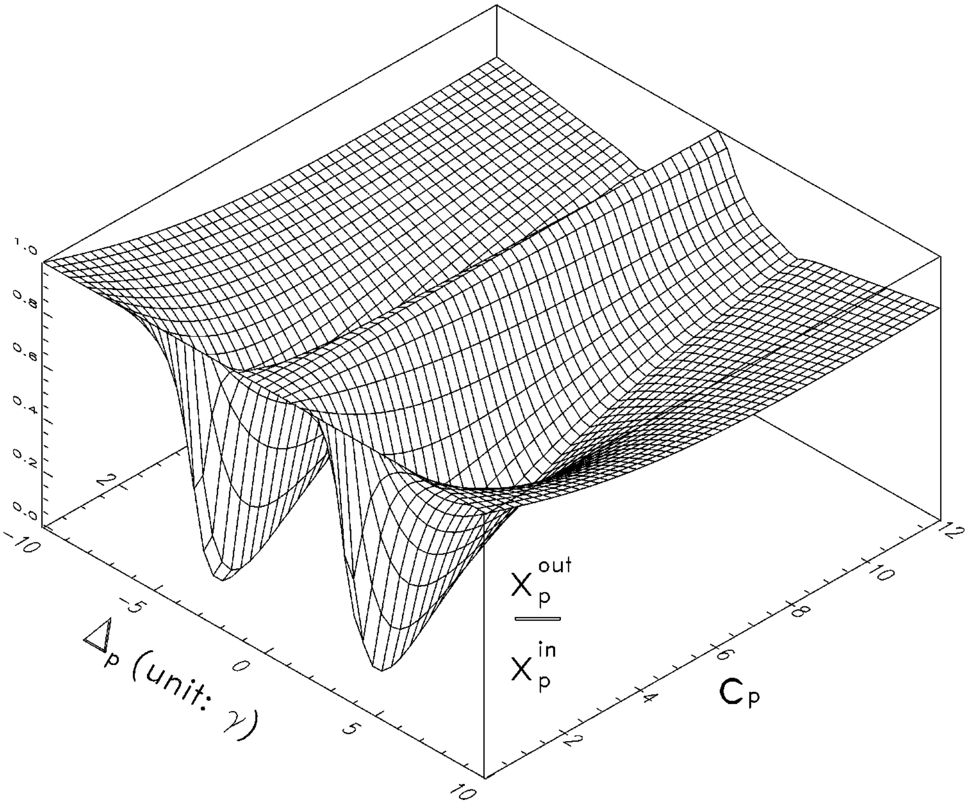

FIG. 2. The transmission of a probe laser with $\gamma_b = 0.5$, $\gamma_c = 1.5$, and $x_p = 0.01$ as a function of a probe detuning and the collective cooperative coefficient of the probe.

$$(2W_{\mathrm{dip}}^0)^2 = (2W_{\mathrm{dip}})^2 + (2C_p + C_p^2)\gamma^2. \tag{41}$$

If $x_p \ll x_c$ and $x_p \ll \gamma^2$, $2W_{\mathrm{dip}} \approx \gamma$ and $2W_{\mathrm{dip}} \approx (1 + C_p)\gamma$, which means that the width of the absorption doublet is broadened by a factor of C_p due to the contribution of dispersion, cavity and dense medium. Second, we further suppose $x_c \gg \gamma^2$, but $x_c \ll (2W_{\mathrm{dip}}^0)^2 \approx (1 + C_p)^2\gamma^2$ (for example, $C_p = 50$). In this case, we have

$$W_{T\text{-window}}^0 = 2\left[\sqrt{(x_p + x_c)\sqrt{1+\alpha} + (W_{\mathrm{dip}}^0)^2} - W_{\mathrm{dip}}^0\right] \approx \frac{x_p + x_c}{(1 + C_p)\gamma}. \tag{42}$$

This width of the transparency window may be less than the total Rabi frequency $2\sqrt{x_p + x_c}$, or even less than the natural linewidth γ, even though it is still determined by the total Rabi frequency (or power broadening).

Finally, we must determine what are the results in the two small sideband dips? To this end, $\beta(\Delta_p)$ cannot be neglected, and must be taken into account in all aspects. Generally, but not always, $\beta(\Delta_p)$ has maximum values at about $\pm(x_p + x_c)\sqrt{1+\alpha}$ but is negative so that the denominator of Eq. (38) has minimum values there. The resulting consequence is that the absorption spectroscopy may have minimum values at about $\pm(x_p + x_c)\sqrt{1+\alpha}$—two sideband dips. If $\alpha \ll 1$, $\beta(\Delta_p) \sim 0$, so that the two sideband dips will disappear, which is the reason for the two sideband dips being so small when the driving laser is much stronger than the probe. Figure 3 gives representative slices of Fig. 2 in two dimensions, as well as a slice when the collective cooperative coefficient value is extended to 50 which is beyond the route of Figure 2, and the total Rabi frequency is about 3.47 (the unit's natural linewidth). When the cooperative coefficient value is 50, Fig. 3 clearly shows that the central transparent peak has a width of about one-half that of the natural width, which is much less than both the natural width and the power broadening. The above theoretical results may be useful in EIT, LWI, and other experiments related to atomic coherence and interference. For example, when we observe the spectroscopic profile of an atom's line, the resulting pro-

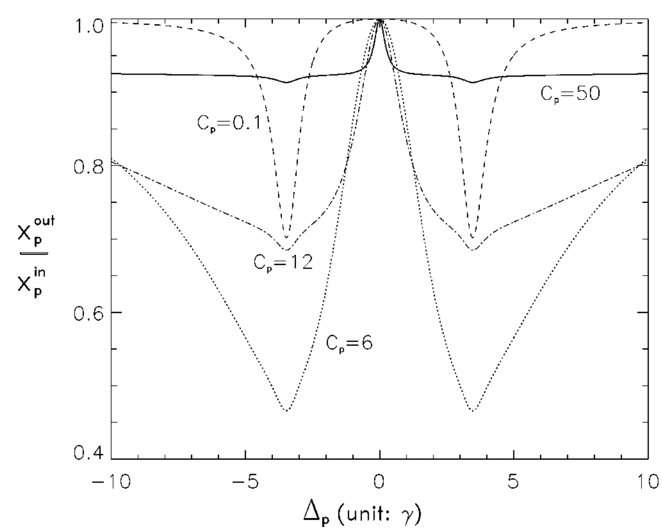

FIG. 3. The transmission of a probe laser with $\gamma_b = 0.5$, $\gamma_c = 1.5$, $x_p = 0.01$, and the collective cooperative coefficient equal to 50 as a function of a probe detuning.

file is the combination of natural line shape and power broadening, and the characteristic width always is larger than both the natural linewidth and power broadening. This situation is much worse if the natural linewidth is very small (for example, the rubidium atom clock case). The existing method to partially remedy this problem is to use as small a probe as possible. Thus the signal-to-noise ratio and the absorption line shape must be balanced. But from the above theory, we may obtain the transparent peak of the probe with a subnatural and power-broadened free width, while we still keep a fairly high signal-to-noise ratio, if another laser is properly driving the atoms. This situation can be realized by simply increasing the effective interacting path through a multipathlike setup. Thus high-resolution spectroscopy is achievable with a subnatural linewidth while without power broadening.

V. CONCLUSION

In this paper, the effects of a cavity and/or a dense medium on EIT are investigated. An optically dense medium or a higher-Q cavity will enhance the contribution of the dispersion and nonlinear gain to the output intensity of the probe so that the nonlinear gain and dispersion terms cannot be neglected. As a consequence of this fact, the absorption profile of the probe will be dramatically modified: The width of the central transparency window will be substantially narrowed and a power-broadened free (and/or subnatural width) EIT may be observed while the width of the absorption doublet will be broadened. The physical picture behind those phenomena is the following (1) The medium is prepared almost in the dark state by a stronger driving field and a weaker probe field so that the absorption of the probe at a central frequency is totally canceled (EIT). (2) A dense medium, a longer optical path, and/or a cavity contribute to the mostly absorbing region of the absorption profile, becoming even more absorbing while maintaining the transparent as fairly transparent. This is an effect of nonlinear gain such as $[C_p \operatorname{Re}(G_{ba})]^2$. (3) Dispersion experienced by the probe will contribute to the output intensity of the probe so that the absorption profile of the probe is further modified. This makes a high-resolution absorption spectroscopic technique accessible and feasible.

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