Classical interventions in quantum systems. II. Relativistic invariance

Asher Peres*
Department of Physics, Technion—Israel Institute of Technology, 32 000 Haifa, Israel
(Received 11 June 1999; published 18 January 2000)

If several interventions performed on a quantum system are localized in mutually spacelike regions, they will be recorded as a sequence of ''quantum jumps'' in one Lorentz frame, and as a different sequence of jumps in another Lorentz frame. Conditions are specified that must be obeyed by the various operators involved in the calculations, so that these two different sequences lead to the same observable results. These conditions are similar to the equal-time commutation relations in quantum field theory. They are sufficient to prevent superluminal signaling. (The derivation of these results does not require most of the contents of the preceding article [Peres, Phys. Rev. A **61**, 022116 (2000)]. What is needed is briefly summarized here, so that the present article is essentially self-contained.)

PACS number(s): 03.65.Bz, 03.30.+p, 03.67.−a

I. THE PROBLEM

Quantum measurements [1] are usually considered quasi-instantaneous processes. In particular, they affect the wave function instantaneously throughout the entire configuration space. Measurements of finite duration [2] cannot alleviate this conundrum. Is this quasi-instantaneous change of the quantum state, caused by a local intervention, consistent with relativity theory? The answer is not obvious. The wave function itself is not a material object forbidden to travel faster than light, but we may still ask how the dynamical evolution of an extended quantum system that undergoes several measurements in distant spacetime regions is described in different Lorentz frames.

Difficulties were pointed out long ago by Bloch [3], Aharonov and Albert [4], and many others [5]. Still before them, in the very early years of quantum mechanics, Bohr and Rosenfeld [6] gave a complete relativistic theory of the measurement of quantum *fields*, but these authors were not concerned about the properties of the new quantum states that resulted from these measurements, and their work did not answer the question that was raised above. Other authors [7,8] considered detectors in relative motion, and therefore at rest in different Lorentz frames. These works also did not give an explicit answer to the above question: a detector in uniform motion is just as good as one that has undergone an ordinary spatial rotation (accelerated detectors involve new physical phenomena [9], and are not considered in this paper). The point is not how individual detectors happen to move, but how the effects due to these detectors are described in different ways in one Lorentz frame or another.

In the preceding paper [10], the notion of measurement was extended to the more general one of *intervention*. An intervention consists of the acquisition and recording of information by a measuring apparatus, possibly followed by the emission of classical signals for controlling the execution of further interventions. More generally, a consequence of the intervention may be a change of the environment in which the quantum system evolves. These effects are the *output* of the intervention. These notions are refined in Sec. II of the present paper so as to be applicable to relativistic situations.

A relativistic treatment is essential to analyze spacelike separated interventions, such as in Bohm's version of the Einstein-Podolsky-Rosen ''paradox'' [11,12] (hereafter EPRB) which is sketched in Fig. 1, with two coordinate systems in relative motion. In that experiment, a pair of spin-$\frac{1}{2}$ particles is prepared in a singlet state at time t_0 (referred to one Lorentz frame) or t_0' (referred to another Lorentz frame). The particles move apart and are detected by two observers. Each observer measures a spin component along an arbitrarily chosen direction. The two interventions are mutually spacelike as shown in the figure. Event A occurs first in t time, and event B is the first one in t' time. The evolution of the quantum state of this bipartite system appears to be genuinely different when recorded in two Lorentz frames in rela-

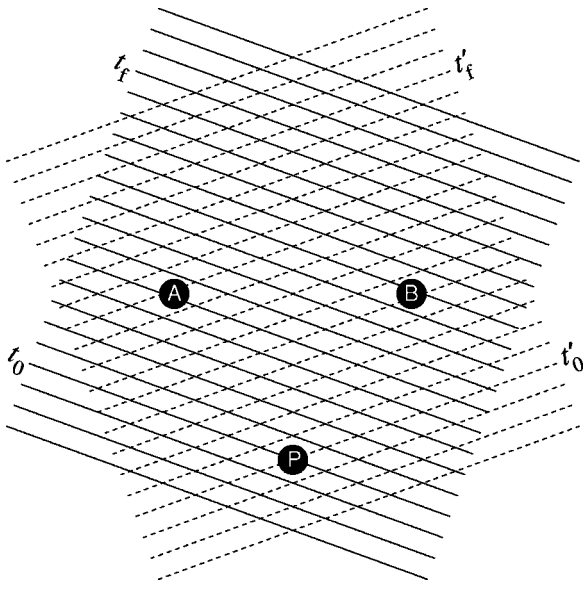

FIG. 1. A quantum system is prepared at point P. Interventions A and B are mutually spacelike. The solid and dotted lines represent equal times t and t' respectively, in two Lorentz frames in relative motion. Event A occurs first in t time, and event B is the first one in t' time.

*Electronic address: peres@photon.technion.ac.il

tive motion. The quantum states are not Lorentz transforms of each other. However, all the observable results are the same. The consistency of the theoretical formalism imposes definite relationships between the various operators used in the calculations. These are investigated in Sec. III.

Another example, this one taken from real life, is the detection system in the experimental facility of a modern high-energy accelerator [13]. Following a high-energy collision, thousands of detection events occur in locations that may be mutually spacelike. However, some of the detection events are mutually timelike, for example when the world line of a charged particle is recorded in an array of wire chambers. High-energy physicists use a language which is different from the one in the present paper. For them, an ''event'' is one high-energy collision together with all the subsequent detections that are recorded. This ''event'' is what I call here an experiment (while they called ''experiment'' the complete experimental setup that may be run for many months). Their ''detector'' is a huge machine weighing thousands of tons, while here the term detector means each elementary detecting element, such as a new bubble in a bubble chamber or a small segment of wire in a wire chamber. (A typical wire chamber records only which wire was excited. However, it is in principle possible to approximately locate the place in that wire where the electric discharge occurred, if we wish to do so.) Apart from the above differences in terminology, the events that follow a high-energy collision are an excellent example of the circumstances discussed in the present paper.

Returning to the Einstein-Podolsky-Rosen conundrum, we must analyze whether it actually involves a genuine quantum nonlocality. Such a claim has led some authors to suggest the possibility of superluminal communication. This would have disastrous consequences for relativistic causality [14]. Bell's theorem [15] asserts that it is impossible to mimic quantum correlations by classical local ''hidden'' variables, so that any classical imitation of quantum mechanics is necessarily nonlocal. However Bell's theorem does not imply the existence of any nonlocality in quantum theory itself. It is shown in Sec. IV that quantum measurements do not allow any information to be transmitted faster than the characteristic velocity that appears in the Green's functions of the particles involved in the experiment. In a Lorentz invariant theory, this limit is the velocity of light, of course. Section V is devoted to a few concluding remarks.

II. RELATIVISTIC INTERVENTIONS

This section includes a brief summary of some parts of the preceding paper [10], and contains all the material necessary to make the present one self-contained. Besides this summary, notions are introduced to cope with the relativistic nature of the phenomena under discussion.

First, recall that each intervention is described by a set of *classical* parameters [16,17]. The latter include the location of that intervention in space-time, referred to an arbitrary coordinate system. The coordinates are classical numbers, just as time in the Schrödinger equation is a classical parameter. We also have to specify the speed and orientation of the apparatus in that coordinate system and various other *input* parameters that define the experimental conditions under which the measuring apparatus operates. The input parameters are determined by classical information received from the past light cone at the point of intervention, or they may be chosen arbitrarily (in a random way) by the observer and/or the apparatus.

I just mentioned the existence of a past light cone. Actually, the only notion needed at the present stage is a *partial ordering* of the interventions: that is, there are no closed causal loops. This property defines the terms earlier and later. The input parameters of an intervention are deterministic (or possibly stochastic) functions of the parameters of earlier interventions, but not of the stochastic outcomes resulting from later interventions, as explained below.

In the conventional presentation of nonrelativistic quantum mechanics, each intervention has a (finite) number of *outcomes*, which are also known as ''results of measurements'' (for example, this or that detector clicks). In a relativistic treatment, the spatial separation of the detectors is essential, and each detector corresponds to a different intervention. The reason for this is that if several detectors are set up so that they act at a given time in one Lorentz frame, they would act at different times in another Lorentz frame. However, a knowledge of the time ordering of events is essential in our dynamical calculations, so that we want the parameters of an intervention to refer unambiguously to only one time (indeed to only one spacetime point). Therefore, an intervention can involve only one detector and it can have only two possible outcomes: either there was a ''click'' or there was not.

Note that the *absence* of a click, while a detector was present, is also a valid result of an intervention. The state of the quantum system does not remain unchanged: it has to change to respect unitarity. The mere presence of a detector that could have been excited implies that there has been an interaction between that detector and the quantum system. Even if the detector has a finite probability of remaining in its initial state, the quantum system correlated to the latter acquires a different state [18]. The absence of a click, when there could have been one, is also an event, and is part of the historical record.

The effect of an intervention on a quantum system initially prepared in the state ρ was given by Eq. (20) in the preceding paper:

$$\rho \to \rho'_\mu = \sum_m A_{\mu m} \rho A_{\mu m}^\dagger , \tag{1}$$

where μ is a label that indicates which detector was involved and whether or not it was activated. The initial ρ is assumed to be normalized to unit trace, and the trace of ρ'_μ is the probability of occurrence of outcome μ. Each symbol $A_{\mu m}$ in the above equation represents a *matrix* (not a matrix element). These may be rectangular matrices where the number of rows depends on μ. The number of columns is of course equal to the order of the initial ρ. Thus the Hilbert space of the resulting quantum system may have a different number of dimensions than the initial one. A quantum system whose description starts in a given Hilbert space may evolve in a

way that requires a set of Hilbert spaces with different dimensions. If one insists on keeping the same Hilbert space for the description of the entire experiment, with all its possible outcomes, this can still be achieved by defining it as a Fock space.

Each experiment yields a *record* that comprises a complete list of which detectors were available (including when and where), and whether these detectors reacted. Such a record is objective: everyone agrees on what happened (e.g., which detectors clicked) irrespective of the state of motion of the observers who read these records. Therefore, everyone agrees on the relative frequency of each type of record among all the records that are observed if the experiment is repeated many times, and the theoretical probabilities also have to be the same for everyone.

What is the role of relativity theory here? We may likewise ask what is the role of translation and/or rotation invariance in a nonrelativistic theory. The point is that the rules for computing quantum probabilities involve explicitly the space-time coordinates of the interventions. Lorentz invariance (or rotation invariance, as a special case) states that if the classical space-time coordinates are subjected to a particular linear transformation, then the probabilities remain the same. This invariance is not trivial because the rule for computing the probability of occurrence of a given record involves a sequence of mathematical operations corresponding to the time-ordered set of all the relevant interventions. If we only consider the Euclidean group, all we have to know is how to transform the classical parameters, and the wave function, and the various operators, under translations and rotations of the coordinates. However, when we consider genuine Lorentz transformations, we not only have to Lorentz transform the above symbols, but we are faced with a new problem: the natural way of calculating the result of a sequence of interventions, namely, by considering them in chronological order, is different for different inertial frames. The issue is not only a matter of covariance of the symbols at each intervention and between consecutive interventions. There are genuinely different prescriptions for choosing the sequence of mathematical operations in our calculation. The principle of relativity asserts that there are no privileged inertial frames. Therefore, these different orderings ought to give the same set of probabilities, and this demand is not trivial.

The experimental records are the only real thing we have to consider. Their observed relative frequencies are objective numbers and are Lorentz invariant. On the other hand, wave functions and operators are mathematical concepts useful for computing quantum probabilities, but they have no real existence [19]. All the difficulties that have been associated with a relativistic theory of quantum measurements are due to attributing a real nature to the symbols that represent quantum states.

Note also that while interventions are localized in space-time, quantum systems are pervasive. In each experiment, irrespective of its history, there is only one quantum system. The latter typically consists of several particles or other subsystems, some of which may be created or annihilated by the various interventions. The next two sections of this paper are concerned with sharp localized interventions on quantum systems that freely evolve throughout space-time between these interventions, and in particular with the Lorentz covariance of the results.

III. TWO MUTUALLY SPACELIKE INTERVENTIONS

Consider again the EPRB *gedankenexperiment* which was depicted in Fig. 1, with two coordinate systems in relative motion. There exists a Lorentz transformation connecting the initial states ρ (at time t_0) and ρ' (at time t_0') before the two interventions, and likewise there is a Lorentz transformation connecting the final states at times t_f and t_f' after completion of the two interventions. On the other hand, there is no Lorentz transformation relating the states at intermediate times represented by the lines that pass between interventions A and B [3,4]. This may be contrasted with the ontology of classical relativistic theory. Classical theory asserts that fields, velocities, etc., transform in a definite way and that the equations of motion of particles and fields behave covariantly. For example if the expression for the Lorentz force is written $f_\mu = F_{\mu\nu}u^\nu$ in one frame, the same expression is valid in any other frame. These symbols (f_μ, etc.) have objective values. They represent entities that really exist, according to the theory. On the other hand, wave functions have no objective value. They do not transform covariantly when there are interventions. Only the classical parameters attached to each intervention transform covariantly. However, in spite of the noncovariance of ρ, the final results of the calculations (the probabilities of specified sets of events) are Lorentz invariant.

Note that each line in Fig. 1 represents one instant of the time coordinate, as in the ordinary non-relativistic formulation of quantum mechanics. There is no way of defining a relativistic proper time for a quantum system which is spread all over space. It is possible to define a proper time for each *apparatus*, which has classical coordinates and follows a continuous world line. However, this is not necessary. We are only interested in a *discrete* set of interventions, and the latter are referred to a common coordinate system that covers the whole of space-time. There is no role for the private proper times that might be attached to the apparatuses' world lines.

If we attempt to generalize the parallel straight lines in Fig. 1 to a spacelike foliation in a curved space-time, as we would have in general relativity, we encounter the difficulty that no such foliation may exist globally. However, there is no need for such a global foliation, and in particular we do not assume the validity of a Schwinger-Tomonaga equation $i\delta\Psi/\delta\sigma = H(\sigma)\Psi$, as can be found in the work of Aharonov and Albert [4]. The only condition that we need is the absence of closed timelike curves. That is, if two events can be connected by continuous timelike (or null) curves, without past-future zigzags, then all these curves have the same orientation.

Returning to special relativity, consider the evolution of the quantum state in the Lorentz frame where intervention A is the first one to occur and has outcome μ, and B is the second intervention, with outcome ν. Between these two

events, nothing actually happens in the real world. It is only in our mathematical calculations that there is a deterministic evolution of the state of the quantum system. This evolution is *not* a physical process. For example, the quantum state of Schrödinger's legendary cat, doomed to be killed by an automatic device triggered by the decay of a radioactive atom, evolves into a superposition of ''live'' and ''dead'' states. This is a manifestly absurd situation for a real cat. The only meaning that such a quantum state can have is that of a mathematical tool for statistical predictions on the fates of numerous cats subjected to the same cruel experiment.

What distinguishes the intermediate evolution *between* interventions from the one occurring *at* an intervention is the unpredictability of the outcome of the latter: either there is a click or there is no click of the detector. This unpredictable macroscopic event starts a new chapter in the history of the quantum system which acquires a new state, according to Eq. (1). As long as there is no such branching, the quantum evolution will be called *free*, even though it may depend on external classical fields that are specified by the classical parameters of the preceding interventions.

Quantum mechanics asserts that during the free evolution of a closed quantum system, its state undergoes a unitary transformation generated by a Hamiltonian. The latter depends in a prescribed way on the preceding outcome(s) according to the protocol that has been specified for the experiment. The unitary operator for the evolution following intervention A with outcome μ, and ending at intervention B, will be denoted by U_{BA_μ}. (More generally, it is possible to consider an evolution which is continuously perturbed by the environment, as in the last section of the preceding paper [10]. In that case, the unitary evolution would be replaced by a more general continuous completely positive map, so that instead of U_{BA_μ} there would be Kraus operators with additional indices to be summed over. I shall refrain from using this more general formalism so as not to get into an unnecessarily complicated argument. In any case, the presence of such a pervasive environment would break Lorentz invariance.)

Note that the chronological order of the indices in U_{BA_μ} is from right to left (just as is the order for consecutive applications of a product of linear operators), and in particular that U_{BA_μ} does not depend on the future outcome at intervention B. Likewise, there is a unitary operator U_{A0} for the evolution that precedes event A, and an operator U_{fB_ν} for the final evolution that follows outcome ν of intervention B. The final quantum state at time t_f is given by a generalization of Eq. (1):

$$\rho_f = \sum_{m,n} K_{mn} \rho K_{mn}^\dagger , \tag{2}$$

where

$$K_{mn} = U_{fB_\nu} B_{\nu n} U_{BA_\mu} A_{\mu m} U_{A0} . \tag{3}$$

The same events can also be described in the Lorentz frame where B occurs first. We have, with the primed variables,

$$\rho'_f = \sum_{m,n} L'_{mn} \rho' L'^\dagger_{mn} , \tag{4}$$

where

$$L'_{mn} = V'_{fA_\mu} A'_{\mu m} V'_{AB_\nu} B'_{\nu n} V'_{B0} . \tag{5}$$

Here the unitary operator for the free evolution between the two interventions has been denoted by V'_{AB_ν}. It is not related in any obvious way to the operator U_{BA_μ}. These operators indeed correspond to different slabs of space-time. Likewise the other evolution operators in the primed coordinates have been called V' with appropriate subscripts. Note that $\mathrm{Tr}(\rho_f) = \mathrm{Tr}(\rho'_f)$ is the joint probability of occurrence of the records μ and ν during the experiment.

Einstein's principle of relativity asserts that there is no privileged inertial frame, and therefore both descriptions given above are equally valid. Formally, the states ρ_f (at time t_f) and the state ρ'_f (at time t'_f) have to be Lorentz transforms of each other. This requirement imposes severe restrictions on the various matrices that appear in the preceding equations. In order to investigate this problem, consider a continuous Lorentz transformation from the primed to the unprimed frame. As long as the order of occurrence of A and B is not affected by this continuous transformation of the spacetime coordinates, the latter is implemented in the quantum formalism by unitary transformations of the various operators. These unitary transformations obviously do not affect the observable probabilities.

Therefore, in order to investigate the issue of relativistic invariance, it is sufficient to consider two Lorentz frames where A and B are almost simultaneous: either A occurs just before B, or just after B. There is of course no real difference in the actual physical situations and the Lorentz ''transformation'' between these two arbitrarily close frames (primed and unprimed) is performed by the unit operator. In particular, $U_{BA_\mu} = \mathbf{1} = V'_{AB_\nu}$, since there is no finite time lapse for any evolution to occur between the two events. The only difference resides in our method for calculating the final quantum state: first A then B, or first B then A. Consistency of the two results is obviously achieved if

$$A_{\mu m} B_{\nu n} = B_{\nu n} A_{\mu m} \tag{6}$$

or

$$[A_{\mu m}, B_{\nu n}] = 0. \tag{7}$$

This equal-time commutation relation, which was derived here as a sufficient condition for consistency of the calculations, is always satisfied if the operators $A_{\mu m}$ and $B_{\nu n}$ are direct products of operators pertaining to the two subsystems:

$$A_{\mu m} = a_{\mu m} \otimes \mathbf{1} \quad \text{and} \quad B_{\nu n} = \mathbf{1} \otimes b_{\nu n} , \tag{8}$$

where $\mathbf{1}$ now denotes the unit matrix of each subsystem. This relationship is obviously fulfilled if there are two distinct apparatuses whose dynamical variables commute, and more-

over if *the dynamical variables of the quantum subsystems commute*. This is indeed a necessary condition for legitimately calling them subsystems.

The analogy with relativistic quantum field theory is manifest: field operators belonging to points at spacelike distances commute (or anticommute in the case of fermionic fields). Quantum field theory mostly uses the Heisenberg picture or the interaction picture, while in the present work it is the Schrödinger picture that is employed. This makes no difference in Eq. (7), which applies to equal times. Could we have here too anticommutation relations? It is easily seen that it is possible to introduce a minus sign on the right-hand side of Eq. (6), or even an arbitrary phase factor $e^{i\phi_{AB}}$. However, this generalization will not be investigated in the present paper, whose subject is quantum mechanics, not quantum field theory.

One may wonder whether the result expressed in Eq. (8) is trivial. Direct products were postulated in the very early years of quantum mechanics by Weyl [20] as the only reasonable way for describing composite systems. Here this representation was derived from an argument involving Lorentz invariance. However, such a proof may well be circular [21]: it assumes a relativistic partial ordering of events, i.e., the impossibility of superluminal signaling, while this impossibility is proved in quantum field theory by assuming the tensor product representation for composite systems. This issue was also investigated by Rosen [22] in the context of molecular biology. According to Rosen, while any microphysical system can be expressed as a composite of subsystems, there is no reason to suppose that such a factorization is unique, because rings of operators may in general be factored in many distinct ways. Only if it were found that the factorization is unique, this would imply that there is only one way in which the state of a system can be synthetized from the states of simpler subsystems.

Returning to Eq. (8), it is important to remember that an intervention can change the dimensions of the quantum system. Here is a simple example. The quantum system initially consists of a pair of spin-$\frac{1}{2}$ particles, as in the EPRB experiment. The two observers are called Alice and Bob, as usual. Alice, who intervenes at *A*, uses an apparatus that contains a subsystem *S* prepared as an entangled state of a spin-$\frac{1}{2}$ particle and a particle of spin 1. She receives a particle of spin $\frac{1}{2}$ (that is, one of the two particles of the quantum system under observation) and she measures the Bell operator [23] of the composite system formed by that particle and the spin-$\frac{1}{2}$ particle in *S*. That measurement can have four different outcomes, and according to its result Alice performs one of four specified unitary rotations on the spin-1 particle of *S*. She then discards everything but that particle of spin 1, and she releases the latter for future experiments. In this way, Alice's intervention converts an incoming spin-$\frac{1}{2}$ system into an outgoing spin 1 system.

Likewise, Bob's intervention, located spacelike with respect to Alice's, outputs a spin-2 particle when Bob receives one with spin $\frac{1}{2}$. How shall we describe the sequence of events in the frame where Alice is the first one to act, and in the frame where Bob is first?

Alice's $A_{\mu m}$ matrices are direct products of a matrix of dimensions 3×2 and the two-dimensional unit matrix, as in Eq. (8). Thereafter, there is a free unitary evolution, where U_{BA_μ} has rank 6. Then Bob's $B_{\nu n}$ matrices are direct products of a three-dimensional unit matrix and one of dimension 5×2. The final ρ is 15-dimensional (the final quantum system consists of a particle of spin 1 and a particle of spin 2). A similar description holds, *mutatis mutandis*, in the frame where Bob acts first (this frame is denoted by primes). The unitary matrix V'_{AB_ν} for the free evolution from *B* to *A* is of order 10, while U_{BA_μ} was of order 6. Obviously these cannot be Lorentz transforms of each other. They would not be Lorentz transforms even if dimensions were the same. However, the final ρ_f and ρ'_f have to be Lorentz transforms of each other.

Are $A_{\mu m}$ and $A'_{\mu m}$ related by a Lorentz transformation? We have seen that $A_{\mu m}$ is a direct product of a matrix of dimension 3×2 and the two-dimensional unit matrix. On the other hand, $A'_{\mu m}$ is a direct product of a matrix of dimension 3×2 and the five-dimensional unit matrix (the latter acts on the spin 2 particle that Bob has produced). Then the nontrivial parts of $A_{\mu m}$ and $A'_{\mu m}$, both rectangular 3×2 matrices, are Lorentz transforms of each other. We may also, if we wish, call the complete $A_{\mu m}$ and $A'_{\mu m}$ matrices ''Lorentz transforms'' if we accept that unit matrices of any order be considered as Lorentz transforms of each other.

IV. SUPERLUMINAL COMMUNICATION?

Bell's theorem [15] has led some authors to suggest the feasibility of superluminal communication by means of quantum measurements performed on correlated systems far away from each other [24,25]. It will now be shown that such a possibility is ruled out by the present relativistic formalism. We have already assumed that there exists a partial ordering of events. Superluminal communication would mean that the deliberate choice [26] of the test performed by an observer (or the random choice of the test performed by his apparatus) could influence in a deterministic way, at least statistically, the outputs of tests located at a spacelike distance from that observer (or apparatus) and having a later time coordinate. If this were true for any pair of spacelike separated events, this would lead to the possibility of propagating information backwards in time between events with timelike separation. For example, we may have *A* in the past light cone of *B*, and both *A* and *B* spacelike with respect to *C*. Then *B* could superluminally influence *C* in the frame where *B* occurs earlier than *C*, and in another frame *C* would likewise influence *A*, so that *B* could indirectly influence *A*. Therefore the assumption of Lorentz invariance, and the existence of random inputs, and the restriction of causal relationships between timelike related events to the future direction, are incompatible with causal relationships at spatial distances.

All this was discussed ad nauseam at the classical level many years ago, when tachyons were popular [27,28]. More recently, superluminal group velocities have actually been observed in barrier tunneling in condensed matter [29,30].

However, special relativity does not forbid the *group* velocity to exceed c. It is the *front* velocity of a wave packet that is the relevant criterion for signal transmission, and the front velocity never exceeds c. What novelty does quantum theory bring to this issue? The common wisdom is that the measuring process creates a ''reality'' that did not exist objectively before the intervention [31]. Let us examine this claim more carefully.

Consider a *classical* situation analogous to the EPRB setup: a bomb, initially at rest, explodes into two fragments carrying opposite angular momenta. Alice and Bob, far away from each other, measure arbitrarily chosen components of $\mathbf{J}_1$ and $\mathbf{J}_2$. (They can measure all the components, since these have objective values.) However, Bob's measurement tells him nothing of what Alice did, nor even whether she did anything at all. He can only know with certainty what *would* be the result found by Alice *if* she measures her $\mathbf{J}$ along the same direction as he, and make statistical inferences for other possible directions of Alice's measurement.

In the quantum world, consider two spin-$\frac{1}{2}$ particles in a singlet state. Alice measures σ_z and finds, say, $+1$. This tells her what the state of Bob's particle is, namely the probabilities that Bob *would* obtain $+1$ *if* he measures (or has measured, or will measure) $\boldsymbol{\sigma}$ along any direction he chooses. This is manifestly counterfactual information: nothing changes at Bob's location until he performs the experiment himself, or receives a classical message from Alice telling him the result that she found. No experiment performed by Bob can tell him whether Alice has measured (or will measure) her half of the singlet. The rules are exactly the same as in the classical case. It does not matter at all that quantum correlations are stronger than classical ones, and violate the Bell inequality.

A seemingly paradoxical way of presenting these results is to ask the following naive question: suppose that Alice finds that $\sigma_z=1$, while Bob does nothing. When does the state of Bob's particle, far away, become the one for which $\sigma_z=-1$ with certainty? Though this question is meaningless, it has a definite answer: Bob's particle state changes instantaneously. In which Lorentz frame is this instantaneous? In *any* frame. Whatever frame is chosen for defining simultaneity, the experimentally observable result is the same, owing to Eq. (7). This does not violate relativity because relativity is built in that equation, as will now be shown in a formal way.

Consider again Eqs. (2) and (3) which give the final (unnormalized) ρ_f following two interventions in which Alice obtains the result μ, and then Bob obtains the result ν. The probability for that pair of results is $\mathrm{Tr}\,(\rho_f)$. If event B lies in the future light cone of A, there can be ordinary classical communication from A to B and there is no causality controversy. We are interested here in the case where B is spacelike with respect to A. The problem is to prove that the probability of Bob's outcome ν is independent of whether or not Alice intervenes before him (in any Lorentz frame). Note that the unitary matrices in Eq. (3) are the Green's functions for the propagation of the *complete* quantum system, and that its subsystems may interact in a nontrivial way even when they are macroscopically separated (for example, these may be charged particles).

Fortunately, we do not need to know these Green's functions explicitly. We simply note that the probabilities that we are seeking are invariant under unitary transformations of the various operators in Eq. (3). In particular, they are not affected by transforming the initial U_{A0} and final U_{fB_ν}. There still is the intermediate unitary operator U_{BA_μ} for the propagation of the composite quantum system between times t_A and t_B. That quantum system is not a localized object. Its velocity is not a well-defined concept, and it is meaningless to argue that it is less than the velocity of light. However, it is possible to eliminate U_{BA_μ} by using the same stratagem as in Sec. III: we perform a Lorentz transformation of the space-time coordinates, which is implemented by a unitary transformation of the quantum operators (so that all probabilities are invariant), in such a way that the time elapsing between interventions A and B is arbitrarily small, and therefore $U_{BA_\mu}\to\mathbf{1}$.

The probability that Bob obtains a result ν, irrespective of Alice's result, thus is

$$p_\nu=\sum_\mu \mathrm{Tr}\left(\sum_{m,n} B_{\nu n}A_{\mu m}\rho A_{\mu m}^\dagger B_{\nu n}^\dagger\right). \tag{9}$$

We now employ Eq. (7) to exchange the positions of $A_{\mu m}$ and $B_{\nu n}$, and likewise those of $A_{\mu m}^\dagger$ and $B_{\nu n}^\dagger$, and then we move $A_{\mu m}$ from the first position to the last one in the product of operators in the traced parentheses. We thereby obtain expressions

$$\sum_m A_{\mu m}^\dagger A_{\mu m}=E_\mu\,. \tag{10}$$

As explained in Ref. [10], these are elements of a positive operator valued measure (POVM) that satisfy $\Sigma_\mu E_\mu=\mathbf{1}$. Therefore Eq. (9) reduces to

$$p_\nu=\mathrm{Tr}\left(\sum_n B_{\nu n}\rho B_{\nu n}^\dagger\right), \tag{11}$$

from which all expressions involving Alice's operators $A_{\mu m}$ have totally disappeared. The statistics of Bob's result are not affected at all by what Alice may do at a spacelike distance, so that no superluminal signaling is possible.

Note that in order to obtain meaningful results the entire experiment has to be considered as a whole: that is, what was prepared in the past light cone of *all* the interventions, and the complete set of results that were obtained, and are known in their joint future light cone. It is tempting, and often possible, to dissect an experiment into consecutive steps, just as it is often possible to discuss separately the properties of entangled particles. However, if ambiguities (or conflicting predictions, or any other ''paradoxes'') are encountered, what has to be done is to consider the whole entangled system and the whole experiment. Contrary to naive intuition, there is no physical state vector that interpolates between the initial and final states. Such interpolations can formally be

written, but they are not unique, not Lorentz covariant, and therefore physically meaningless.

However, there is an important exception to the above rule: if there exists a space-time point such that there are interventions in the past and future light cones of that point, but no intervention is spacelike with respect to it, then it is possible to divide the experiment into two steps, before and after that point. It is then meaningful to define not only an initial state ρ_0 and a final state ρ_f, but also an intermediate state ρ_i *at that point*. It is conventional to refer such a state to a spacelike hyperplane that passes through the point, but actually the only role of that hyperplane is to define the Lorentz frame in which we write a mathematical description of the state.

It thus appears that the notion of quantum state should be reassessed. There are two types of states: first, there are physically meaningful states, attached to space-time points with respect to which no classical intervention has a spacelike location. Then, between any two such points, we may draw a continuous timelike curve and try to attach a quantum state to each one of the points of that curve. These interpolating states can indeed be defined, as shown in the present paper, by considering a set of parallel spacelike hyperplanes. However, states defined in such a way are merely formal mathematical expressions, and they have no invariant physical meaning.

In summary, relativistic causality cannot be violated by quantum measurements. The fundamental physical assumption that was needed in the above proof was that Lorentz transformations of the space-time coordinates are implemented in quantum theory by *unitary* transformations of the various operators. This is the same as saying that the Lorentz group is a valid symmetry of the physical system.

V. CONCLUDING REMARKS

In the present paper it has been shown that a careful treatment, avoiding any speculations that have no experimental support, leads to the ''peaceful coexistence'' [32] of quantum mechanics and special relativity. The space-time coordinates of the observers' interventions are classical parameters subject to ordinary (classical) Lorentz transformations. The latter are implemented in quantum mechanics by unitary transformations of the operators. There are no essentially new features in the causality issue that arise because of quantum mechanics. Quantum correlations do not carry any information, even if they are stronger than Bell's inequality allows. The information has to be carried by material objects, quantized or not.

The issue of information transfer is essentially nonrelativistic. Replace ''superluminal'' by ''supersonic'' and the argument is exactly the same. The maximal speed of communication is determined by the dynamical laws that govern the physical infrastructure. In quantum field theory, the field excitations are called ''particles,'' and their speed over macroscopic distances cannot exceed the speed of light. In condensed-matter physics, linear excitations are called phonons and the maximal speed is that of sound.

The classical-quantum analogy (with bomb fragments carrying opposite angular momenta $\mathbf{J}_1 = -\mathbf{J}_2$) becomes complete if we use statistical mechanics for treating the classical case. The distribution of bomb fragments is given by a Liouville function in phase space. When Alice measures $\mathbf{J}_1$, the Liouville function for $\mathbf{J}_2$ is instantly altered, however far Bob is from Alice. No one would find this surprising, since it is universally agreed that a Liouville function is only a mathematical tool representing our statistical knowledge. Likewise, the wave function ψ, or the corresponding Wigner function [33] which is the quantum analog of a Liouville function, should be considered as mere mathematical tools for computing probabilities. It is only when they are regarded as physical objects that superluminal paradoxes arise.

The essential difference between the classical and quantum functions which change instantaneously as the result of measurements is that the classical Liouville function is attached to objective properties that are only imperfectly known. On the other hand, in the quantum case, the probabilities are attached to *potential* outcomes of mutually incompatible experiments, and these outcomes do not exist ''out there'' without the actual interventions. Unperformed experiments have no results [34].

ACKNOWLEDGMENTS

I am grateful to the California Institute of Technology, where this research program began, for its hospitality, and in particular to Chris Fuchs for many helpful comments and an inexhaustible supply of references. I also had fruitful discussions with Dagmar Bruß, Rainer Plaga, Barbara Terhal, and Daniel Terno. This work was supported by the Gerard Swope Fund and the Fund for Encouragement of Research.

[1] *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).

[2] A. Peres and W. K. Wootters, Phys. Rev. D **32**, 1968 (1985).

[3] I. Bloch, Phys. Rev. **156**, 1377 (1967).

[4] Y. Aharonov and D. Z. Albert, Phys. Rev. D **24**, 359 (1981); **29**, 228 (1984).

[5] A. Peres, in *Fundamental Problems in Quantum Theory*, edited by D. M. Greenberger and A. Zeilinger [Ann. (N.Y.) Acad. Sci. **755**, 445 (1995)], and references therein.

[6] N. Bohr and L. Rosenfeld, Mat. Fys. Medd. Dan. Vidensk. Selsk. **12** (8) (1933); an English translation appears in Ref. [1].

[7] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1993), p. 154.

[8] A. Suarez and V. Scarani, Phys. Lett. A **232**, 9 (1997).

[9] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[10] A. Peres, preceding paper, Phys. Rev. A **61**, 022116 (2000).

[11] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[12] D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951), p. 614.
[13] K. Ahmet *et al.*, Nucl. Instrum. Methods Phys. Res. A **305**, 275 (1991).
[14] J. Berkovitz, Stud. Hist. Philos. Mod. Phys. **29**, 183 (1998); **29**, 509 (1998).
[15] J. S. Bell, Physics (Utrecht) **1**, 195 (1964).
[16] Ph. Blanchard and A. Jadczyk, Found. Phys. **26**, 1669 (1996); Int. J. Theor. Phys. **37**, 227 (1998).
[17] I. C. Percival, Phys. Lett. A **244**, 495 (1998).
[18] R. H. Dicke, Am. J. Phys. **49**, 925 (1981).
[19] H. P. Stapp, Am. J. Phys. **40**, 1098 (1972).
[20] H. Weyl, *Gruppentheorie und Quantenmechanik* (Hirzel, Leipzig, 1928); translated by H. P. Robertson, *The Theory of Groups and Quantum Mechanics* (Methuen, London, 1931; reprinted by Dover, New York), p. 91.
[21] J. B. Kennedy, Philos. Sci. **62**, 543 (1995).
[22] R. Rosen, Bull. Math. Biophys. **22**, 227 (1960).
[23] S. L. Braunstein, A. Mann, and M. Revzen, Phys. Rev. Lett. **68**, 3259 (1992).
[24] N. Herbert, Found. Phys. **12**, 1171 (1982).
[25] A. Garuccio, in *Quantum Interferometry*, edited by by F. De Martini, G. Denardo, and Y. Shih (VCH, Weinheim, 1996).
[26] A. Peres, Found. Phys. **16**, 573 (1986).
[27] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962).
[28] G. Feinberg, Phys. Rev. **159**, 1089 (1967).
[29] R. Y. Chiao and A. M. Steinberg, in *Progress in Optics XXXVII*, edited by E. Wolf (Elsevier, Amsterdam, 1997); Phys. Scr. **T76**, 61 (1998).
[30] J. C. Garrison, M. W. Mitchell, R. Y. Chiao, and E. L. Bolda, Phys. Lett. A **245**, 19 (1998).
[31] J. A. Wheeler, in *Mathematical Foundations of Quantum Theory*, edited by A. R. Marlow (Academic Press, New York, 1978), pp. 9–48.
[32] A. Shimony, Internat. Philos. Quarterly **18**, 3 (1978).
[33] E. Wigner, Phys. Rev. **40**, 749 (1932).
[34] A. Peres, Am. J. Phys. **46**, 745 (1978).