Control of decoherence and relaxation by frequency modulation of a heat bath

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We demonstrate, in a very general fashion, the considerable slowing down of decoherence and relaxation by fast frequency modulation of the system-heat–bath coupling. The slowing occurs as the decoherence rates are now determined by the spectral components of bath correlations which are shifted due to fast modulation. We present several examples including the slowing down of the heating of a trapped ion, where the system-bath interaction is not necessarily Markovian.

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In recent times the decoherence of a coherent superposition state has acquired a new dimension [1–5] because of the requirement of the stability of such a superposition. The stability has been investigated for certain systems. The decoherence rates have been calculated and even measured [6] in the context of Schrödinger-cat states [7] for the radiation field in a cavity. The decoherence issues are also very significant in the context of quantum computation [8–10]. Clearly the stability of coherent superpositions requires methods for slowing down the decoherence. Several proposals exist in the literature [11–13]. These involve, for example, the use of a sequence of pulses [12] or engineering of the density of states associated with the reservoir or even changing the reservoir interaction from a single-photon to multiphoton (or more generally multiboson) interaction [11,14]. Other proposals involve feedback methods [13]. It may be added that spontaneous emission in many systems is also a cause of decoherence. We now understand reasonably well how to inhibit spontaneous emission either by manipulating the density of states [15] or by using external fields [16,17]. The methods based on external fields could be especially useful for slowing down decoherence and decay.

In this paper, we discuss a method based on the frequency modulation [18–20] of the system-heat–bath (environment) coupling. We specifically assume a large frequency modulation, and take the modulation index m to have a value given by $J_0(m)=0$. Under these conditions, we demonstrate a considerable slowing down of the decay and decoherence rates. We present a physical basis for this slowing down. We present several examples including the heating of a trapped ion. *Our method is useful only if the correlation time* κ^{-1} *of the heat bath is larger than the rate of frequency modulation, i.e.,* ν^{-1}. In fact this kind of condition is inherent in all proposals based on the use of coherent interactions, the time scales of which have to be smaller than the bath correlation time. It must be noted that a recent experimental proposal to control decoherence [6(b)] also depends on a coherent coupling with a bath (second single-mode cavity) and works under similar conditions [12,21,22].

In order to appreciate the basic idea of using frequency modulation, we consider a two-state system, where the states $|a\rangle$ and $|b\rangle$ are coupled by some field. We also assume that state $|b\rangle$ decays at a rate 2κ. This simple model (Fig. 1) can describe many physical situations. For example, it can represent an excited atom in a cavity [23], in which the photon leaks out at the rate 2κ. In this case states $|a\rangle$ and $|b\rangle$ will correspond to $|e,o\rangle$ and $|g,1\rangle$, where $|o\rangle$ and $|1\rangle$ represent vacuum and one-photon states, respectively and where $|e\rangle$ and $|g\rangle$ represent the excited and ground states of the atom. It can also describe a situation where the state $|b\rangle$ could be an excited state coupled to ionization continuum. The probability amplitudes C_a and C_b for the states $|a\rangle$ and $|b\rangle$ obey the equations

$$\dot{C}_a = -igC_b, \qquad \dot{C}_b = -\kappa C_b - ig^* C_a. \tag{1}$$

We have removed any fast time dependence by working in an appropriate frame. If κ is large, then, as is well known,

$$|C_a|^2 \cong \exp\{-2\Gamma t\}, \qquad \Gamma = |g|^2/\kappa; \quad \kappa \gg g. \tag{2}$$

The decay of state $|a\rangle$ arises from the decay of state $|b\rangle$. In the opposite limit ($\kappa \to 0$) one obtains an oscillatory behavior, which in the cavity context is known as the vacuum field Rabi oscillation. We now consider the effect of a phase modulation $m \sin \nu t$ on the decay of the state $|a\rangle$: we assume a modulation of the coupling constant,

$$g \to g \exp\{-im \sin \nu t\}. \tag{3}$$

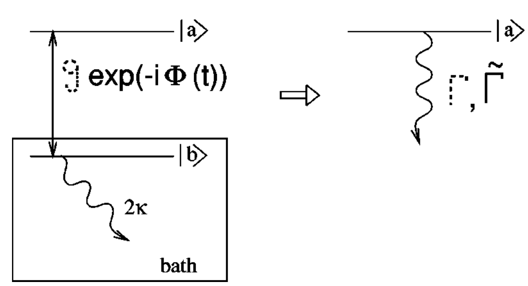

FIG. 1. Schematic representation of a generic two-level system with lower-level decay into a bath. A strong modulation of the coupling slows the decay of the upper level.

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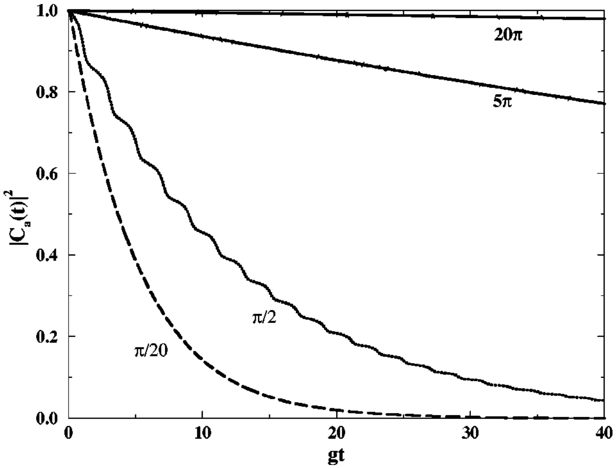

FIG. 2. The population in the excited state $|C_a(t)|^2$ [Eqs. (1) and (3)] as a function of t for different values of the modulation frequency and for $\kappa=10g$. The modulation index is chosen to be the fifth zero of Eq. (4). The curves from top to bottom are for $\nu/\pi g=20,5,0.5$, and 0.05. The curve for $m=0$ is hardly distinguishable from the curve for $\nu/\pi g=0.05$.

Here m and ν give the amplitude and the frequency of the modulation respectively. Equation (1) is no longer amenable to analytical solutions. In Fig. 2, we display the excited-state population for different values of ν and m chosen to be a zero of the Bessel function of order zero

$$J_0(m)=0. \tag{4}$$

This choice of m will become clear in the analysis to follow. In Fig. 2 we also show the behavior in the absence of modulation. We observe that under condition (4) the decay of the excited-state population is considerably slowed as the modulation frequency increases. This clearly demonstrates *how a frequency modulation can slow down the effects of decay*. We thus have a *method of controlling relaxation or decay* by frequency modulation. We now explain the observed numerical behavior for large ν. Using Eq. (1), we can easily derive the following integrodifferential equation for the amplitude of the excited state:

$$\dot{C}_a\equiv-|g|^2e^{-i\Phi(t)}\int_0^t e^{-\kappa(t-\tau)+i\Phi(\tau)}C_a(\tau)d\tau. \tag{5}$$

We use

$$e^{-i\Phi(t)}=\sum_{l=-\infty}^{+\infty}J_l(m)e^{-il\nu t}, \tag{6}$$

and we assume that (i) ν is large and (ii) $C_a(\tau)$ varies slowly with τ, and carry out a long-time average denoted by an overbar to obtain

$$\frac{\partial}{\partial t}(\ln C_a)\equiv-|g|^2\int_0^t e^{-\kappa\tau}\overline{e^{-i\Phi(t)}e^{i\Phi(t-\tau)}}d\tau,$$

$$\equiv-|g|^2\sum_p J_p^2(m)(\kappa+ip\nu)^{-1}. \tag{7}$$

In order to slow down the decoherence, we need to remove the $\nu=0$ term in Eq. (7). This can be achieved by imposing the condition (4), after which Eq. (7) reduces to

$$\frac{\partial}{\partial t}(\ln C_a)\approx-2|g|^2J_1^2(m)(\kappa^2+\nu^2)^{-1}\kappa. \tag{8}$$

Therefore the decay of the excited state occurs at a modified rate $2\tilde{\Gamma}$ with

$$\frac{\tilde{\Gamma}}{\Gamma}\cong J_1^2(m)\left(\frac{2\kappa^2}{\kappa^2+\nu^2}\right). \tag{9}$$

The decay factor (9) agrees very well with the behavior shown in Fig. 2 for 20π, as then $\nu\gg\kappa$. The very fast oscillations do not show up on the scale of the Fig. 2. Result (9) can be understood by noting that (i) the factor in the parentheses in Eq. (9) is just the factor that one would have obtained with a detuned interaction between the states $|a\rangle$ and $|b\rangle$, and (ii) the Bessel function represents the strength of the first sideband.

We next demonstrate that the above idea applies rather generally. We consider the usual microscopic treatment of the heat bath [24] with the *modulation of the system–heat-bath coupling*. For the purpose of illustration, we consider a spin system (raising and lowering operators S^+ and S^-) interacting, say, with dc and ac magnetic fields in the Z direction, so that the unperturbed Hamiltonian is $(\omega_0-m\nu\cos\nu t)S^z$. The energy separation is modulated—such modulations are routinely used (see, e.g., Noel *et al.* [18]). In the interaction picture the interaction with the heat bath can be written as

$$H_I(t)=(S^+e^{+i\omega_0 t-i\Phi(t)}R^-(t)+\text{H.c.}), \tag{10}$$

where $R^-(t)$ is the appropriate operator for the heat bath. As usual [24], we will assume that the coupling of the bath to the system is weak. The heat bath is characterized in terms of the correlation functions:

$$\begin{aligned}&\langle R^-(t)\rangle=0,\\&\langle R^+(t+\tau)R^-(t)\rangle=C^{+-}(\tau),\\&\langle R^-(t+\tau)R^+(t)\rangle=C^{-+}(\tau),\\&\langle R^-(t+\tau)R^-(t)\rangle=0.\end{aligned} \tag{11}$$

The Fourier transforms of C^{+-} and C^{-+} are related via the fluctuation dissipation theorem. We can now do the standard calculation [24] to derive a master equation for the reduced density matrix ρ of the system alone. We quote the result of this calculation:

$$\frac{\partial\rho}{\partial t} = -(S^+S^-\rho - S^-\rho S^+) \times \int_0^t d\tau C^{-+}(\tau)e^{+i\omega_0\tau}e^{-i\Phi(t)}e^{i\Phi(t-\tau)}, -(\rho S^-S^+ - S^+\rho S^-) \times \int_0^t d\tau C^{+-}(-\tau)e^{+i\omega_0\tau}e^{i\Phi(t-\tau)-i\Phi(t)}, +(\text{terms with subscripts } \pm\to\mp, \omega_0\to-\omega_0, \Phi\to-\Phi). \tag{12}$$

First of all we note, that if the bath correlations were like δ-function correlations $C^{-+}(\tau)=2\delta(\tau)C^{-+}$, then the master equation (12) does *not depend on the modulation* Φ. Clearly, the *bath correlation time* τ_c *has to be at least of the order of the time* associated with the modulation. Under the fundamental condition (4), the time average in Eq. (12) can be *approximated by*

$$\overline{e^{-i\Phi(t)+i\Phi(t-\tau)}} \cong 2J_1^2(m)\cos\nu\tau, \tag{13}$$

and then Eq. (12) reduces to

$$\frac{\partial\rho}{\partial t} \equiv -2(S^+S^-\rho - S^-\rho S^+) \times \int_0^\infty d\tau C^{-+}(\tau)e^{+i\omega_0\tau}\cos\nu\tau J_1^2(m), -2(\rho S^-S^+ - S^+\rho S^-) \times \int_0^\infty d\tau C^{+-}(-\tau)e^{+i\omega_0\tau}\cos\nu\tau J_1^2(m), +(\text{terms with } \pm\to\mp, \omega_0\to-\omega_0). \tag{14}$$

The standard master equation corresponds to the limits $\nu\to 0$ and $J_1^2\to 1$. It is clear that if ν is large enough compared to frequency scale of $C^{-+}(\tau)e^{i\omega_0\tau}$ then the real part of the integral in Eq. (14) will be approximately zero and *decoherence effectively does not exist*. In particular, if $C^{-+}(\tau) = C_0^{-+}e^{-\kappa\tau-i\omega\tau}, C^{+-}(-\tau)=C_0^{-+}e^{-\kappa\tau-i\omega\tau}$, then

$$\frac{\partial\rho}{\partial t} = -\frac{2(\kappa-i\Delta)J_1^2(m)}{(\kappa-i\Delta)^2+\nu^2}\{C_0^{-+}(S^+S^-\rho - S^-\rho S^+) + C_0^{+-}(\rho S^-S^+ - S^+\rho S^-)\} + \text{c.c.}, \Delta=(\omega_0-\omega). \tag{15}$$

Clearly, the relaxation coefficients in the master equation are modified by factors like Eq. (9). Hence the relaxation is much slower. In particular, the relaxation of the coherence $\langle S^\pm\rangle$ will be on a much longer time scale. For large ν compared to κ and Δ, the relaxation time is very large.

We next consider the application of the above ideas to the decoherence of an ion in a trap. This is important in many applications of ion traps, such as in connection with the production of Schrödinger-cat states and in quantum computation. In particular, we consider the possibility of *reducing* the *heating* of the *ground state of trapped ions*. In a recent paper, James [10] considered a model for heating produced by a stochastic field $E(t)$. In terms of the annihilation and creation operators a and $a^\dagger$ associated with the ionic motion the heating is described by the Hamiltonian

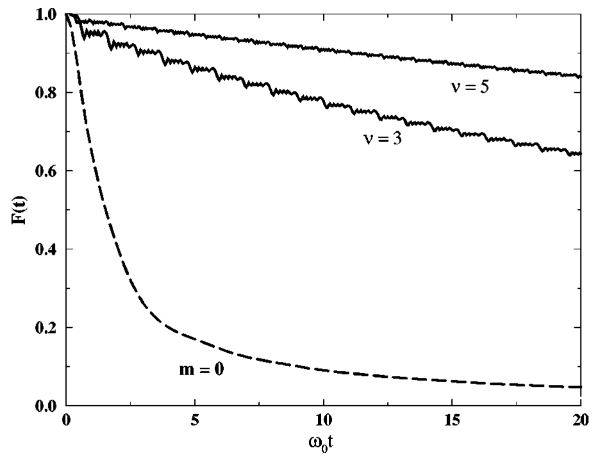

FIG. 3. Fidelity [Eq. (17)] or the heating of an ion in the trap for two different values of the modulation frequency $\nu=5$ and 3. The dashed curve gives the result in the absence of the modulation. Parameters are chosen as $\omega_0/\kappa=1$; Ω [defined by Eq. (19)] $=\sqrt{2}\omega_0$. The wiggles arise from the periodic modulation.

$$H_1 = i\hbar[u(t)a^\dagger - u^*(t)a], \tag{16}$$

where $u(t)=iZE(t)e^{i\omega_0 t}/\sqrt{2M\hbar\omega_0}$. The field $E(t)$ is a Gaussian stochastic process. The time scale of the stochastic field is taken to be comparable to the time scale of the ionic motion. Hence this model is *outside* the usual *Markovian limit*. We now consider the effect of an external modulation so that effectively, $u(t)\to u(t)e^{-i\Phi(t)}$. Following James's work the fidelity $F(t)$ of the ground state is given by

$$F(t) = [1+2\langle|v(t)|^2\rangle + \langle|v(t)|^2\rangle^2 - |\langle v^2(t)\rangle|^2]^{-1/2}, \tag{17}$$

$$v(t) = \frac{iZ}{\sqrt{2M\hbar\omega_0}}\int_0^t E(t')e^{-i\Phi(t')+i\omega_0 t'}dt'. \tag{18}$$

The mean values in Eq. (17) can be obtained from Eq. (18) by assuming exponential correlation for $E(t)$:

$$\langle u(t)u^*(t')\rangle = \frac{\Omega^2}{2}e^{-\kappa|t-t'|},$$

$$\langle|v|^2\rangle = \frac{\Omega^2}{2}\sum_{-\infty}^{+\infty}\sum J_n(m)J_p(m)I(\omega_0-n\nu; -\omega_0+p\nu),$$

$$\langle v^2\rangle = -\frac{\Omega^2}{2}\sum_{-\infty}^{+\infty}\sum J_n(m)J_p(m)I(\omega_0-n\nu; \omega_0-p\nu), \tag{19}$$

where the integral $I(\omega_\alpha,\omega_\beta)$ is found to be

$$I(\omega_\alpha,\omega_\beta)=(i\omega_\alpha+i\omega_\beta)^{-1}[(\kappa+i\omega_\beta)^{-1}e^{i(\omega_\alpha+\omega_\beta)t}-(\kappa-i\omega_\alpha)^{-1}]+(i\omega_\alpha-\kappa)^{-1}(-i\omega_\beta-\kappa)^{-1}\times e^{i\omega_\alpha t-\kappa t}+(\text{terms with } \alpha\Leftrightarrow\beta). \tag{20}$$

Note that $\omega_\alpha+\omega_\beta$ can vanish in which case, a limiting procedure leads to

$$I(\omega_\alpha,-\omega_\alpha)=(\kappa-i\omega_\alpha)^{-1}t+(i\omega_\alpha-\kappa)^{-2}(e^{i\omega_\alpha t-\kappa t}-1)+\text{c.c.} \tag{21}$$

We show the fidelity factor F in Fig. 3, both in the absence and presence of the modulation. We choose a parameter domain in which *fidelity* was being *degraded* rather fast. Clearly, if we assume large frequency modulation and condition (4), then, as the figure shows, there is considerable *improvement in the fidelity* under frequency modulation of the stochastic field $E(t)$ responsible for heating the trapped ion.

In conclusion, we have shown how the appropriate modulation of the system–heat-bath interaction can slow down the decay as well as the decoherence to a very large extent. This happens as generally, the decoherence is determined by the spectral components of the bath correlation functions. If system-bath interaction is modulated, then the decoherence is determined by the spectral components, which are shifted by the multiples of the modulation frequency. If the modulation frequency is large compared to the width of the bath correlations, then we would get much smaller decoherence rate. Finally, note that we have a method to control the effects of decoherence since the modulation depth and frequency can be varied.

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