Exact diagonalization of two quantum models for the damped harmonic oscillator

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The damped harmonic oscillator is a workhorse for the study of dissipation in quantum mechanics. However, despite its simplicity, this system has given rise to some approximations whose validity and relation to more refined descriptions deserve a thorough investigation. In this work, we apply a method that allows us to diagonalize exactly the dissipative Hamiltonians that are frequently adopted in the literature. Using this method, we derive the conditions of validity of the rotating-wave approximation (RWA) and show how this approximate description relates to more general ones. We also show that the existence of dissipative coherent states is intimately related to the RWA. Finally, through the evaluation of the dynamics of the damped oscillator, we notice an important property of the dissipative model that has not been properly accounted for in previous works, namely the necessity of new constraints to the application of the factorizable initial conditions.

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I. INTRODUCTION

The study of dissipative systems and in particular of Brownian motion has been pursued for a long time in the context of classical [1] and quantum mechanics [2]. Although there have been a number of publications in this area, there are some subtle points that have never been properly investigated in the literature. Among these we could mention three major ones: a careful investigation of the relation between different models [3], the existence of dissipative coherent states [4–7], or the condition for the employment of factorizable initial conditions. These are exactly the issues we shall address in this paper.

Usually the dissipation in the system is described as a consequence of its coupling to a reservoir. The properties of this dissipative system are generally studied through the evaluation of the time evolution of its reduced density operator. This evolution is often described either by a generalization of the Feynman-Vernon approach [2,8–10] or through master equations [4–7,11–15]. In this work the properties of the system will be studied through exact diagonalization of different Hamiltonians of the dissipative models.

We will consider a damped harmonic oscillator. The usual models of dissipation consist of coupling the harmonic oscillator to a reservoir that is conveniently chosen as a group of N noninteracting oscillators. The coupling between the two systems is bilinear in the creation and destruction operators of quanta of energy. Then the Hamiltonian of the total system is given by [4]

$$\hat{H}=\hat{H}_{\rm sis}+\hat{H}_{\rm res}+\hat{H}_{\rm int}, \tag{1.1}$$

being

$$\hat{H}_{\rm sis}=\hbar\omega_o\hat{a}^\dagger\hat{a}, \quad \hat{H}_{\rm res}=\hbar\sum_j \omega_j\hat{b}_j^\dagger\hat{b}_j, \tag{1.2}$$

$$\hat{H}_{\rm int}=\hbar(\hat{a}^\dagger+\hat{a})\sum_j (k_j\hat{b}_j+k_j^*\hat{b}_j^\dagger), \tag{1.3}$$

where we consider a harmonic oscillator with frequency ω_o (the system of interest) interacting with a bath of oscillators with frequencies ω_j through the coupling constants k_j's. We will take the limit of a continuous spectrum of excitations in the reservoir of the Hamiltonian $\hat{H}$. Then we will diagonalize $\hat{H}$ and determine the time evolution of the operator $\hat{a}$ exactly. The analysis of $\hat{a}(t)$ will determine the conditions of validity of the rotating-wave approximation (RWA) which consists of neglecting the terms $k_j\hat{a}\hat{b}_j+k_j^*\hat{a}^\dagger\hat{b}_j^\dagger$ in Eq. (1.2) and writing

$$\hat{H}_{\rm int}^{\rm RWA}=\hbar\sum_j (k_j\hat{a}^\dagger\hat{b}_j+k_j^*\hat{a}\hat{b}_j^\dagger). \tag{1.4}$$

Once this has been accomplished, we will discuss the existence of dissipative coherent states. Some authors [4–7] have stated that the coherent states are special states that remain pure during their decay in dissipative systems. We will show that the existence of these dissipative coherent states is directly related to the RWA; they can only exist at zero temperature and in systems that meet the conditions required for the RWA.

Once we have determined the evolution of the operator $\hat{a}(t)$ of the system, we can determine the evolution of any of its observables. However, the dynamics of these observables will depend on the specific form adopted for the coupling constants k_j as functions of the frequencies ω_j. Our method holds for an arbitrary form, but in order to compare our results with the Caldeira-Leggett model [2] we will reduce our results to the case where the function becomes the same as the one they have adopted. Then, as in Refs. [9,10], we

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will determine the evolution of the mean value of the position operator $\langle \hat{q}(t) \rangle$ of the damped oscillator. The result of this calculation reveals a very special need to carefully treat the initial time of the motion. We propose a simple initial condition that eliminates the initial transient that would appear in the evolution of $\langle \hat{q}(t) \rangle$ and we believe that it is enough to eliminate most of or maybe all the initial transients (in a certain time scale) which were noticed in this system in previous works [11,12,14].

The paper is organized as follows. In Sec. II we write the Hamiltonian (1.1) in the limit of a continuous spectrum for the reservoir excitations and we diagonalize it exactly within and without the RWA. We compare the model given by the Hamiltonian (1.1) with the dissipative model presented in [2] in Sec. III. Here, we also determine the relation between the coupling function $|\nu(\omega)|^2$, introduced in Sec. II, and the spectral function $J(\omega)$ introduced in [8]. In Sec. IV we analyze the relevance of the different terms that appear in the calculation of the evolution of the operator $\hat{a}(t)$ with relation to the intensity of the dissipation in the system. In Sec. V we show under which conditions the evolution of the operator $\hat{a}(t)$ is reduced to that given in the RWA. In Sec. VI we show that the existence of dissipative coherent states is only possible within the RWA. In Sec. VII we present the calculation of the evolution of the mean value of the position of the damped harmonic oscillator. In Sec. VIII we discussed the physical meaning of the initial condition proposed in Sec. VII. Finally, we discuss the main results and conclusions in Sec. IX.

II. DIAGONALIZATION OF THE DISSIPATIVE HAMILTONIANS

A. Treating a reservoir with a continuous spectrum

We can rewrite the Hamiltonian (1.1) considering a continuous spectrum of excitations in the reservoir by making use of the transformation between the discrete boson operators $\hat{b}_j$ and the continuous ones $\hat{b}_\Omega$ [16],

$$\hat{b}_j = \sqrt{g(\Omega_j)} \int_{1/g(\Omega_j)} d\Omega \, \hat{b}_\Omega , \tag{2.1}$$

where $g(\Omega_j)d\Omega_j$ is the number of modes in the reservoir with frequencies between Ω_j and $\Omega_j + d\Omega_j$, and $\int_{1/g(\Omega_j)} d\Omega$ represents an integration in a band of width $1/g(\Omega_j)$ around Ω_j. The operators $\hat{b}_\Omega$ then satisfy the commutation relation

$$[\hat{b}_\Omega , \hat{b}^\dagger_{\tilde{\Omega}}] = \delta(\Omega - \tilde{\Omega}), \tag{2.2}$$

and all other commutators vanish.

Under the transformation (2.1), we find

$$\hat{H}_{\rm int} = \hbar(\hat{a}^\dagger + \hat{a}) \int d\Omega \sqrt{g(\Omega)} [k(\Omega)\hat{b}_\Omega + k^*(\Omega)\hat{b}^\dagger_\Omega], \tag{2.3}$$

where we considered that $g(\Omega_j)$ and $k(\Omega_j)$ are constant inside the interval $1/g(\Omega_j)$ and that $\Sigma_j \int_{1/g(\Omega_j)} d\Omega$ is nothing but $\int d\Omega$, where this last integral covers the whole spectrum of excitations of the reservoir. Then the total Hamiltonian of our system is given by

$$\hat{H} = \hbar\omega_o \hat{a}^\dagger \hat{a} + \hbar \int \Omega \hat{b}^\dagger_\Omega \hat{b}_\Omega \, d\Omega + \hbar(\hat{a}^\dagger + \hat{a}) \int [\nu(\Omega)\hat{b}_\Omega + \nu^*(\Omega)\hat{b}^\dagger_\Omega] d\Omega, \tag{2.4}$$

where

$$\nu(\Omega) = \sqrt{g(\Omega)} k(\Omega). \tag{2.5}$$

B. The Hamiltonian within the rotating-wave approximation

We will now perform a canonical transformation and apply the procedure proposed by Fano [17] in order to diagonalize the Hamiltonian of our global system in the RWA, which is written as

$$\hat{H}^{\rm RWA} = \hbar\omega_o \hat{a}^\dagger \hat{a} + \hbar \int \Omega \hat{b}^\dagger_\Omega \hat{b}_\Omega \, d\Omega + \hbar \int [\nu(\Omega)\hat{a}^\dagger \hat{b}_\Omega + \nu^*(\Omega)\hat{a}\hat{b}^\dagger_\Omega] d\Omega. \tag{2.6}$$

The diagonalization procedure presented in the sequel is basically a review of the method presented in [18]. Our goal is to find an operator that satisfies the eigenoperator equation

$$[\hat{A}_\omega , \hat{H}^{\rm RWA}] = \hbar\omega \hat{A}_\omega , \tag{2.7}$$

and therefore has its evolution trivially given by $\hat{A}_\omega(t) = \hat{A}_\omega e^{-i\omega t}$.

The new operator $\hat{A}_\omega$ can be written in terms of the operator $\hat{a}$ of the system and of the operators $\hat{b}_\Omega$ of the reservoir in the form

$$\hat{A}_\omega = \alpha_\omega \hat{a} + \int d\Omega \, \beta_{\omega,\Omega} \hat{b}_\Omega . \tag{2.8}$$

Substituting this expression for $\hat{A}_\omega$ as well as Eq. (2.6) for $\hat{H}^{\rm RWA}$ in Eq. (2.7) and calculating the commutators, we have

$$\omega_o \alpha_\omega \hat{a} + \alpha_\omega \int d\Omega \, \nu(\Omega)\hat{b}_\Omega + \int d\Omega \, \Omega \beta_{\omega,\Omega} \hat{b}_\Omega + \int d\Omega \, \nu^*(\Omega)\beta_{\omega,\Omega}\hat{a} = \omega \left(\alpha_\omega \hat{a} + \int d\Omega \, \beta_{\omega,\Omega} \hat{b}_\Omega \right). \tag{2.9}$$

Now, taking the commutator of this expression with $\hat{a}^\dagger$ and $\hat{b}^\dagger_\Omega$, we obtain

$$\omega_o \alpha_\omega + \int d\Omega \, \nu^*(\Omega)\beta_{\omega,\Omega} = \omega\alpha_\omega , \tag{2.10}$$

$$v(\Omega)\alpha_\omega+\Omega\beta_{\omega,\Omega}=\omega\beta_{\omega,\Omega}, \tag{2.11}$$

respectively. Imposing

$$[\hat{A}_\omega,\hat{A}^\dagger_{\tilde{\omega}}]=\delta(\omega-\tilde{\omega}), \tag{2.12}$$

we have

$$\alpha_\omega\alpha^*_{\tilde{\omega}}+\int d\Omega\ \beta_{\omega,\Omega}\beta^*_{\tilde{\omega},\Omega}=\delta(\omega-\tilde{\omega}). \tag{2.13}$$

The system of equations (2.10), (2.11), and (2.13) is identical to the one presented in [17]. The solution is given by

$$|\alpha_\omega|^2=\frac{|v(\omega)|^2}{[\omega-\omega_o-F(\omega)]^2+[\pi|v(\omega)|^2]^2}, \tag{2.14}$$

with an arbitrary phase of α_ω, and

$$\beta_{\omega,\Omega}=\left[\mathrm{P}\frac{1}{\omega-\Omega}+\frac{\omega-\omega_o-F(\omega)}{|v(\omega)|^2}\delta(\Omega-\omega)\right]v(\Omega)\alpha_\omega, \tag{2.15}$$

where

$$F(\omega)=\mathrm{P}\int\frac{|v(\Omega)|^2}{\omega-\Omega}d\Omega, \tag{2.16}$$

and P denotes the principal part.

We can calculate the evolution of the operator $\hat{a}$ of the system expressing it as a function of the operators $\hat{A}_\omega$. We can write $\hat{a}$ as a function of $\hat{A}_\omega$ in the following way:

$$\hat{a}=\int d\omega\ f_\omega\hat{A}_\omega. \tag{2.17}$$

Taking the commutator $[\hat{a},\hat{A}^\dagger_\omega]$, first using Eq. (2.8) and then Eq. (2.17), we obtain $f_\omega=\alpha^*_\omega$. Therefore, the evolution of the operator $\hat{a}$ is given by

$$\hat{a}(t)=\int d\omega\ \alpha^*_\omega\hat{A}_\omega e^{-i\omega t}. \tag{2.18}$$

Substituting the expression for $\hat{A}_\omega$ in this equation and using Eq. (2.15), we obtain

$$\begin{aligned}\hat{a}(t)=&\int d\omega|\alpha_\omega|^2e^{-i\omega t}\hat{a}\\&+\int d\Omega\,v(\Omega)\left\{\int d\omega|\alpha_\omega|^2\mathrm{P}\frac{1}{\omega-\Omega}e^{-i\omega t}\right.\\&\left.+\frac{|\alpha_\Omega|^2}{|v(\Omega)|^2}[\Omega-\omega_o-F(\Omega)]e^{-i\Omega t}\right\}\hat{b}_\Omega.\end{aligned} \tag{2.19}$$

C. The Hamiltonian without the rotating-wave approximation

Now we will present the diagonalization of the Hamiltonian (2.4) without the RWA. The procedure that we will present is similar to the one adopted in [19].

Again we want to find an operator $\hat{A}_\omega$ that satisfies Eq. (2.7), with $\hat{H}$ in the place of $\hat{H}^{\mathrm{RWA}}$, and Eq. (2.12). Then we write $\hat{A}_\omega$ in the form

$$\hat{A}_\omega=\alpha_\omega\hat{a}+\int d\Omega\ \beta_{\omega,\Omega}\hat{b}_\Omega+\chi_\omega\hat{a}^\dagger+\int d\Omega\ \sigma_{\omega,\Omega}\hat{b}^\dagger_\Omega. \tag{2.20}$$

Imposing Eqs. (2.7), and (2.12), we obtain (see Appendix A)

$$|\alpha_\omega|^2=\left(\frac{\omega+\omega_o}{2\omega_o}\right)^2\frac{1}{|v(\omega)|^2[\pi^2+z^2(\omega)]}, \tag{2.21}$$

$$\beta_{\omega,\Omega}=\left[\mathrm{P}\frac{1}{\omega-\Omega}+z(\omega)\delta(\omega-\Omega)\right]\frac{2\omega_o}{\omega+\omega_o}v(\Omega)\alpha_\omega, \tag{2.22}$$

$$\chi_\omega=\frac{\omega-\omega_o}{\omega+\omega_o}\alpha_\omega, \tag{2.23}$$

$$\sigma_{\omega,\Omega}=\frac{1}{\omega+\Omega}\frac{2\omega_o}{\omega+\omega_o}v^*(\Omega)\alpha_\omega, \tag{2.24}$$

where

$$z(\omega)=\frac{\omega^2-\omega_o^2-2\omega_oH(\omega)}{2\omega_o|v(\omega)|^2} \tag{2.25}$$

and

$$H(\omega)=F(\omega)-G(\omega)=\mathrm{P}\int\frac{|v(\Omega)|^2}{\omega-\Omega}d\Omega-\int\frac{|v(\Omega)|^2}{\omega+\Omega}d\Omega. \tag{2.26}$$

We can express $\hat{a}$ as a function of $\hat{A}_\omega$ and $\hat{A}^\dagger_\omega$ in the following way:

$$\hat{a}=\int d\omega\ \phi_\omega\hat{A}_\omega+\int d\omega\ \varphi_\omega\hat{A}^\dagger_\omega. \tag{2.27}$$

Now taking, again, the commutators $[\hat{a},\hat{A}^\dagger_\omega]$ and $[\hat{a},\hat{A}_\omega]$, we obtain $\phi_\omega=\alpha^*_\omega$ and $\varphi_\omega=-\chi_\omega$. Substituting the expression (2.20) for $\hat{A}_\omega$ in Eq. (2.27), the time evolution of the operator a can be easily written as

$$\hat{a}(t)=\int \frac{d\omega}{\pi}|L(\omega)|^2\{A(\omega)\cos(\omega t)\hat{a} - i[B(\omega)\hat{a}+C(\omega)\hat{a}^\dagger]\sin(\omega t)\} + \int \frac{d\Omega}{\pi}B_1(\Omega;t)\hat{b}_\Omega+\int \frac{d\Omega}{\pi}B_2(\Omega;t)\hat{b}^\dagger_\Omega, \tag{2.28}$$

where

$$A(\omega)=2\omega, \quad B(\omega)=\frac{\omega^2+\omega_o^2}{\omega_o}, \quad C(\omega)=\frac{\omega^2-\omega_o^2}{\omega_o}, \tag{2.29}$$

$$B_1(\Omega;t)=v(\Omega)\{(\omega_o+\Omega)[X(\Omega;t)+Z(\Omega)e^{-i\Omega t}] - iY_{(+)}(\Omega;t)\}, \tag{2.30}$$

$$B_2(\Omega;t)=v^*(\Omega)\{(\omega_o-\Omega)[X(\Omega;t)+Z(\Omega)e^{i\Omega t}] - iY_{(-)}(\Omega;t)\}, \tag{2.31}$$

with

$$X(\Omega;t)=\mathrm{P}\int d\omega\frac{2|L(\omega)|^2}{\omega^2-\Omega^2}\omega\cos(\omega t), \tag{2.32}$$

$$Y_{(\pm)}(\Omega;t)=\mathrm{P}\int d\omega\frac{2|L(\omega)|^2}{\omega^2-\Omega^2}(\omega^2\pm\omega_o\Omega)\sin(\omega t), \tag{2.33}$$

$$Z(\Omega)=\frac{|L(\Omega)|^2}{|v(\Omega)|^2}\left[\frac{\Omega^2-\omega_o^2}{2\omega_o}-H(\Omega)\right], \tag{2.34}$$

$$|L(\omega)|^2=\frac{2\pi\omega_o|v(\omega)|^2}{[\omega^2-\omega_o^2-2\omega_oH(\omega)]^2+[2\pi\omega_o|v(\omega)|^2]^2}. \tag{2.35}$$

III. THE MODEL OF COORDINATE-COORDINATE COUPLING

The expressions obtained for the evolution of the operator $\hat{a}(t)$, within or without the RWA, remained written in terms of the coupling function $|v(\omega)|^2$. Therefore, the choice of the function $|v(\omega)|^2$ will determine the dynamics of the damped oscillator. We will choose the function $|v(\omega)|^2$ by comparing the dissipation model corresponding to the Hamiltonian (2.4) to the one presented in [2] that corresponds to the following Hamiltonian:

$$\hat{H}=\frac{\hat{p}^2}{2M}+V(\hat{q})+\sum_j\left(\frac{\hat{p}_j^2}{2m_j}+\frac{m_j\omega_j^2}{2}\hat{q}_j^2\right) - \sum_j C_j\hat{q}_j\hat{q}+V_R(\hat{q}), \tag{3.1}$$

where the counterterm $V_R(\hat{q})$, which cancels the additional contribution to $V(\hat{q})$ due to the coupling of the system to the reservoir, is given by

$$V_R(\hat{q})=\sum_j\frac{C_j^2}{2m_j\omega_j^2}\hat{q}^2. \tag{3.2}$$

The spectral function $J(\omega)$ is defined by

$$J(\omega)=\frac{\pi}{2}\sum_j\frac{C_j^2}{m_j\omega_j}\delta(\omega-\omega_j)=\frac{\pi}{2}\frac{g(\omega)C_\omega^2}{m_\omega\omega}, \tag{3.3}$$

where we have taken the limit of a continuous spectrum and used $g(\omega)$ from Eq. (2.1). For Ohmic dissipation,

$$J(\omega)=\begin{cases}2M\gamma\omega & \text{if } \omega<\Omega_c,\\ 0 & \text{if } \omega>\Omega_c,\end{cases} \tag{3.4}$$

where Ω_c is a cutoff frequency, much larger than the natural frequencies of the motion of the system of interest. But in our calculations we will conveniently use the Drude form

$$J(\omega)=\frac{2M\gamma\omega}{(1+\omega^2/\Omega_c^2)}. \tag{3.5}$$

We are treating a damped harmonic oscillator so $V(\hat{q})=1/2M\omega_o^2\hat{q}^2$. Applying the usual definitions of the operators $\hat{a}$ and $\hat{b}_j$,

$$\hat{a}=\sqrt{\frac{M\omega_o}{2\hbar}}\left(\hat{q}+\frac{i}{M\omega_o}\hat{p}\right), \quad \hat{b}_j=\sqrt{\frac{m_j\omega_j}{2\hbar}}\left(\hat{q}_j+\frac{i}{m_j\omega_j}\hat{p}_j\right), \tag{3.6}$$

we can rewrite Eq. (3.1), initially without the inclusion of the counterterm $V_R(\hat{q})$, as

$$\hat{H}=\hbar\omega_o\hat{a}^\dagger\hat{a}+\sum_j\hbar\omega_j\hat{b}_j^\dagger\hat{b}_j - \frac{\hbar}{2}\sqrt{\frac{1}{M\omega_o}}(\hat{a}+\hat{a}^\dagger)\sum_j\frac{C_j}{\sqrt{m_j\omega_j}}(\hat{b}_j+\hat{b}_j^\dagger) \tag{3.7}$$

(measuring the energy of the system from the energy of the vacuum). Now we can use the transformation (2.1) in order to consider a continuous spectrum for the excitations of the reservoir. The second term on the right-hand side of Eq. (3.7) becomes

$$\hat{H}_{\mathrm{res}}=\hbar\int\omega\hat{b}_\omega^\dagger\hat{b}_\omega\,d\omega \tag{3.8}$$

and its last term can be written in the following way:

$$\hat{H}_{\mathrm{int}}=-\frac{\hbar}{2}\sqrt{\frac{1}{M\omega_o}}(\hat{a}+\hat{a}^\dagger)\int d\omega\sqrt{\frac{g(\omega)}{m_\omega\omega}}C_\omega(\hat{b}_\omega+\hat{b}_\omega^\dagger). \tag{3.9}$$

Now comparing Eqs. (3.7)–(3.9) with Eq. (2.4), we see that both Hamiltonians will be equivalent if we employ

$$v(\omega)=-\frac{1}{2}\sqrt{\frac{g(\omega)}{M\omega_o m_\omega \omega}}C_\omega. \tag{3.10}$$

Taking the square of Eq. (3.10) and comparing it with Eq. (3.3), we obtain

$$v^2(\omega)=\frac{1}{2\pi}\frac{J(\omega)}{M\omega_o}. \tag{3.11}$$

Adopting the Drude form (3.5), $|v(\omega)|^2$ is given by

$$|v(\omega)|^2=\frac{\gamma\omega}{\pi\omega_o}\frac{1}{(1+\omega^2/\Omega_c^2)}, \tag{3.12}$$

which is defined only for $\omega\geqslant 0$.

Now that we have established the form of $|v(\omega)|^2$ corresponding to the Caldeira-Leggett model [2], we can determine $H(\omega)$ through Eq. (2.26). A simple calculation shows that $H(\omega)$ will be given by

$$H(\omega)=-\frac{\gamma\Omega_c}{\omega_o}\frac{1}{(1+\omega^2/\Omega_c^2)}. \tag{3.13}$$

We can also diagonalize the Hamiltonian (3.1) considering the inclusion of the counterterm $V_R(\hat{q})$ (see Appendix A). The result is that all the equations (2.20)–(2.35) will remain valid with the following substitution: whenever the function $H(\omega)$ appears, it should be replaced by

$$H_R(\omega)=H(\omega)+\frac{\Delta\omega^2}{2\omega_o}, \tag{3.14}$$

where the frequency shift $\Delta\omega^2$ is defined as [2]

$$\frac{\Delta\omega^2}{2\omega_o}=\frac{1}{2\omega_o M}\sum_{j=1}^{N}\frac{C_j^2}{m_j\omega_j^2}=2\int d\omega\frac{|v(\omega)|^2}{\omega}=\frac{\gamma\Omega_c}{\omega_o}. \tag{3.15}$$

Whenever a function appears with the subindex ''*R*'' it means that we are considering the introduction of the counterterm.

The spectral function (3.5) is appropriate to the description of the reservoir since we consider $\Omega_c\gg\omega_o,\gamma$. So, in order to simplify and also obtain the exact function associated to the Ohmic dissipation, we will take the limit $\Omega_c\rightarrow\infty$ in the expression for $|L(\omega)|_R^2$. To do so, first we consider the renormalized function $|L(\omega)|_R^2$ given by

$$|L(\omega)|_R^2=\frac{2\pi\omega_o|v(\omega)|^2}{[\omega^2-\omega_o^2-2\omega_o H_R(\omega)]^2+[2\pi\omega_o|v(\omega)|^2]^2}. \tag{3.16}$$

Once

$$\lim_{\Omega_c\rightarrow\infty}H_R(\omega)=0 \quad \text{and} \quad \lim_{\Omega_c\rightarrow\infty}|v(\omega)|^2=\frac{\gamma\omega}{\pi\omega_o}, \tag{3.17}$$

we obtain

$$\lim_{\Omega_c\rightarrow\infty}|L(\omega)|_R^2=\frac{2\gamma\omega}{(\omega^2-\omega_o^2)^2+(2\gamma\omega)^2}. \tag{3.18}$$

Thus, we see that

$$\lim_{\Omega_c\rightarrow\infty}|L(\omega)|_R^2=M\chi''(\omega), \tag{3.19}$$

where $\chi''(\omega)$ is the imaginary part of the response function of a damped harmonic oscillator.

In the limit $\gamma\ll\omega_o$, we can write

$$|L(\omega)|_R^2\simeq\frac{2\omega_o\gamma}{[2\omega_o(\omega-\omega_o)]^2+(2\omega_o\gamma)^2}$$

$$=\frac{1}{2\omega_o}\frac{\gamma}{(\omega-\omega_o)^2+\gamma^2}, \tag{3.20}$$

which corresponds to a Lorentzian distribution of width γ.

For the function $|L(\omega)|^2$, without the renormalization, we have $H(\omega\ll\Omega_c)\simeq-\gamma\Omega_c/\omega_o$ and therefore for $\Omega_c\gg\omega_o,\gamma$ we obtain

$$|L(\omega)|^2=\frac{2\gamma\omega}{(\omega^2-\omega_o^2+2\gamma\Omega_c)^2+(2\gamma\omega)^2}. \tag{3.21}$$

In this case we should have $\omega_o^2>2\gamma\Omega_c$, because, without the renormalization, we must have [19]

$$\omega_o^2>|\Delta\omega^2| \tag{3.22}$$

for the diagonalization to be consistent.

IV. ANALYSIS OF THE EVOLUTION OF $\hat{a}(t)$

Now we can analyze in detail the time evolution of the operator $\hat{a}$ associated with the system. We will analyze each term of the expression for $\hat{a}(t)$ in Eq. (2.28). We will be interested in the relation between the degree of dissipation in our system and the importance of each one of those terms.

Initially we will analyze the coefficients associated to the operators $\hat{a}$ and $\hat{a}^\dagger$. The fastest and most efficient way to understand the behavior of each one of them is through graphs.

The graphs in Fig. 1(a) present the behavior of $|L(\omega)|_R^2$ for three values of γ: $\gamma_1=0.1\omega_o$, $\gamma_2=\omega_o$, and $\gamma_3=10\omega_o$. We see that for $\gamma_1=0.1\omega_o$, $|L(\omega)|_R^2$ presents a narrow peak centered approximately about ω_o [we showed that in the limit $\gamma\ll\omega_o$ the function $|L(\omega)|_R^2$ tends to a Lorentzian centered at ω_o and with width γ]. As γ increases ($\gamma_2=\omega_o$), the function $|L(\omega)|_R^2$ broadens and becomes centered at progressively lower frequencies. For γ still larger ($\gamma_3=10\omega_o$),$|L(\omega)|_R^2$ narrows again, but its peak is about very low frequencies.

The graphs in Fig. 1(b) present the behavior of the functions $A(\omega)$, $B(\omega)$, and $C(\omega)$ that appear multiplying

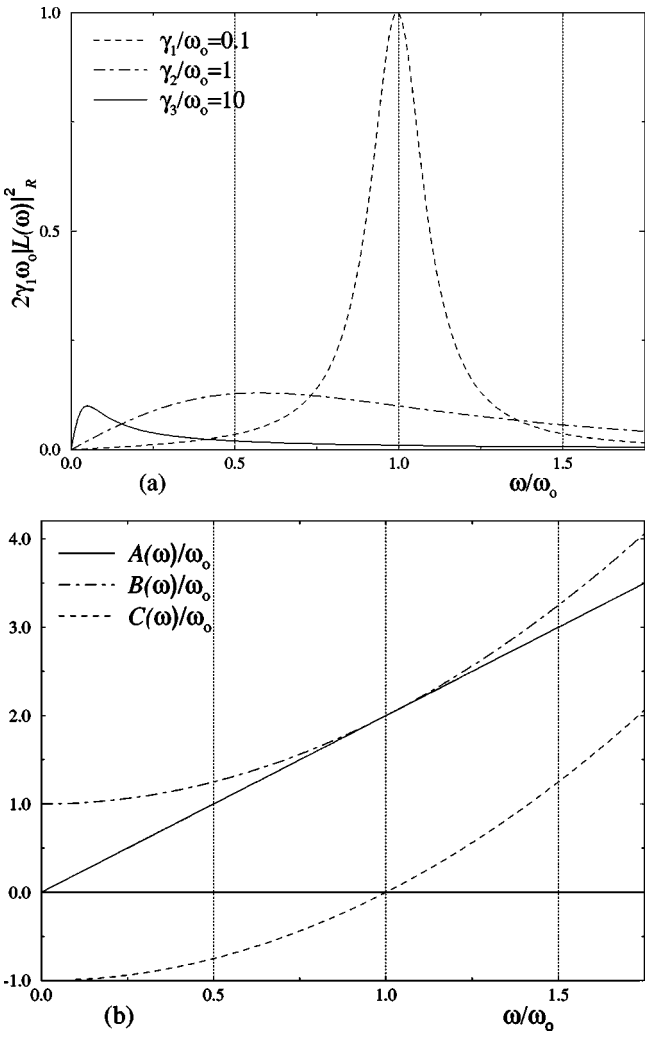

FIG. 1. (a) Graph of $|L(\omega)|^2_R$ for different rations γ/ω_o. (b) Graph of the functions $A(\omega)$, $B(\omega)$, and $C(\omega)$ that appear multiplying $|L(\omega)|^2_R$ in the different terms of the expression for $\hat{a}(t)$.

$|L(\omega)|^2_R$ in the different terms of the expression for $\hat{a}(t)$. Simultaneously observing Figs. 1(a) and 1(b), we conclude that when $\gamma \ll \omega_o$, the function $C(\omega)|L(\omega)|^2_R$ has a negligible amplitude if compared to the functions $A(\omega)|L(\omega)|^2_R$ and $B(\omega)|L(\omega)|^2_R$, because in this case $|L(\omega)|^2_R$ is very sharp and centered at ω_o whereas $C(\omega_o)=0$. As γ/ω_o increases, $|L(\omega)|^2_R$ has its peak broadened and moved away from ω_o. The function $C(\omega)|L(\omega)|^2_R$ becomes comparable to the others, and in the limit $\gamma \gg \omega_o$ it is of the same order of $B(\omega)|L(\omega)|^2_R$ whereas $A(\omega)|L(\omega)|^2_R$ becomes very small.

It remains to analyze the coefficients $B_{1,R}(\Omega;t)$ and $B_{2,R}(\Omega;t)$ of $\hat{b}_\Omega$ and $\hat{b}^\dagger_\Omega$, respectively, in the expression (2.28) for $\hat{a}(t)$. We know that in the limit $\gamma \ll \omega_o$ the function $|L(\omega)|^2_R$ tends to a Lorentzian centered at ω_o and with width γ. Therefore, the function $(\omega_o-\Omega)Z_R(\Omega)$ that appears in the expression (2.31) for $B_{2,R}(\Omega;t)$ is, in this limit, negligible if compared to the function $(\omega_o+\Omega)Z_R(\Omega)$ in the expression (2.30) for $B_{1,R}(\Omega;t)$. The evaluation of $X_R(\Omega;t)$ results in

$$X_R(\Omega;t)=\frac{-\pi}{(\Omega^2-\omega'^2+\gamma^2)+(2\gamma\omega')^2}\times\frac{d}{dt}\left\{\left[\frac{\Omega^2-\omega'^2+\gamma^2}{\omega'}\sin(\omega' t)+2\gamma\cos(\omega' t)\right]e^{-\gamma t}\right\}-\frac{2\gamma\Omega\sin(\Omega t)}{(\Omega^2-\omega_o^2)+(2\gamma\Omega)^2} \tag{4.1}$$

for $\gamma<\omega_o$, where $\omega'=\sqrt{\omega_o^2-\gamma^2}$. We see that in the limit $\gamma \ll \omega_o$, the function $X_R(\Omega;t)$ will also be very sharply peaked around ω_o. Therefore, the function $(\omega_o-\Omega)X_R(\Omega;t)$ in the expression (2.31) for $B_{2,R}(\Omega;t)$ is also negligible if compared to the function $(\omega_o+\Omega)X_R(\Omega;t)$ in the expression (2.30) for $B_{1,R}(\Omega;t)$. Similarly it can be shown that, in this limit, the function $Y_{(-),R}(\Omega;t)$ is negligible in relation to the function $Y_{(+),R}(\Omega;t)$. We conclude that in the limit $\gamma \ll \omega_o$, the coefficient $B_{2,R}(\Omega;t)$ is negligible in comparison to the coefficient $B_{1,R}(\Omega;t)$. As the ratio γ/ω_o increases and the function $|L(\omega)|^2_R$ changes its shape, the coefficient $B_{2,R}(\Omega;t)$ becomes comparable to $B_{1,R}(\Omega;t)$.

So far we have analyzed the relevance of the terms associated to $\hat{a}^\dagger$ and $\hat{b}^\dagger_\Omega$ in the expression (2.28) for $\hat{a}(t)$ considering the inclusion of the counterterm $V_R(\hat{q})$ in our model. We showed that these terms are negligible in the limit $\gamma \ll \omega_o$, but become important as the dissipation increases and the function $|L(\omega)|^2_R$ becomes broader and is no longer centered at ω_o. Now if we had not considered the inclusion of the counterterm in the interaction Hamiltonian, we would have $|L(\omega)|^2$ given by Eq. (3.21) instead of $|L(\omega)|^2_R$. In this case, we see that the condition for the function $|L(\omega)|^2$ to be centered very close to ω_o is that $2\gamma\Omega_c \ll \omega_o^2$ or

$$\frac{\gamma}{\omega_o} \ll \frac{\omega_o}{\Omega_c} \quad (\ll 1). \tag{4.2}$$

Therefore, the condition $\gamma/\omega_o \ll 1$ would not be enough for us to ignore the terms associated to $\hat{a}^\dagger$ and $\hat{b}^\dagger_\Omega$ in the expression for $\hat{a}(t)$. These terms can only be neglected if the condition (4.2), which limits our system to a much weaker dissipation, is satisfied.

We notice that a system subject to a weak dissipation ($\gamma \ll \omega_o$ in our case) does not guarantee that its frequency shift ($\Delta\omega^2=2\gamma\Omega_c$) is also small. We will see later, in more detail, that for a system subject to very weak dissipation, the damping coefficient γ will be given by $\pi|v(\omega_o)|^2$ and the frequency shift by $H(\omega_o)$. Observing the expression (2.26) for $H(\omega)$, we clearly see that the relation between these functions depends on the form adopted for the function $|v(\omega)|^2$. Therefore, $\pi|v(\omega_o)|^2 \ll \omega_o$ does not guarantee that we will have $H(\omega_o) \ll \omega_o$ [as we have seen to be the case for $|v(\omega)|^2$ given by Eq. (3.12)], although this can happen for some functions $|v(\omega)|^2$.

V. REDUCTION TO THE MODEL WITH THE ROTATING-WAVE APPROXIMATION

Now let us consider the situation in which the following conditions are satisfied:

$$\pi|\nu(\omega)|^2 \ll \omega_o \quad \text{for} \quad \omega \sim \omega_o , \tag{5.1}$$

$$H(\omega) \ll \omega_o \quad \text{for} \quad \omega \sim \omega_o . \tag{5.2}$$

Under these conditions the function $|L(\omega)|^2$ will be a function well-peaked around ω_o. Therefore, we can ignore the terms associated with $\hat{a}^\dagger$ and $\hat{b}_\Omega^\dagger$ in the expression for $\hat{a}(t)$. Even the expressions for the coefficients of $\hat{a}$ and $\hat{b}_\Omega$ can be approximated considering that $|L(\omega)|^2$ will only be appreciable, in this case, for $\omega \simeq \omega_o$. We can write

$$A(\omega)|L(\omega)|^2 \simeq B(\omega)|L(\omega)|^2 \simeq 2\omega_o|L(\omega)|^2, \tag{5.3}$$

$$B_\Omega^{(1)} \simeq \nu(\Omega)\left\{ \int d\omega\, 2|L(\omega)|^2 \mathrm{P}\frac{\omega_o}{\omega-\Omega} e^{-i\omega t} + 2\omega_o \frac{|L(\Omega)|^2}{|\nu(\Omega)|^2}[\Omega-\omega_o-H(\Omega)]e^{-i\Omega t}\right\}, \tag{5.4}$$

and finally

$$\hat{a}(t) = \int d\omega |\tilde{\alpha}_\omega|^2 e^{-i\omega t}\hat{a} + \int d\Omega\, \nu(\Omega)\left\{ \int d\omega |\tilde{\alpha}_\omega|^2 \mathrm{P}\frac{1}{\omega-\Omega} e^{-i\omega t} + \frac{|\tilde{\alpha}_\Omega|^2}{|\nu(\Omega)|^2}[\Omega-\omega_o-H(\Omega)]e^{-i\Omega t}\right\}\hat{b}_\Omega , \tag{5.5}$$

where the function $|\tilde{\alpha}_\omega|^2$ comes from the approximation of $|L(\omega)|^2$ considering Eqs. (5.1) and (5.2),

$$\frac{2\omega_o}{\pi}|L(\omega)|^2 \simeq \frac{|\nu(\omega)|^2}{[\omega-\omega_o-H(\omega)]^2+[\pi|\nu(\omega)|^2]^2} = |\tilde{\alpha}_\omega|^2 . \tag{5.6}$$

Now let us compare Eqs. (5.5) and (5.6) with the expressions (2.14) and (2.19), previously obtained in the RWA. The only difference between these expressions is given by the presence of $H(\omega)$ instead of $F(\omega)$. Once $H(\omega)-F(\omega) = -G(\omega)$, we would have, for $\omega \sim \omega_o$, $H(\omega) \simeq F(\omega)$ if $G(\omega) \ll F(\omega)$. There can be functions $|\nu(\omega)|^2$ that satisfy this requirement. However, most of the physically reasonable functions $|\nu(\omega)|^2$ do not; for example, if $|\nu(\omega)|^2$ is given by Eq. (3.12), we have $G(\omega)/F(\omega) \simeq -1$ for $\omega \sim \omega_o$. In this case, $H(\omega_o) \simeq 2F(\omega_o)$ yielding twice the frequency shift given by the model within the RWA [22]. The same relation is found whenever $|\nu(\omega)|^2$ extends to frequencies much larger than ω_o with non-negligible values, for then

$$H(\omega_o) \simeq -2\mathrm{P}\int \frac{|\nu(\Omega)|^2}{\Omega} d\Omega \simeq 2F(\omega_o). \tag{5.7}$$

This larger frequency shift can be easily understood through a perturbative analysis. Let us consider a system described by Eqs. (1.1) and (1.2) and having $\hat{H}_{\mathrm{int}}$ within the RWA (1.4). It can be shown that, in second order, the perturbed levels of the oscillator remain equidistant with an apparent frequency $\omega_o + \Delta^{\mathrm{RWA}}\omega$, where [16]

$$\Delta^{\mathrm{RWA}}\omega = \mathrm{P}\sum_j \frac{|k_j|^2}{\omega_o-\omega_j}. \tag{5.8}$$

Taking the continuous limit and using Eq. (2.5), we see that this expression is nothing but $F(\omega_o)$, which really represents the frequency shift in the weak dissipation limit. Now it is easy to show that if we consider $\hat{H}_{\mathrm{int}}$ given by Eq. (1.3) without the RWA, we have in second order in the perturbation,

$$\Delta\omega = \mathrm{P}\sum_j \frac{|k_j|^2}{\omega_o-\omega_j} - \mathrm{P}\sum_j \frac{|k_j|^2}{\omega_o+\omega_j}. \tag{5.9}$$

This expression, in the continuum limit, is merely $H(\omega_o)$. Therefore, we see that the substitution of $F(\omega_o)$ by $H(\omega_o)$ could already be foreseen by a simple perturbative theory. The same perturbative analysis can be used to understand why the counter-rotating term is not important in the calculation of the decay rate of the system in the weak dissipation limit. In first order, the decay rate of the system is given by Fermi's golden rule, for which only the terms of $\hat{H}_{\mathrm{int}}$ that directly conserve energy in the transition are relevant. This is not done by the counter-rotating terms. In fact, it is only done by the rotating terms that create or destroy energy quanta such that $\omega_j = \omega_o$. This is the reason for the dependence only on $|\nu(\omega_o)|^2$ that appears in the very weak dissipation calculations.

In a model that takes the counterterm into account, we automatically have $H_R(\omega) = 0$ and the expression (5.5) can be substituted by

$$\hat{a}(t) = \int d\omega |\alpha_\omega|_R^2 e^{-i\omega t}\hat{a} + \int d\Omega\, \nu(\Omega) \times\left[\int d\omega |\alpha_\omega|_R^2 \mathrm{P}\frac{1}{\omega-\Omega} e^{-i\omega t} + \frac{|\alpha_\Omega|_R^2}{|\nu(\Omega)|^2}(\Omega-\omega_o)e^{-i\Omega t}\right]\hat{b}_\Omega , \tag{5.10}$$

where

$$\frac{2\omega_o}{\pi}|L(\omega)|_R^2 \simeq \frac{|\nu(\omega)|^2}{(\omega-\omega_o)^2+[\pi|\nu(\omega)|^2]^2} = |\alpha_\omega|_R^2 . \tag{5.11}$$

Therefore, the RWA leads us to the correct results, with regard to the decay rate of the system (related to $|\alpha_\omega|_R^2$), *if and only if the condition of weak dissipation (5.1) is satisfied*. Regarding the frequency shift [associated to $F(\omega_o)$], we see that its agreement with that given in the limit of weak dissipation, in a model without the counterterm, strongly de-

pends on the function $|v(\omega)|^2$ adopted. For functions $|v(\omega)|^2$ that extend to frequencies much larger than ω_o, we have twice the shift foreseen in the RWA. Besides, it is also necessary that the condition (5.2) be satisfied in order to guarantee that this shift is much smaller than ω_o [and we can neglect the terms in $\hat{a}^\dagger$ and $\hat{b}_\Omega^\dagger$ in the expression for $\hat{a}(t)$].

In the case of Ohmic dissipation, the conditions (5.1) and (5.2) are reduced to

$$\gamma \ll \omega_o, \tag{5.12}$$

once in this case

$$\pi|v(\omega_o)|^2 = \gamma \quad \text{and} \quad H_R(\omega) = 0,$$

in the limit $\Omega_c \to \infty$.

VI. EVOLUTION OF A COHERENT STATE

We showed that if our system satisfies the conditions of weak dissipation (5.1) and small frequency shift (5.2), the evolution of the operator $\hat{a}(t)$ can be reduced to the expression given by Eq. (5.5). Now we will suppose that initially our system is in a coherent state $|\alpha\rangle$ and that the reservoir is in the vacuum state $|0\rangle$ corresponding to a reservoir at zero temperature. In this case we have

$$\hat{a}(t)|\alpha,0\rangle = \int d\omega |\tilde{\alpha}_\omega|^2 e^{-i\omega t}\alpha|\alpha,0\rangle. \tag{6.1}$$

Therefore, in this particular case, a coherent state stays as such during its evolution with eigenvalue $\alpha(t)$ given by

$$\alpha(t) = \alpha \int d\omega |\tilde{\alpha}_\omega|^2 e^{-i\omega t}. \tag{6.2}$$

We can also calculate the evolution of the operator $\hat{b}_\Omega(t)$ of the reservoir. Then in the case of weak dissipation and small frequency shift we can show that the modes of the reservoir also evolve from the vacuum state to coherent states with eigenvalues given by

$$\beta_\Omega^{(\mathcal{R})}(t) = \alpha\left[\mathrm{P}\int d\omega \frac{|\tilde{\alpha}_\omega|^2}{\omega-\Omega} e^{-i\omega t} + \frac{\Omega-\omega_o-H(\Omega)}{|v(\Omega)|^2}|\tilde{\alpha}_\Omega|^2 e^{-i\Omega t}\right] v^*(\Omega). \tag{6.3}$$

Still under the conditions (5.1) and (5.2) we can further approximate $|\tilde{\alpha}_\omega|^2$ by

$$|\tilde{\alpha}_\omega|^2 \simeq \frac{\pi|v(\omega_o)|^2}{[\omega-\omega_o-H(\omega_o)]^2+[\pi|v(\omega_o)|^2]^2} \tag{6.4}$$

and also extend the lower limit of the frequency integral in Eq. (6.2) to $-\infty$ introducing a negligible error. Then we have

$$\alpha(t) = \alpha e^{-i[\omega_o+\Delta\omega]t} e^{-\pi|v(\omega_o)|^2 t} \quad \text{where} \quad \Delta\omega = H(\omega_o). \tag{6.5}$$

In the case of Ohmic dissipation with the inclusion of the counterterm, we have

$$\alpha(t) = \alpha e^{-i\omega_o t} e^{-\gamma t}. \tag{6.6}$$

Now it is clear that when Eqs. (5.1) and (5.2) are not satisfied making the terms associated to the operators $\hat{a}^\dagger$ and $\hat{b}_\Omega^\dagger$ in the expression (2.28) for $\hat{a}(t)$ no longer negligible, $|\alpha,0\rangle$ will not be an eigenstate of $\hat{a}(t)$ because $|\alpha\rangle$ and $|0\rangle$ are not eigenstates of $\hat{a}^\dagger$ and $\hat{b}_\Omega^\dagger$, respectively. Therefore, we see that an initial coherent state $|\alpha\rangle$, interacting with a reservoir even at temperature $T=0$, will not remain a coherent state during its decay unless we have a system subject to very weak dissipation.

The previous works that emphasized the existence of dissipative coherent states [4–7], in models described by the $\hat{H}_{\mathrm{int}}$ (1.3), were based on master equations obtained through a method that is appropriate only in the limit of weak dissipation. However, we saw that in this limit the corresponding model (1.3) is reduced to the RWA model (1.4) that really preserves the coherent states. We believe that the implicit assumption of weak dissipation is the reason why these authors have obtained the dissipative coherent states. Our result agrees with the one presented in [13], where it was shown that the model (3.1) presents the coherent states as the initial states of the system that produce the least amount of entropy as time evolves.

VII. EVOLUTION OF THE CENTER OF A WAVE PACKET

We can also study the evolution of the operator $\hat{q}$ associated to the position of the particle. Once the operators $\hat{q}$ and $\hat{p}$ are related to the operator $\hat{a}$ by Eq. (3.6), we obtain from Eq. (2.28) the following expression for $\hat{q}(t)$:

$$\hat{q}(t) = \mathcal{G}_{\mathcal{S}}(\hat{q},\hat{p};t) + \mathcal{F}_{\mathcal{R}}(\hat{q}_\Omega,\hat{p}_\Omega;t), \tag{7.1}$$

where

$$\mathcal{G}_{\mathcal{S}}(\hat{q},\hat{p};t) = \hat{q}\frac{d}{dt}\mathcal{L}(t) + \frac{\hat{p}}{M}\mathcal{L}(t), \tag{7.2}$$

$$\mathcal{F}_{\mathcal{R}}(\hat{q}_\Omega,\hat{p}_\Omega;t) = 2\omega_o \int \frac{d\Omega}{\pi} v(\Omega)\sqrt{\frac{m_\Omega\Omega}{M\omega_o}}\left\{\left[\frac{d}{dt}W_R(\Omega,t) + Z_R(\Omega)\cos(\Omega t)\right]\hat{q}_\Omega + [\Omega W_R(\Omega,t) + Z_R(\Omega)\sin(\Omega t)]\right\}\frac{\hat{p}_\Omega}{m_\Omega\Omega}, \tag{7.3}$$

$$\mathcal{L}(t) = 2\int \frac{d\omega}{\pi}|L(\omega)|_R^2 \sin(\omega t), \tag{7.4}$$

$$W_R(\Omega,t) = \mathrm{P}\int d\omega \frac{2|L(\omega)|_R^2}{\omega^2-\Omega^2}\sin(\omega t), \tag{7.5}$$

with $Z(\Omega)$ defined in Eq. (2.34).

Now we suppose that the initial density operator of our global system can be written in the factorizable form

$$\rho_T = \rho_{\mathcal{S}} \otimes \rho_{\mathcal{R}}, \tag{7.6}$$

where $\rho_{\mathcal{S}}$ and ρ_R are, respectively, the density operators of the system and reservoir when they are isolated. Then we have

$$\begin{aligned}\langle \hat{q}(t) \rangle &= \mathrm{Tr}_{\mathcal{S}}[\mathcal{G}_{\mathcal{S}}(\hat{q},\hat{p};t)\rho_{\mathcal{S}}] + \mathrm{Tr}_{\mathcal{R}}[\mathcal{F}_{\mathcal{R}}(\hat{q}_\Omega,\hat{p}_\Omega;t)\rho_{\mathcal{R}}] \\ &= \mathcal{G}_{\mathcal{S}}(\langle \hat{q} \rangle_{\mathcal{S}}, \langle \hat{p} \rangle_{\mathcal{S}};t) + \mathcal{F}_{\mathcal{R}}(\langle \hat{q}_\Omega \rangle_{\mathcal{R}}, \langle \hat{p}_\Omega \rangle_{\mathcal{R}};t). \end{aligned} \tag{7.7}$$

Assuming that the initial state of the reservoir is such that

$$\langle \hat{q}_j \rangle_{\mathcal{R}} = \langle \hat{p}_j \rangle_{\mathcal{R}} = 0, \tag{7.8}$$

which in the continuum limit corresponds to $\langle \hat{q}_\Omega \rangle_{\mathcal{R}} = \langle \hat{p}_\Omega \rangle_{\mathcal{R}} = 0$, we obtain the following expression for $\langle \hat{q}(t) \rangle$:

$$\langle \hat{q}(t) \rangle = \langle \hat{q} \rangle_{\mathcal{S}} \frac{d}{dt}\mathcal{L}(t) + \frac{\langle \hat{p} \rangle_{\mathcal{S}}}{M}\mathcal{L}(t), \tag{7.9}$$

where

$$\mathcal{L}(t) = \begin{cases} \dfrac{1}{\omega'}\sin(\omega' t)e^{-\gamma t} & \text{for } \gamma < \omega_o, \\ t e^{-\gamma t} & \text{for } \gamma = \omega_o, \\ \dfrac{1}{\gamma_2 - \gamma_1}e^{-\gamma_1 t} + \dfrac{1}{\gamma_1 - \gamma_2}e^{-\gamma_2 t} & \text{for } \gamma > \omega_o, \end{cases} \tag{7.10}$$

with $\omega' = \sqrt{\omega_o^2 - \gamma^2}$ and $\gamma_{1,2} = \gamma \pm \sqrt{\gamma^2 - \omega_o^2}$. The expression (7.9) was also obtained by Grabert and collaborators [9], by the method of functional integration. They affirmed that it would correspond to the classical trajectory of a damped harmonic oscillator. However, it is easy to see that this is not true. If the initial state of the system presents an initial average momentum $\langle \hat{p} \rangle_{\mathcal{S}} = p_o$ and an initial average position $\langle \hat{q} \rangle_{\mathcal{S}} = q_o$, then according to Eq. (7.9), $\langle \hat{q}(t) \rangle$ would evolve as

$$\langle \hat{q}(t) \rangle = q_o \left[\cos(\omega' t) - \frac{\gamma}{\omega'}\sin(\omega' t) \right] e^{-\gamma t} + \frac{p_o}{M\omega'}\sin(\omega' t)e^{-\gamma t} \tag{7.11}$$

for $\gamma < \omega_o$. However the classical trajectory is known to be

$$q(t)_{\mathrm{clas}} = q_o \left[\cos(\omega' t) + \frac{\gamma}{\omega'}\sin(\omega' t) \right] e^{-\gamma t} + \frac{p_o}{M\omega'}\sin(\omega' t)e^{-\gamma t}. \tag{7.12}$$

Thus, we see that there is a phase difference between Eqs. (7.11) and (7.12) if the oscillator has an initial displacement q_o.

Let us now suppose that the initial state of the reservoir is such that

$$\langle \hat{q}_j \rangle_{\mathcal{R}} = \frac{C_j}{m_j\omega_j^2}\langle \hat{q} \rangle_{\mathcal{S}}, \quad \langle \hat{p}_j \rangle_{\mathcal{R}} = 0. \tag{7.13}$$

We can write the expression (7.3) in the discrete limit, replace Eq. (7.8) by Eq. (7.13), and return to the continuum limit. Then we obtain (see Appendix B) the following expression for $\langle \hat{q}(t) \rangle$:

$$\langle \hat{q}(t) \rangle = \langle \hat{q} \rangle_{\mathcal{S}} \left[\frac{d}{dt}\mathcal{L}(t) + 2\gamma\mathcal{L}(t) \right] + \frac{\langle \hat{p} \rangle_{\mathcal{S}}}{M}\mathcal{L}(t). \tag{7.14}$$

In this case if the initial state of the system presents an initial average momentum $\langle \hat{p} \rangle_{\mathcal{S}} = p_o$ and an initial average position $\langle \hat{q} \rangle_{\mathcal{S}} = q_o$, Eq. (7.14) becomes

$$\langle \hat{q}(t) \rangle = q_o \left[\cos(\omega' t) + \frac{\gamma}{\omega'}\sin(\omega' t) \right] e^{-\gamma t} + \frac{p_o}{M\omega'}\sin(\omega' t)e^{-\gamma t} \tag{7.15}$$

for $\gamma < \omega_o$, which corresponds to the correct classical trajectory.

Thus, we see that the classical evolution is not obtained with the initial condition (7.8) but with the initial condition (7.13). We can understand why this happens through the classical analysis of the model (3.5) presented in the next section.

VIII. CLASSICAL ANALYSIS AND DISCUSSION

In this section we will accomplish a classical analysis of the model used. Our objective is to obtain a physical intuition on the effect that causes the difference between Eqs. (7.11) and (7.15) and then on the meaning of the initial condition (7.13). This procedure can be justified by the equivalence of the classical and quantum dynamics of this model [20].

The Hamiltonian (3.1) can be written as [21]

$$H = \frac{p^2}{2M} + V(q) + \sum_j \left[\frac{p_j^2}{2m_j} + \frac{m_j\omega_j^2}{2}\left(q_j - \frac{C_j}{m_j\omega_j^2}q \right)^2 \right]. \tag{8.1}$$

The equations of motion of this system are given by

$$M\ddot{q}(t) + V'(q) = \sum_j C_j \left[q_j(t) - \frac{C_j}{m_j\omega_j^2}q(t) \right], \tag{8.2}$$

$$m_j\ddot{q}_j(t) + m_j\omega_j^2 q_j(t) = C_j q(t). \tag{8.3}$$

If $q_j(0)$ and $\dot{q}_j(0)$, are the initial conditions the solution of the homogeneous part of Eq. (8.3) will be

$$q_j^H(t)=q_j(0)\cos(\omega_j t)+\frac{\dot{q}_j(0)}{\omega_j}\sin(\omega_j t). \tag{8.4}$$

The particular solution, considering the presence of the force $C_j q(t)$, can be obtained by taking the Fourier transform of Eq. (8.3). Then we have

$$\begin{aligned} q_j^P(t)&=\frac{C_j}{m_j\omega_j}\int_0^t dt'\ q(t')\sin[\omega_j(t-t')] \\ &=\frac{C_j}{m_j\omega_j^2}\Bigg\{q(t)-q(0)\cos(\omega_j t) \\ &\quad-\int_0^t dt'\ \dot{q}(t')\cos[\omega_j(t-t')]\Bigg\}. \end{aligned} \tag{8.5}$$

Using the definition of the spectral function $J(\omega)$ [Eqs. (3.3) and (3.4)], it can be shown that in the limit $\Omega_c\to\infty$ we have

$$\sum_j \frac{C_j^2}{m_j\omega_j^2}\int_0^t dt'\ \cos[\omega_j(t-t')]\dot{q}(t')=2M\gamma\dot{q}(t). \tag{8.6}$$

Therefore, the general solution of Eq. (8.3), $q_j(t)=q_j^H(t)+q_j^P(t)$, when substituted in Eq. (8.2) results in the following Langevin equation:

$$M\ddot{q}(t)+V'(q)+2M\gamma\dot{q}(t)=F(t),$$

where

$$F(t)=\sum_j\ C_j\tilde{q}_j(0)\cos(\omega_j t)+\sum_j\ \frac{C_j}{\omega_j}\dot{q}_j(0)\sin(\omega_j t) \tag{8.7}$$

is the fluctuating force and we have redefined the position of the oscillators of the bath [23],

$$\tilde{q}_j(0)=q_j(0)-\frac{C_j}{m_j\omega_j^2}q(0). \tag{8.8}$$

Supposing that the bath is initially in thermodynamic equilibrium in relation to the coordinates $\tilde{q}_j(0)$, we have, in the classical limit,

$$\langle\tilde{q}_j(0)\rangle=\langle\dot{q}_j(0)\rangle=\langle\tilde{q}_j(0)\dot{q}_{j'}(0)\rangle=0, \tag{8.9}$$

$$\langle\tilde{q}_j(0)\tilde{q}_{j'}(0)\rangle=\frac{kT}{m_j\omega_j^2}\delta_{jj'},\quad \langle\dot{q}_j(0)\dot{q}_{j'}(0)\rangle=\frac{kT}{m_j}\delta_{jj'}. \tag{8.10}$$

The physical meaning of this initial condition written in terms of the relative coordinates $\tilde{q}_j$ has already been analyzed by Zwanzig [24] some time ago. Using Eqs. (8.9) and (8.10) and after some algebraic manipulations, it is shown that $\langle F(t)\rangle=0$ and $\langle F(t)F(t')\rangle\simeq 4M\gamma kT\delta(t-t')$, which correspond to the expressions that characterize the Brownian motion.

On the other hand, if we had adopted the initial condition

$$\langle q_j(0)\rangle=\langle\dot{q}_j(0)\rangle=\frac{1}{m_j}\langle p_j(0)\rangle=0, \tag{8.11}$$

we would have

$$\begin{aligned} \langle F(t)\rangle&=-q(0)\sum_j\ \frac{C_j^2}{m_j\omega_j^2}\cos(\omega_j t) \\ &=-4M\gamma q(0)\frac{1}{\pi}\int_0^{\Omega_c}d\omega\cos\omega t \\ &=-4M\gamma q(0)\delta(t), \end{aligned} \tag{8.12}$$

where we have used Eqs. (3.3), (3.4), and taken the limit $\Omega_c\to\infty$. Therefore, we would not have $\langle F(t)\rangle=0$, but the presence of a δ force at $t=0$. Physically what happens is that if the oscillators of the bath are not ''appropriately'' distributed around the particle [as in the initial condition (8.11)], when it is inserted in the bath, these oscillators will ''pull'' the particle until they reach this ''appropriate'' distribution. This force will act on the particle during a time interval of the order $1/\Omega_c$. Therefore, in the limit $\Omega_c\to\infty$ we will have a δ force that will cause a phase difference in the evolution of the system. This phase difference is the difference between Eqs. (7.11) and (7.12), which is corrected in Eq. (7.15) by the adoption of the initial condition (7.13) [quantum analog of Eq. (8.9)] instead of Eq. (7.8) [quantum analog of Eq. (8.11)]. As far as we know, the need to use the initial condition (7.13) in place of Eq. (7.8) in the quantum treatment of this model has not been noticed in previous works. In Ref. [2] the authors make some approximations which are equivalent to regarding the initial time as $t=0^+(t\sim 1/\Omega_c)$. So the initial conditions are established at this instant although the coupling between particle and bath is switched on at $t=0$ and gives rise to a δ-type force at this instant. The inclusion of $t=0$ in propagator methods must be accompanied by the above-mentioned modification of the factorizable initial condition. However, it must be emphasized that we are not addressing here the question of the generalized initial condition [9,10]. Actually the point we have raised is clearly responsible for the disagreement between $\langle\hat{q}(t)\rangle$ found in these references. In Ref. [9] the authors reproduced the dephased $\langle\hat{q}(t)\rangle$ [cf. Eq. (7.9) above], whereas in Ref. [10] this time evolution is the correct one as in Eq. (7.15). The origin of the discrepancy is the use of $t=0$ or $t=0^+$ as the initial instant together with the factorizable initial condition.

We would like to take advantage of this opportunity to correct a mistake that was made in Ref. [10], of which one of us is coauthor. The referred article considers an initial condition of the system when the bath of oscillators meets thermodynamical equilibrium with the particle at the position it is placed in the bath. In this case one obtains mean values of the position $\langle\hat{q}(t)\rangle$ and momentum $\langle\hat{p}(t)\rangle$ which depend on

the temperature of the reservoir and that do not exactly coincide with their classical counterparts. This disagreement was justified within a classical analysis of the model. In this analysis it was affirmed that the classical initial condition equivalent to the proposed quantum initial state, which corresponds exactly to $\langle \tilde{q}_j(0)\rangle = 0$, would imply a classical solution of the model different from the trajectories of a damped harmonic oscillator. We saw in the present work that this is not true and therefore this argument cannot be used. We believe that the origin of the disagreement when adopting a nonfactorizable initial condition is the impossibility of describing the evolution of the system through an independent sum of functions of the system and reservoir variables as in Eq. (7.7). The quantum effects of the correlation between the variables of the system and reservoir prevent a direct comparison of the quantum mean values with the values obtained through the classical analysis of the model. Accordingly, it can be shown that the discrepancy vanishes in the classical limit ($kT \gg \hbar\omega_o$).

After we had made the above analysis, we became aware that in previous works [11,12] the authors had also noticed the existence of initial kicks and jolts in this system when the initial condition implied by Eqs. (7.6) and (7.8) is used. In both of them the existence of an initial kick, given by Eq. (8.12) in the limit $\Omega_c \to \infty$, is noticed [see their Eqs. (3.2) and (45), respectively]. However, the existence of this initial transient is considered as a characteristic of the model to be taken into account. In our analysis we see that, although the existence of the kick given by Eq. (8.12) is a real characteristic of the model when it is subject to the above-mentioned initial condition, it is an undesirable feature that should be corrected. Fortunately, this correction can be made even with an improved factorizable initial condition, that is, Eqs. (7.6) and (7.13).

The authors of [11] and [12] have recognized that the presence of initial jolts, in their master equation coefficients, generates certain nonphysical effects and so they suppose that they are due to the adoption of a factorizable initial condition. In [14] the evolution of the system is analyzed for a nonfactorizable initial condition, similar to the one used in [9] and [10], in which the initial position of the particle is defined by a measurement process in a state in thermodynamic equilibrium with the bath. However, the initial jolts in the time scale $1/\Omega_c$ still persist. We believe that this happens because this initial condition does not satisfy Eq. (7.13) when the initial mean values of the position of the particle and the oscillators in the bath are calculated. Thus, in this aspect, it is less general than the improved factorizable initial condition that we considered.

Actually the initial jolt, at least the one they attribute to the decoherence process (however, see the discussion below), does not appear in the more general initial condition later adopted in [15]. It is an initial condition prepared by a dynamic process in a finite time t_p. In this case we can consider that the condition (7.13) will be satisfied since $t_p \gg 1/\Omega_c$. Indeed, in this situation ($t_p \gg 1/\Omega_c$), it was shown that the initial jolt does not appear.

Thus, we believe that the initial condition (7.13) is enough to eliminate most of or maybe all the initial transients that would appear in this system in the characteristic time scale $1/\Omega_c$. However, another initial transient in this system is also known. In [25], it was shown that for a factorizable initial state in the high-temperature limit ($kT \gg \hbar\Omega_c$) of the master equation it presents an initial transient within the time scale of the internal decoherence of the initial wave packet. If applied to times shorter than this, it can lead to nonsensical results. We believe that this pathology can only be really corrected with the adoption of nonfactorizable initial conditions.

IX. CONCLUSION

In this paper we have applied the Fano diagonalization procedure to two Hamiltonians commonly used as models for dissipative systems in quantum optics and in condensed matter systems; the rotating wave and the coordinate-coordinate coupling models, respectively.

By exactly diagonalizing these two models, we have succeeded in showing how the RWA turns out to be the extremely underdamped limit of the more general coordinate-coordinate coupling model. We have also been able to analyze the role played by the counterterm in this limiting procedure from the latter to the RWA. We have shown through the evaluation of the destruction operator $\hat{a}(t)$ of the system that the RWA is a good approximation for Eq. (1.1) if and only if the conditions (5.1) and (5.2) are satisfied. For certain choices of $|v(\omega)|^2$, we have $H(\omega_o) \approx \pi |v(\omega_o)|^2$ and the fulfillment of Eq. (5.1) automatically implies Eq. (5.2). However, for other choices, we can have $H(\omega_o) \gg \pi |v(\omega_o)|^2$ and Eq. (5.2) limits the validity of the approximation. Once these conditions are satisfied, we have shown that the time evolution of the system is identical to that determined within the RWA, with the exception of the frequency shift. We have found that this shift will be given by $H(\omega_o)$ instead of $F(\omega_o)$. As we have shown, these functions usually have the same order of magnitude, but they are not identical. For functions $|v(\omega)|^2$ that extend to frequencies much larger than ω_o, we have $H(\omega_o) \simeq 2F(\omega_o)$.

The comparison of the Hamiltonian (2.4) with the Hamiltonian of the coordinate-coordinate coupling model established the relation (3.11) between the spectral function $J(\omega)$ of this model and the coupling function $|v(\omega)|^2$. In the case of Ohmic dissipation and considering the inclusion of the counterterm, we find that $H_R(\omega) = 0$ in the limit $\Omega_c \to \infty$. Then the only condition required for the RWA to be valid is

$$\gamma \ll \omega_o . \tag{9.1}$$

As an application of this method, we have studied the existence of dissipative coherent states and concluded that they can only exist within the RWA and when thermal fluctuations are negligible. When these conditions are not met, the initial state will in the long run become a statistical mixture.

Finally, we have also addressed the question of the discrepancies in the time evolution of the observables of the system that arise when the factorizable initial conditions are not properly accounted for. We have shown how to deal with

this problem by using the appropriate improved factorizable initial condition (7.13) rather than (7.8).

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APPENDIX A: DIAGONALIZATION WITHOUT THE RWA

Here, the procedure used in the diagonalization of the Hamiltonian (2.4) will be presented. We want to find the operator $\hat{A}_\omega$ that allows us to write Eq. (2.4) in the diagonal form. We write $\hat{A}_\omega$ in its general form (2.20) and then we impose the commutation relation (2.7),

$$[\hat{A}_\omega,\hat{H}]=\hbar\omega\hat{A}_\omega. \tag{A1}$$

Replacing Eqs. (2.4) and (2.20) in Eq. (A1) and taking the commutators of the expression obtained with $\hat{a}^\dagger$, $\hat{a}$, $\hat{b}_\Omega$, and $\hat{b}_\Omega^\dagger$, we have, respectively,

$$\omega\alpha_\omega=\omega_o\alpha_\omega+\int[\beta_{\omega,\Omega}v^*(\Omega)-\sigma_{\omega,\Omega}v(\Omega)]d\Omega, \tag{A2}$$

$$\omega\chi_\omega=-\omega_o\chi_\omega+\int[\beta_{\omega,\Omega}v^*(\Omega)-\sigma_{\omega,\Omega}v(\Omega)]d\Omega, \tag{A3}$$

$$\omega\beta_{\omega,\Omega}=(\alpha_\omega-\chi_\omega)v(\Omega)+\Omega\beta_{\omega,\Omega}, \tag{A4}$$

$$\omega\sigma_{\omega,\Omega}=(\alpha_\omega-\chi_\omega)v^*(\Omega)-\Omega\sigma_{\omega,\Omega}. \tag{A5}$$

Subtracting Eq. (A2) from Eq. (A3), we have

$$\chi_\omega=\frac{\omega-\omega_o}{\omega+\omega_o}\alpha_\omega. \tag{A6}$$

Replacing Eq. (A6) in Eq. (A4) we obtain

$$\beta_{\omega,\Omega}=\left[\mathrm{P}\frac{1}{\omega-\Omega}+z(\omega)\delta(\omega-\Omega)\right]\frac{2\omega_o}{\omega+\omega_o}v(\Omega)\alpha_\omega, \tag{A7}$$

where $z(\omega)$ is a function to be determined. Similarly, substituting Eq. (A6) in Eq. (A5), we have

$$\sigma_{\omega,\Omega}=\frac{1}{\omega+\Omega}\frac{2\omega_o}{\omega+\omega_o}v^*(\Omega)\alpha_\omega. \tag{A8}$$

Now, substituting Eqs. (A7) and (A8) in Eq. (A2), we obtain $z(\omega)$ given by Eq. (2.25).

It remains to determine α_ω. For this we impose the condition (2.12), which results in

$$\alpha_\omega\alpha_{\tilde{\omega}}^*+\int d\Omega\ \beta_{\omega,\Omega}\beta_{\tilde{\omega},\Omega}^*-\chi_\omega\chi_{\tilde{\omega}}^*-\int d\Omega\ \sigma_{\omega,\Omega}\sigma_{\tilde{\omega},\Omega}^*=\delta(\omega-\tilde{\omega}). \tag{A9}$$

Using Eqs. (A6) and (A8) we obtain, respectively,

$$\alpha_\omega\alpha_{\tilde{\omega}}^*-\chi_\omega\chi_{\tilde{\omega}}^*=\frac{2\omega_o(\omega+\tilde{\omega})}{(\omega+\omega_o)(\tilde{\omega}+\omega_o)}\alpha_\omega\alpha_{\tilde{\omega}}^* \tag{A10}$$

and

$$\int d\Omega\ \sigma_{\omega,\Omega}\sigma_{\tilde{\omega},\Omega}^*=\frac{(2\omega_o)^2}{(\omega+\omega_o)(\tilde{\omega}+\omega_o)}\frac{G(\tilde{\omega})-G(\omega)}{\omega-\tilde{\omega}}\alpha_\omega\alpha_{\tilde{\omega}}^*, \tag{A11}$$

where $G(\omega)$ is given by Eq. (2.26). Now, using Eq. (A7), as well as the property

$$P\frac{1}{\omega-\omega'}\,P\,\frac{1}{\tilde{\omega}-\omega'}=P\frac{1}{\omega-\tilde{\omega}}\left(P\,\frac{1}{\tilde{\omega}-\omega'}-P\frac{1}{\omega-\omega'}\right)+\pi^2\delta(\omega-\tilde{\omega})\delta\left[\omega'-\frac{1}{2}(\omega+\tilde{\omega})\right], \tag{A12}$$

we obtain

$$\int d\Omega\,\beta_{\omega,\Omega}\beta_{\tilde{\omega},\Omega}^*=\frac{(2\omega_o)^2}{(\omega+\omega_o)(\tilde{\omega}+\omega_o)}\left\{\frac{1}{\omega-\tilde{\omega}}\left[\frac{\tilde{\omega}^2-\omega^2}{2\omega_o}+G(\tilde{\omega})-G(\omega)\right]+[\pi^2+z^2(\omega)]|v(\omega)|^2\delta(\omega-\tilde{\omega})\right\}. \tag{A13}$$

Then substituting Eqs. (A10), (A11), and (A13) in Eq. (A9), we have

$$\alpha_\omega\alpha_{\tilde{\omega}}^*\frac{(2\omega_o)^2|v(\omega)|^2}{(\omega+\omega_o)(\tilde{\omega}+\omega_o)}[\pi^2+z^2(\omega)]\delta(\omega-\tilde{\omega})=\delta(\omega-\tilde{\omega}) \tag{A14}$$

and, therefore, we should have $|\alpha_\omega|^2$ given by Eq. (2.21).

In the calculations presented above, we supposed that $|v(\omega)|$ is a continuous function such that $|v(0)|=0$. In this way we guarantee that $\int_0^\infty d\Omega\, f(\Omega)|v(\Omega)|^2\delta(\Omega-\omega)=f(\omega)|v(\omega)|^2$ for any nonsingular function $f(\omega)$ within the whole interval $(0,\infty)$.

We can also diagonalize the Hamiltonian (3.1) considering the introduction of the counterterm $V_R(\hat{q})$. Rewriting it in terms of the operators $\hat{a}$ and $\hat{b}_j$, defined in Eq. (3.6), we have

$$\hat{H}=\hbar\omega_o\hat{a}^\dagger\hat{a}+\hbar\frac{\Delta\omega^2}{4\omega_o}(\hat{a}+\hat{a}^\dagger)^2+\sum_j\hbar\omega_j\hat{b}_j^\dagger\hat{b}_j-\frac{\hbar}{2}\sqrt{\frac{1}{M\omega_o}}(\hat{a}+\hat{a}^\dagger)\sum_j\frac{C_j}{\sqrt{m_j\omega_j}}(\hat{b}_j+\hat{b}_j^\dagger). \tag{A15}$$

Writing Eq. (A15) in the continuum limit and following the same procedure as adopted above, we will see that the equations (A2) and (A3) will be substituted now by the equations

$$\omega\alpha_\omega=\left(\omega_o+\frac{\Delta\omega^2}{2\omega_o}\right)\alpha_\omega-\frac{\Delta\omega^2}{2\omega_o}\chi_\omega + \int[\beta_{\omega,\Omega}v^*(\Omega)-\sigma_{\omega,\Omega}v(\Omega)]d\Omega, \quad \text{(A16)}$$

$$\omega\chi_\omega=-\left(\omega_o+\frac{\Delta\omega^2}{2\omega_o}\right)\chi_\omega+\frac{\Delta\omega^2}{2\omega_o}\alpha_\omega + \int[\beta_{\omega,\Omega}v^*(\Omega)-\sigma_{\omega,\Omega}v(\Omega)]d\Omega, \quad \text{(A17)}$$

respectively. Equations (A4) and (A5) will stay the same. Thus, it can be easily shown that all the other previous equations will not change, with the only difference being that the function $H(\omega)$ should be substituted by $H_R(\omega)$ given in Eq. (3.14).

APPENDIX B: CALCULATION OF $\mathcal{F}_\mathcal{R}(\langle\hat{q}_j\rangle_\mathcal{R},\langle\hat{p}_j\rangle_\mathcal{R};t)$

Expression (7.3) for $\mathcal{F}_\mathcal{R}(\hat{q}_\Omega,\hat{p}_\Omega;t)$ can be written as

$$\mathcal{F}_\mathcal{R}(\hat{q}_\Omega,\hat{p}_\Omega;t)=2\omega_o\int\frac{d\Omega}{\pi}\sqrt{\frac{m_\Omega\Omega}{M\omega_o}}v(\Omega)\left[\mathcal{J}(\Omega;t)\hat{q}_\Omega+\mathcal{K}(\Omega;t)\frac{\hat{p}_\Omega}{m_\Omega\Omega}\right], \quad \text{(B1)}$$

where the expressions for $\mathcal{J}(\Omega;t)$ and $\mathcal{K}(\Omega;t)$ are obtained by direct comparison between Eqs. (B1) and (7.3). Now we can substitute the expression (3.10) for $v(\Omega)$ in Eq. (B1) and write the expression obtained in the discrete limit,

$$\mathcal{F}_\mathcal{R}(\hat{q}_\Omega,\hat{p}_\Omega;t) =-\frac{1}{M}\sum_j\frac{C_{\Omega_j}}{\pi}\left[\mathcal{J}(\Omega_j;t)\sqrt{g(\Omega_j)}\int_{1/g(\Omega_j)}d\Omega\ \hat{q}_\Omega+\frac{\mathcal{K}(\Omega_j;t)}{m_{\Omega_j}\Omega_j}\sqrt{g(\Omega_j)}\int_{1/g(\Omega_j)}d\Omega\ \hat{p}_\Omega\right]. \quad \text{(B2)}$$

Recalling the relation (2.1) between the discrete and continuous operators, we obtain

$$\mathcal{F}_\mathcal{R}(\hat{q}_j,\hat{p}_j;t)=-\frac{1}{M}\sum_j\frac{C_j}{\pi}\left[\mathcal{J}(\Omega_j;t)\hat{q}_j+\mathcal{K}(\Omega_j;t)\frac{\hat{p}_j}{m_{\Omega_j}\Omega_j}\right]. \quad \text{(B3)}$$

Employing the initial condition (7.13), we have

$$\mathcal{F}_\mathcal{R}(\langle\hat{q}_j\rangle_R,\langle\hat{p}_j\rangle_\mathcal{R};t)=-\frac{1}{M\pi}\sum_j\frac{C_j^2}{m_j\Omega_j^2}\mathcal{J}(\Omega_j;t)\langle\hat{q}\rangle_\mathcal{S} =\mathcal{H}(t)\langle\hat{q}\rangle_\mathcal{S}, \quad \text{(B4)}$$

with

$$\mathcal{H}(t)=-4\omega_o\int\frac{d\Omega}{\pi}\frac{|v(\Omega)|^2}{\Omega}\mathcal{J}(\Omega;t), \quad \text{(B5)}$$

where we used again the relation (3.10). Writing $\mathcal{H}(t)$ as

$$\mathcal{H}(t)=I_1(t)+I_2(t), \quad \text{(B6)}$$

we have

$$I_1(t)=-4\omega_o\int\frac{d\Omega}{\pi}\frac{|v(\Omega)|^2}{\Omega}\frac{d}{dt}W_R(\Omega,t), \quad \text{(B7)}$$

$$I_2(t)=-4\omega_o\int\frac{d\Omega}{\pi}\frac{|v(\Omega)|^2}{\Omega}Z_R(\Omega)\cos(\Omega t). \quad \text{(B8)}$$

The calculation of $I_1(t)$ is a somewhat lengthy but straightforward calculation and results in $I_1(t)=0$. So all that is left is

$$\mathcal{H}(t)=I_2(t)=4\gamma\int\frac{d\Omega}{\pi}\frac{\omega_o^2-\Omega^2}{(\Omega^2-\omega_o^2)^2+(2\gamma\Omega)^2}\cos(\Omega t). \quad \text{(B9)}$$

The evaluation of this last integral can also be accomplished by the method of residues and yields

$$\mathcal{H}(t)=2\gamma\mathcal{L}(t) \quad \text{(B10)}$$

for $t>0$.

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