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Electric-field influence on doubly excited Feshbach resonance states of the positronium negative ion below the $N=3$ threshold of the positronium atom

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We present a study of the influence of the constant uniform electric field on the doubly excited resonances of the positronium negative ion associated with the $N=3$ positronium threshold. The calculation is performed in the framework of the complex coordinate rotation method which obtains results both for the real part of the energy (position of the resonance) and for the imaginary part of the energy (related to the width of the resonance). Results are given for the $M=0$ Stark components of the Feshbach ${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances.

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I. INTRODUCTION

This work presents a theoretical study of an external dc electric-field influence on doubly excited states of the positronium negative ion, Ps$^-$. The positronium negative ion is a three-body system consisting of two electrons and a positron interacting through Coulomb forces. This system is known to have only one truly bound state—the $1s^2\ {}^1S^e$ state [1]. It does not have any singly excited bound states. Its spectrum contains, however, rich resonance structures of doubly excited states, lying in the e-Ps scattering continua, and in the vicinity of the excited states of the target positronium atom [2–5].

The positronium negative ion was experimentally observed by Mills [6]. Subsequently, he measured its annihilation rate [7]. For early developments of the studies of this system, readers are referred to reviews [8,9].

The influence of the dc electric field on doubly excited states has been much better studied, both theoretically and experimentally, for another two-electron system which, as well as the positronium negative ion, can be considered as an example of a highly correlated atomic system—the hydrogen negative ion, H$^-$.

A unique experimental setup was made at the Los Alamos Meson Physics Facility (LAMPF) by researchers led by the University of New Mexico team [10]. The H$^-$ beam accelerated to the energy of about 800 MeV was intersected by a laser beam with various intersecting angles. Under such an experimental setup the influence of the strong dc electric field on the doubly excited ${}^1P^o$ states of the hydrogen negative ion has been measured. In a series of publications the Stark effect on the lowest ${}^1P^o$ Feshbach resonance of H$^-$ below the $N=2$ hydrogenic threshold has been investigated [11] as well as the influence of the dc electric field on the ${}^1P^o$ shape resonance above the $N=2$ hydrogenic threshold [12]. Furthermore, studies of the influence of the dc electric field on the $N=3$ Feshbach ${}^1P^o(1)$ resonance [13] and the ${}^1P^o$ resonances associated with higher hydrogenic thresholds have also been reported [14].

As far as the $N=2\ {}^1P^o$ Feshbach resonance is concerned, since it lies very close to the second member of the series of the resonances of the ${}^1S^e$ symmetry, the ${}^1S^e(2)$ resonance, even a weak external electric field causes a strong mixing of these two states and results in splitting of the ${}^1P^o(1)$ state into two components, exhibiting the linear Stark effect for the ${}^1P^o(1)$ and ${}^1S^e(2)$ doubly excited states.

We shall outline briefly some known facts used for the theoretical description of the properties of the atomic system placed in the electric field. Under the influence of the dc electric field the truly bound states of the atoms/ions become quasibound states, since the electrons are able to tunnel through the potential barrier formed by the combined Coulomb and external electric fields. As is well known, this phenomenon can be described theoretically by introducing the complex energy eigenvalues, the real part of a complex energy representing the shifted resonance position and the imaginary part being connected to the lifetime of the quasibound state. Analogously, only slightly more complex is the situation for the autoionizing states of two-electron systems subject to the influence of the external electric field, the difference between the autoionizing and bound states being that even without the external electric field the former undergo the process of autoionization.

The opening of the new tunneling channels due to the external electric field provides an alternative route for the autoionization processes. The width (or lifetime) of such a state is therefore a result of competition between the autoionization process due to the electron-electron Coulomb interaction, tunneling due to the external electric field, and autoionization into other field-induced angular momentum continua.

We have recently carried out theoretical investigations of the influence of the dc electric-field on the doubly excited resonance states of H$^-$. The method of complex-coordinate rotation [15–17] has been used. In Ref. [18] we have reported results for the $N=3\ {}^1P^o(1)$ resonance. dc electric-field influence on the Feshbach resonances below the hygrogenic ($N=2$) threshold and on the $N=2\ {}^1P^o$ shape resonance was studied in Refs. [19] and [20], respectively.

The influence of the electric field on the $N=2\ {}^1P^o$ shape resonance was studied also by Wendoloski and Reinhardt

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[21]. The stabilization method was used to investigate the Stark effect for the $N=2$ Feshbach resonances of H^- by Callaway and Rau [22].

In the present work we employ the method of the complex-coordinate rotation to study the influence of the dc electric field on the Feshbach resonances of Ps^- below the $N=3$ threshold of the positronium atom. In particular, we present results for the $M=0$ components of the lowest resonances of symmetries ${}^1S^e$, ${}^1P^o$, and ${}^1D^e$ associated with the $N=3$ positronium threshold—${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances.

II. DETAILS OF THE CALCULATIONAL PROCEDURE

Introducing as independent variables the relative coordinates $\vec{r}_1,\vec{r}_2$ of two electrons with respect to the positron, and the vector of the distance $\vec{r}_{12}$ between two electrons, the Hamiltonian of the positronium negative ion in the presence of a constant uniform electric field can be written as (Rydberg units are used throughout the paper)

$$\hat{H}=-2\nabla_1^2-2\nabla_2^2-2\nabla_1\cdot\nabla_2-\frac{2}{r_1}-\frac{2}{r_2}+\frac{2}{r_{12}}+\vec{F}\cdot(\vec{r}_1+\vec{r}_2), \tag{1}$$

where $\vec{F}$ is the external electric field measured in Rydberg units (a Rydberg unit of electric field strength is 2.57×10^9 V/cm).

The theoretical foundation and detailed description of the complex coordinate method can be found elsewhere [16]; here we shall give only briefly some details of the computational procedure used in the present calculation.

In the complex-rotation method [16], the radial coordinates in the Hamiltonian (1) are transformed according to the rule

$$r\to r\exp(i\theta), \tag{2}$$

where θ is the so-called rotation angle. Under this transformation the Hamiltonian given by Eq. (1) assumes the form

$$\hat{H}=-2(\nabla_1^2+\nabla_2^2+\nabla_1\cdot\nabla_2)\exp(-2i\theta)-2\left(\frac{1}{r_1}+\frac{1}{r_2}-\frac{1}{r_{12}}\right)\exp(-i\theta)+\vec{F}\cdot(\vec{r}_1+\vec{r}_2)\exp(i\theta). \tag{3}$$

The eigenvalues are obtained by solving the complex-eigenvalue problem

$$E=\frac{\langle\Psi\hat{H}\Psi\rangle}{\langle\Psi\Psi\rangle}, \tag{4}$$

with $E=E_r-i\Gamma/2$, E_r and Γ giving the position of the resonance and its width, respectively. As the basis functions we used in the present work the products of the Slater-type orbitals (properly symmetrized and coupled to correspond to given rotational and transpositional symmetries)

$$\Psi(\vec{r}_1,\vec{r}_2)=\phi_{n_1l_1}(r_1)\phi_{n_2l_2}(r_2)Y^{L,M}_{l_1,l_2}(1,2)S(\sigma_1,\sigma_2)-\phi_{n_1l_1}(r_2)\phi_{n_2l_2}(r_1)Y^{L,M}_{l_1,l_2}(2,1)S(\sigma_2,\sigma_1), \tag{5}$$

where

$$\phi_{nl}(r)=r^n\exp(-\xi_l r) \tag{6}$$

and $Y^{L,M}_{l_1,l_2}(1,2)$ and $S(\sigma_2,\sigma_1)$ are the eigenfunctions of total angular and spin momenta, respectively. They are constructed in the usual way, e.g., for $Y^{L,M}_{l_1,l_2}$ one has

$$Y^{L,M}_{l_1,l_2}(1,2)=\sum_{m_{l_1}m_{l_2}}C(l_1,l_2,L,m_{l_1},m_{l_2},M)\times Y_{l_1m_{l_1}}(1)Y_{l_2m_{l_2}}(2), \tag{7}$$

where $C(l_1,l_2,L,m_{l_1},m_{l_2},M)$ are the Clebsch-Gordan coefficients.

The parameters n, l of the Slater orbitals can be varied within the limits $0\leqslant l\leqslant N_{max}$, $l\leqslant n\leqslant N_{max}$, where the parameter N_{max} has the same value for both Slater orbitals in the formula (5). This parameter thus governs the number of the basis functions of a given symmetry which participate in the construction of the Hamiltonian matrix.

The parameters ξ_l in Eq. (6) can be chosen independently for different l and can be used in the calculation as additional nonlinear variational parameters. In the present calculation of the lowest resonances associated with the $N=3$ positronium threshold, the nonlinear parameters ξ_l's are chosen to correspond to the ''positronium'' values, i.e., $\xi_o=\xi_1=\xi_2=1/6$, $\xi_l=0.5/(l+1)$ for $l>2$. Such choice was motivated by the results of our previous calculation of these resonances in the field-free case [5], where it was found to give satisfactory results.

Due to the well-known selection rules for the matrix elements of the electric dipole operator the matrix of the Hamiltonian (1) calculated in the basis given by Eq. (5) assumes a blocklike structure (see Fig. 1), where diagonal blocks corresponding to different total angular momentum states are coupled together by the external electric field. In principle the blocks with arbitrary large angular momenta are present in the matrix of the Hamiltonian (1). Of course, doing practical calculations one should truncate the Hamiltonian matrix restricting the number of the angular momentum blocks taken into account. Since the problems of pure numerical origin put the limitations on the maximum matrix size which can be handled, one should find a compromise between the sizes of different individual angular momentum blocks and the number of different angular momentum blocks taken into account in the calculation. If the external electric field is not very large so that the arguments borrowed from the ordinary perturbation theory are at least qualitatively valid, then it is clear that in determining the resonance parameters of a resonance having a given value L of the total angular momentum

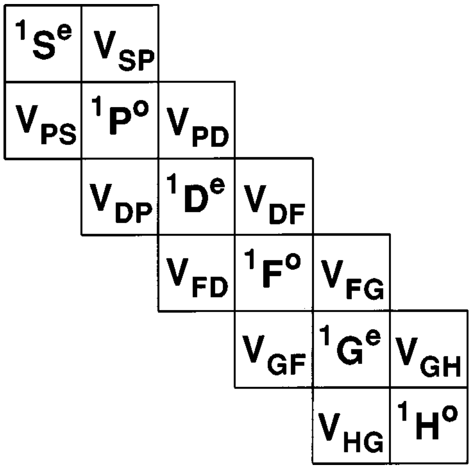

FIG. 1. Blocklike structure of the Hamiltonian matrix for the states with $M=0$.

in the field-free case it is the states with angular momenta that do not differ much from L that play the most important role.

In the present calculation we concentrated our attention on the three resonances $^1S^e(1)$, $^1P^o(1)$, and $^1D^e(1)$ associated with the $N=3$ positronium threshold. In the classification scheme based on the picture of the independent electrons in a central field, these states could be classified as $3s^2\ ^1S$, $3s3p\ ^1P$, and $3p^2\ ^1D$, respectively.

The block matrices up to $L_{\max}=5$ have been taken into account in the calculation. As for the sizes of the different angular momentum blocks within the Hamiltonian matrix, we actually ran three separate calculations, to be referred to below as calculations *A*, *B*, and *C*. In each of them an accent was on put on the states of one of the symmetries mentioned above ($^1S^e$, $^1P^o$, and $^1D^e$). More specifically, in our first calculation (calculation *A*) the dimensions of the angular momentum blocks on Fig. 1 were chosen as follows: $^1S^e$, 1140 basis functions; $^1P^o$, 1120 basis functions; $^1D^e$, 781; $^1F^o$ 212; $^1G^e$, 63; and $^1H^o$, 40 basis functions, making thus the total matrix size 3356. In our second calculation (calculation *B*), where the emphasis was put on the states of the $^1P^o$ symmetry, the dimensions of the angular momentum blocks on Fig. 1 were chosen as follows: $^1S^e$, 560 basis functions; $^1P^o$, 1360 basis functions; $^1D^e$, 1261; $^1F^o$, 212; $^1G^e$, 63; and $^1H^o$, 40 basis functions, which gave totally 3496 basis functions. Finally, in our third calculation (calculation *C*) we put emphasis on the states of the $^1D^e$ symmetry and the dimensions of the angular momentum blocks on Fig. 1 were chosen as follows: $^1S^e$, 455 basis functions; $^1P^o$, 920 basis functions; $^1D^e$, 1261; $^1F^o$, 464; $^1G^e$, 123; and $^1H^o$, 40 basis functions, making totally 3253 basis functions. The three calculations were different also in another respect. An important condition of the success of a complex rotation calculation is the optimal choice of the rotation angle θ in formula (3). When doing calculations where not a very large basis set is involved, the usual practice is to carry out calculations for different values of the rotation angle and to pick up the value for which the complex energy of a given resonance is least sensitive to the variations of the rotation angle, or to put it in more strict terms, the optimal θ value is chosen so that the absolute value of the derivative $\partial E/\partial\theta$ assumes the least possible value.

In the present calculation we dealt with too large matrices to follow this strategy, the diagonalization of 3000×3000 matrix being a rather time-consuming procedure. We adopted, therefore, another strategy. In each of the three calculations described above the θ value was chosen to be equal to the optimum θ value for the calculation of the complex energies of the resonances $^1S^e(1)$, $^1P^o(1)$, and $^1D^e(1)$ correspondingly, in the field-free case. With the size of the Hamiltonian matrix being in the field-free case not very big, the optimum θ values were found by repeating the calculations for different rotation angles and picking up the optimal value as described above. This procedure gave the following values for the rotation angle. Calculation *A*: $\theta=0.25$; calculation *B*: $\theta=0.30$; calculation *C*: $\theta=0.33$.

TABLE I. Results of three different calculations (marked in the table as *A*, *B*, and *C*) of the energies of the $M=0$ Stark components of $^1S^e(1)$ and $^1D^e(1)$ resonances associated with the $N=3$ Ps threshold for different values of the external electric-field strength.

F (Ry)		E_r (Ry)	Γ (Ry) $^1S^e(1)$	E_r (Ry)	Γ (Ry) $^1D^e(1)$
5×10^{-5}	*A*	-0.07076152	1.49269×10^{-4}	-0.0679214	4.662×10^{-5}
	B	-0.07076176	1.47931×10^{-4}	-0.0679226	4.654×10^{-5}
	C	-0.07076179	1.47944×10^{-4}	-0.0679224	4.694×10^{-5}
1×10^{-4}	*A*	-0.07099945	1.50290×10^{-4}	-0.0682030	5.67×10^{-5}
	B	-0.07099969	1.49022×10^{-4}	-0.0682074	5.47×10^{-5}
	C	-0.07099973	1.49027×10^{-4}	-0.0682073	5.51×10^{-5}
1.5×10^{-4}	*A*	-0.07141450	1.5241×10^{-4}	-0.0686890	1.120×10^{-4}
	B	-0.07141480	1.5149×10^{-4}	-0.0686985	1.129×10^{-4}
	C	-0.07141484	1.5145×10^{-4}	-0.0686984	1.132×10^{-4}
2×10^{-4}	*A*	-0.0720427	1.9529×10^{-4}	-0.0693636	3.518×10^{-4}
	B	-0.0720430	1.9418×10^{-4}	-0.0693768	3.652×10^{-4}
	C	-0.0720431	1.9416×10^{-4}	-0.0693766	3.657×10^{-4}

TABLE II. Comparison of the results of the calculations A,B,C and A',B',C' for the positions of the $M=0$ Stark components of ${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances associated with the $N=3$ Ps threshold for different values of the external electric field.

F (Ry)		E_r (Ry) ${}^1S^e(1)$	E_r (Ry) ${}^1P^o(1)$	E_r (Ry) ${}^1D^e(1)$
2×10^{-4}	A	-0.0720427	-0.064265	-0.069363
	A'	-0.0720430	-0.064235	-0.069368
	B	-0.0720430	-0.064276	-0.069376
	B'	-0.0720422	-0.064337	-0.069408
	C	-0.0720419	-0.064275	-0.069376
	C'	-0.0720431	-0.064231	-0.069372
3×10^{-4}	A	-0.073908	-0.065420	-0.070995
	A'	-0.073915	-0.065427	-0.070985
	B	-0.073910	-0.065426	-0.071003
	B'	-0.073904	-0.065359	-0.070992
	C	-0.073910	-0.065428	-0.071002
	C'	-0.073915	-0.065439	-0.070979

Thus three calculations (A, B, C) differed in two respects. First, the composition of the Hamiltonian matrix was different for each of these calculations (angular momentum blocks on Fig. 1 had different sizes). Second, the value of the rotation angle in formula (3) was different in each of these calculations. The comparison of these calculations can therefore give an estimation of the accuracy of the present calculation.

This comparison is presented in Tables I, II, and III. In Table I we give the results of the three calculations A, B, and C described above for real and imaginary parts of the energy of the $M=0$ Stark components of the resonances ${}^1S^e(1)$ and ${}^1D^e(1)$ for several values of the external electric-field strength. To save space we give a comparison of the three calculations mentioned above only for these two states, the calculation of the ${}^1P^o(1)$ resonance giving the same order of magnitude for the estimated error of our calculation. From the data presented in Table I we would estimate the error due to the truncation of the S, P, and D blocks in the energy matrix to finite-size blocks as being not larger than 7×10^{-6} Ry both for real and imaginary parts of energy for all three resonances considered.

This is, however, not the only possible source of numerical error that might be present in our calculation. Another possible source of numerical errors may be due to the fact that in calculations of this type the energy matrix itself must be truncated to include some finite number of different angular momentum blocks. We recall that in our calculations the angular momentum blocks with angular momentum up to $L=5$ were retained in the energy matrix. The convergence of our results with respect to the omission of angular momentum blocks with large angular momentum in the energy matrix can be studied if one performs a separate calculation with some of the large angular momentum blocks omitted.

TABLE III. Comparison of the results of the calculations A,B,C and A',B',C' for the widths of the $M=0$ Stark components of ${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances associated with the $N=3$ Ps threshold for different values of the external electric field.

F (Ry)		Γ (Ry) ${}^1S^e(1)$	Γ (Ry) ${}^1P^o(1)$	Γ (Ry) ${}^1D^e(1)$
2×10^{-4}	A	0.0001953	0.001169	0.000351
	A'	0.0001945	0.001141	0.000332
	B	0.0001942	0.001149	0.000365
	B'	0.0001971	0.001097	0.000381
	C	0.0001931	0.001149	0.000365
	C'	0.0001927	0.001103	0.000330
3×10^{-4}	A	0.000806	0.00192	0.001677
	A'	0.000803	0.00188	0.001681
	B	0.000804	0.00195	0.00170
	B'	0.000808	0.00201	0.00186
	C	0.000805	0.00195	0.001702
	C'	0.000806	0.00192	0.001692

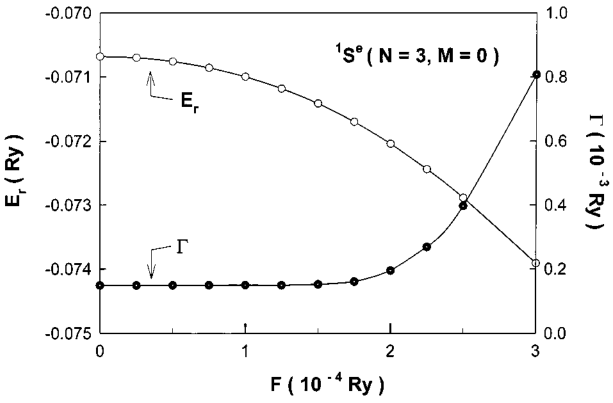

FIG. 2. Position and width of the $^1S^e(1)$ resonance as a function of external electric-field intensity.

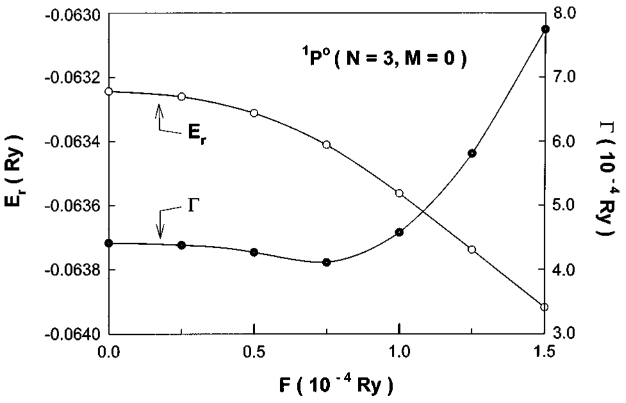

FIG. 3. Position and width of the $^1P^o(1)$ resonance ($M=0$ component) as a function of external electric-field intensity.

The results of such a calculation are presented in Tables II and III. The data given in the tables have been obtained as a result of the ''reduced'' variants of the calculations A, B, C. We recall that in the calculations A, B, C the dimensions of the separate angular momentum blocks in the Hamiltonian matrix were chosen as follows. For the calculation A: $^1S^e$, 1140 basis functions; $^1P^o$, 1120 basis functions; $^1D^e$, 781; $^1F^o$, 212; $^1G^e$, 63; and $^1H^o$, 40 basis functions, thus making the total matrix size 3356. For the calculation B: $^1S^e$, 560 basis functions; $^1P^o$, 1360 basis functions; $^1D^e$, 1261, $^1F^o$, 212; $^1G^e$, 63; and $^1H^o$, 40 basis functions. And finally, for the calculation C: $^1S^e$, 455 basis functions; $^1P^o$, 920 basis functions; $^1D^e$, 1261; $^1F^o$, 464; $^1G^e$, 123; and $^1H^o$, 40 basis functions. While doing the reduced calculations A', B', and C' we removed H and G blocks from the Hamiltonian matrix. Thus, the basis sets of the calculations A', B', and C' are obtained by removing the basis functions of the H and G symmetries from the basis sets of the calculations A, B, and C, respectively.

In Tables II and III we present a comparison of the results of the original calculations A, B, C and the results given by the modified calculations A', B', C' for the resonance positions (Table II) and widths (Table III).

It is clear that this source of possible errors of the calculation starts to play an important role only for large values of the external electric-field strength since, as the perturbation theory arguments suggest, the contribution of large angular momentum blocks for small electric-field values is suppressed. We therefore present in Tables II and III the results for the comparatively large values of the external electric-field strength. Comparison of the data suggests an estimation that for large electric-field values [$\approx(2-3)\times10^{-4}$ Ry] the absolute accuracy of our calculation should be not worse than 5×10^{-5} Ry.

One should emphasize that the estimation presented above should be considered as an upper bound of the numerical error we have in our calculation when external electric field is large. For the smaller electric-field values the error is of course much smaller.

Having established the estimations of the accuracy of our calculation we proceed to a more detailed discussion of our data.

III. DISCUSSION

The results of our calculation of the energies and widths of the $^1S^e(1)$, $^1P^o(1)$, and $^1D^e(1)$ resonances as functions of the external electric-field intensity are presented in Figs. 2–4 and Tables IV and V. An immediate observation one can make from the figures is that when the electric field is turned on, the widths of all of the three resonances stay nearly constant. In particular, the width of the $^1P^o(1)$ resonance decreases slightly in magnitude until F (the intensity of the external electric field) reaches a critical value, F_c with $F_c\approx7.5\times10^{-5}$ Ry. After passing that critical value of the electric-field strength the width starts to increase rapidly. Similarly for the $^1S^e(1)$ and $^1D^e(1)$ states the widths stay nearly constant until the field strength reaches the values $F_c\approx1.5\times10^{-4}$ Ry and $F_c\approx1.25\times10^{-4}$ Ry, respectively. After that the widths start to increase rapidly with the field.

The above results may be interpreted as follows. The $^1S^e(1)$, $^1D^e(1)$, and $^1P^o(1)$ Feshbach resonance states in Ps^- are highly correlated atomic states with strong two-electron ''bonding'' effects. When the external electric-field strength is not strong enough to break the two-electron bonding, the autoionization process proceeds very much like in the field-free case. The fact that the width of the $^1P^o(1)$ state decreases slightly for small electric-field values can be attributed to the process of redistribution of autoionization

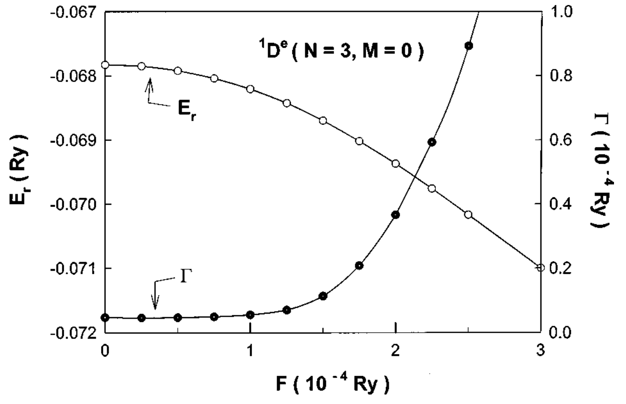

FIG. 4. Position and width of the $^1D^e(1)$ resonance ($M=0$ component) as a function of external electric-field intensity.

TABLE IV. Positions of the $M=0$ Stark components of ${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances associated with the $N=3$ Ps threshold as functions of electric-field strength.

F (Ry)	E_r (Ry) ${}^1S^e(1)$	${}^1P^o(1)$	${}^1D^e(1)$
0	−0.07068379	−0.06324445	−0.06782878
2.5×10^{-5}	−0.0707031	−0.0632609	−0.0678519
5×10^{-5}	−0.0707615	−0.063312	−0.067922
7.5×10^{-5}	−0.0708601	−0.063411	−0.068041
1×10^{-4}	−0.070999	−0.063563	−0.068207
1.25×10^{-4}	−0.071183	−0.063739	−0.068426
1.5×10^{-4}	−0.071414	−0.063918	−0.068698
1.75×10^{-4}	−0.071700	−0.064089	−0.069019
2×10^{-4}	−0.072042	−0.06427	−0.06937
2.25×10^{-4}	−0.072441	−0.06450	−0.06976
2.5×10^{-4}	−0.072889	−0.06477	−0.07017
3×10^{-4}	−0.073910	−0.06543	−0.07100

intensities between different singlet-spin states. When the external electric-field strength reaches the above-mentioned critical value F_c, the electric field becomes strong enough to break the two-electron bonding. The autoionization process then starts to be dominated by the one-electron tunneling effect. Since the thickness of the potential barrier, formed by the combined atomic Coulomb force and the external electric field, is reduced with the increase of the electric-field strength, the electron requires a shorter time to tunnel out, which results in the rapid increase of the width. On this stage, the autoionization process proceeds mostly through tunneling and is quite similar to that for the one-electron case (i.e., the Stark effect in hydrogen). The critical value of the electric-field strength which breaks the two-electron bonding differs from state to state, depending on the ''tightness'' of the doubly excited state relative to the parent positronium threshold.

Thus the observed behavior on Figs. 2–4 of the resonance width staying nearly constant (or even slightly decreasing) for small external electric-field values, followed by a rapid increase of the width when the electric field exceeds some critical value, is due to the fact that the doubly excited positronium negative ion presents an example of a highly correlated atomic system. Analogous behavior of the width as a function of the external electric field has been observed experimentally for the H^- system [13] and predicted theoretically for doubly excited states of neutral helium [23].

Figures 2–4 show also the changes of the resonance positions when the external electric field is turned on. It is seen that the energies of the ${}^1S^e(1)$, ${}^1D^e(1)$, and ${}^1P^o(1)$ resonances are all shifted downward, exhibiting the quadratic Stark effect.

It should be mentioned that in the present work we have not investigated the positron-electron annihilation process for the doubly excited Ps^-. In an earlier study [24] of the positron-electron annihilation rate for some $N=2$ doubly excited ${}^1S^e$ states, it was found that the autoionization rate is greater than the annihilation rate by a factor of about 10^4.

TABLE V. Widths of the $M=0$ Stark components of ${}^1S^e(1)$, ${}^1P^o(1)$, and ${}^1D^e(1)$ resonances associated with the $N=3$ Ps threshold as functions of electric-field strength.

F (Ry)	Γ (Ry) ${}^1S^e(1)$	${}^1P^o(1)$	${}^1D^e(1)$
0	1.493×10^{-4}	4.412×10^{-4}	4.561×10^{-5}
2.5×10^{-5}	1.476×10^{-4}	4.377×10^{-4}	4.56×10^{-5}
5×10^{-5}	1.479×10^{-4}	4.27×10^{-4}	4.7×10^{-5}
7.5×10^{-5}	1.484×10^{-4}	4.11×10^{-4}	4.9×10^{-5}
1×10^{-4}	1.490×10^{-4}	4.57×10^{-4}	5.5×10^{-5}
1.25×10^{-4}	1.496×10^{-4}	5.81×10^{-4}	7.0×10^{-5}
1.5×10^{-4}	1.51×10^{-4}	7.74×10^{-4}	1.13×10^{-4}
1.75×10^{-4}	1.61×10^{-4}	9.78×10^{-4}	2.08×10^{-4}
2×10^{-4}	1.94×10^{-4}	1.15×10^{-4}	3.66×10^{-4}
2.25×10^{-4}	2.69×10^{-4}	1.31×10^{-3}	5.93×10^{-4}
2.5×10^{-4}	3.95×10^{-4}	1.48×10^{-3}	8.9×10^{-4}
3×10^{-4}	8.05×10^{-4}	1.95×10^{-3}	1.70×10^{-3}

External electric field changes both autoionization and annihilation rates. It would be interesting to investigate the influence of the external electric field on the annihilation rates for the various states (ground and autoionizing ones) of Ps^-.

In summary, we have carried out a theoretical investigation of the influence of the electric field on the doubly excited $N=3$ Feshbach resonance states of Ps^-. We hope our work will stimulate further experimental investigations of this three-body atomic system.

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