Gain components in the Autler-Townes doublet from quantum interferences in decay channels

Sunish Menon[1] and G. S. Agarwal[1,2]
[1]*Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India*
[2]*Max-Plank-Institut für Quantenoptik, 85748 Garching, Germany*
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We consider *nondegenerate* pump-probe spectroscopy of V systems under conditions such that interference among decay channels is important. We demonstrate how this interference can result in new gain features instead of the usual absorption features. We relate this gain to the existence of a new vacuum induced quasi-trapped-state. We further show how this also results in a large refractive index with low absorption.

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I. INTRODUCTION

The properties of a medium are significantly altered when the medium is driven by strong, resonant, coherent fields. Mollow first studied in detail the physical characteristics of a two-level system driven by a coherent field of arbitrary strength. He discovered new features in the emission spectra [1]. These new features are best understood in terms of field-dependent eigenstates and eigenvalues or dressed states of the system [2]. Mollow, in a later paper [3], further demonstrated the possibility of amplification of a probe field in a coherently driven two-level system. This gain can also be understood in terms of dressed states [4]. An alternative way of probing a driven two-level model is by coupling a probe field to one of the states (e.g., the ground state) of the strongly driven transition and a different excited state (V system). The absorption spectra will show new resonances related to the dressed states of the strongly driven two-level atom. The strong drive splits the absorption resonance into two components known as the Autler-Townes components [5]. Various experiments in gases [6] and in solid-state systems [7] have confirmed the presence of the Autler-Townes splitting of absorption lines. Such a three-level model is not known to show gain unless additional fields are introduced. For example, an incoherent pumping along the probing transition of a V system will give rise to gain [8,9] . Other three-level models with coherent and incoherent pumping also exhibit gain [10].

In this paper, we demonstrate that quantum interference between different paths of spontaneous emission in a V system can produce *gain under conditions when one would have otherwise observed absorption peaks*. Interference due to spontaneous emission can arise when spontaneous emission from one level can strongly affect a neighboring transition. For example, consider excited levels $|1\rangle$, $|2\rangle$ of the same parity and the ground state $|3\rangle$ of a different parity [see Fig. 1(a)]. Let the spontaneous-emission rates from levels $|1\rangle$ and $|2\rangle$ to level $|3\rangle$ be denoted as $2\gamma_1$ and $2\gamma_2$, respectively. In the interaction picture, the equations of motion for density-matrix elements will be [11]

$$\dot{\rho}_{11}=-2\gamma_1\rho_{11}-\sqrt{\gamma_1\gamma_2}\cos\theta(\rho_{12}e^{-iW_{12}t}+\rho_{21}e^{iW_{12}t}), \tag{1}$$

$$\dot{\rho}_{22}=-2\gamma_2\rho_{22}-\sqrt{\gamma_1\gamma_2}\cos\theta(\rho_{12}e^{-iW_{12}t}+\rho_{21}e^{iW_{12}t}),$$

$$\dot{\rho}_{12}=-(\gamma_1+\gamma_2)\rho_{12}-\sqrt{\gamma_1\gamma_2}\cos\theta e^{iW_{12}t}(\rho_{11}+\rho_{22}),$$

$$\dot{\rho}_{13}=-\gamma_1\rho_{13}-\sqrt{\gamma_1\gamma_2}\cos\theta e^{iW_{12}t}\rho_{23},$$

$$\dot{\rho}_{23}=-\gamma_2\rho_{23}-\sqrt{\gamma_1\gamma_2}\cos\theta e^{-iW_{12}t}\rho_{13}.$$

Here $\hbar W_{12}$ is the energy separation between the excited levels which we keep arbitrary. The above equations are derived without making any kind of secular approximation, and can be solved under the conditions $\rho_{33}+\rho_{22}+\rho_{11}=1$ and $\rho_{ij}=\rho_{ji}^*$. The off-diagonal radiative coupling terms in the equations for diagonal elements of ρ are due to interference among decay channels. Here the parameter θ is the angle between the dipole matrix elements $\vec{d}_{13}$ and $\vec{d}_{23}$, where $\vec{d}_{i3}=\langle i|\mathbf{d}|3\rangle$ $(i=1,2)$ and $\mathbf{d}$ is the dipole moment operator. Note that this interference exists only when $\theta\neq 90°$. A result of such an off-diagonal radiative coupling is that the coherence $\rho_{12}(t)$ will evolve even when $\rho_{12}(0)=0$. The coherence arises from the vacuum of the electromagnetic field. We refer to it as vacuum-induced coherence (VIC). This coherence term will change the steady-state response of the medium and under suitable conditions can *create trapping in a degenerate V system* [11]. The coherence can also modify significantly the emission spectrum of a near-degenerate V system [12]. Recent works [13] generalize Eq. (1) to include thermal photons as well as incoherent pumping. The presence of both thermal photons and VIC leads to additional features in the spectrum. Moreover, it should be borne in

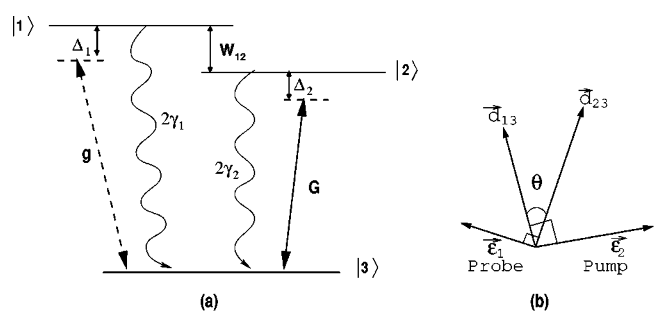

FIG. 1. (a) Schematic diagram of a three-level V system. The pump and probe fields have frequency detunings Δ_2 and Δ_1, respectively. The γ's denote the spontaneous-emission rates from the respective levels. (b) The arrangement of field polarization required for a single field driving one transition if dipoles are nonorthogonal.

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mind that the coherence terms in Eqs. (1) are insignificant when $W_{12} \gg \gamma_1, \gamma_2$, as then the oscillatory terms will average out to zero and such effects vanish.

In the present paper, we study how to probe VIC by using a pump-probe spectroscopy. We demonstrate that the VIC can manifest itself via gain features instead of the traditional absorption features. The organization of this paper is as follows. In Sec. II we present basic equations describing the pump-probe spectroscopy under conditions when interference between decay channels is important. We also point out the crucial difference between the present work and the previous studies. In Sec. III we discuss our numerical results. We show the possibility of new gain features due to VIC. In Sec. IV we report the possibility of quasi-trapped-states that arise strictly from the interference between decay channels. In Sec. V we analyze the effect of this trapping on the absorption and dispersion properties of the pump field. In Sec. VI we explain the numerical results of Sec. III in terms of the trapped states discussed in Sec. IV.

II. BASIC EQUATIONS

Consider the pump-probe setup shown in Fig. 1(a). The transition dipole moments $\vec{d}_{13}$ and $\vec{d}_{23}$ are nonorthogonal. To study the situation as much parallel to the usual case where the pump and probe fields act on two different arms of the V system, we consider an arrangement of field polarization as shown in Fig. 1(b). This enables us to study the VIC effects as well as compare with the usual situation [14]. Thus the pump field ($\vec{E}_2 = \vec{\varepsilon}_2 e^{-i\omega_2 t} + \text{c.c.}$) with a Rabi frequency $2G = 2\vec{d}_{23} \cdot \vec{\varepsilon}_2/\hbar$ drives the $|2\rangle \leftrightarrow |3\rangle$ transition ($\vec{d}_{13} \cdot \vec{\varepsilon}_2 = 0$) and similarly the probe field ($\vec{E}_1 = \vec{\varepsilon}_1 e^{-i\omega_1 t} + \text{c.c.}$) with a Rabi frequency $2g = 2\vec{d}_{13} \cdot \vec{\varepsilon}_1/\hbar$ drives the $|1\rangle \leftrightarrow |3\rangle$ transition ($\vec{d}_{23} \cdot \vec{\varepsilon}_1 = 0$). We note here that for Fig. 1(b) the Rabi frequencies will also depend on the angle θ. But for convenience in comparison with different values of θ, we will keep the Rabi frequencies the same, which in practice can be done by suitably increasing/decreasing the field strength. The Hamiltonian for this system will be

$$H = \hbar W_{13}|1\rangle\langle 1| + \hbar W_{23}|2\rangle\langle 2| - \hbar(G|2\rangle\langle 3| \times e^{-i\omega_2 t} + g|1\rangle\langle 3|e^{-i\omega_1 t} + \text{H.c.}), \tag{2}$$

where $\hbar W_{i3}$ ($i=1,2$) is the energy of the state $|i\rangle$ when measured with respect to the state $|3\rangle$. In the rotating-wave approximation the density-matrix equations with the inclusion of all the decay terms will be

$$\begin{aligned}
\dot{\rho}_{11} &= -2\gamma_1\rho_{11} - \eta(\rho_{12}+\rho_{21}) + ige^{-i\delta t}\rho_{31} - ige^{i\delta t}\rho_{13},\\
\dot{\rho}_{22} &= -2\gamma_2\rho_{22} - \eta(\rho_{12}+\rho_{21}) + iG\rho_{32} - iG\rho_{23},\\
\dot{\rho}_{12} &= -(\gamma_1+\gamma_2+iW_{12})\rho_{12} - \eta(\rho_{11}+\rho_{22}) + ige^{-i\delta t}\rho_{32} - iG\rho_{13},\\
\dot{\rho}_{13} &= -[\gamma_1 + i(\Delta_2 + W_{12})]\rho_{13} - \eta\rho_{23} - iG\rho_{12} + ige^{-i\delta t}(1-2\rho_{11}-\rho_{22}),\\
\dot{\rho}_{23} &= -(\gamma_2 + i\Delta_2)\rho_{23} - \eta\rho_{13} - ige^{-i\delta t}\rho_{21} + iG(1-\rho_{11}-2\rho_{22}),
\end{aligned} \tag{3}$$

where $\delta = \omega_1 - \omega_2$ is the probe-pump detuning. The probe detuning $\Delta_1 = W_{13} - \omega_1$ and the pump detuning $\Delta_2 = W_{23} - \omega_2$ are related by $\Delta_1 - \Delta_2 = W_{12} - \delta$. In deriving Eqs. (3) we have made the canonical transformations so that ρ_{13} and ρ_{23} are obtained by multiplying the solution of Eqs. (3) by $e^{-i\omega_2 t}$. We also use the trace condition $\rho_{11}+\rho_{22}+\rho_{33}=1$. Here $\eta = \sqrt{\gamma_1\gamma_2}\cos\theta$ is the VIC parameter, which is nonzero when $\theta \neq 90°$. Note that for the geometry shown in Fig. 1(b), θ is always nonzero, though it could be small. In the absence of external fields as seen from Eqs. (1), the VIC effect is important when the separation between the two excited levels is of the order of the natural linewidth. However, this condition may be relaxed when the system is being driven by external fields, as we will see later.

Let us first consider the case when $\theta = 90°$. Making a further canonical transformation on ρ_{13} and ρ_{12}, we can get rid of the explicit time dependence. The imaginary part of ρ_{13} yields the probe absorption. In the limit of a weak probe field ($g \ll \gamma_1, \gamma_2$), we obtain

$$\rho_{13} = \frac{g\{(\gamma_2^2+\Delta_2^2+G^2)[\Delta_2-\Delta_1+i(\gamma_1+\gamma_2)] + G^2(\Delta_2 - i\gamma_2)\}}{(\gamma_2^2+\Delta_2^2+2G^2)\{G^2 + (\Delta_1 - i\gamma_1)[\Delta_2-\Delta_1+i(\gamma_1+\gamma_2)]\}}. \tag{4}$$

In the limit of vanishing γ's and large G, the above expression shows that two complex poles exist at $\Delta_1 = [\Delta_2 + i\gamma_2 + 2i\gamma_1 \pm \sqrt{(\Delta_2 + i\gamma_2)^2 + 4G^2}]/2$. The probe absorption as a function of Δ_1, i.e. as a function of probe frequency will have two resonances at $\Delta_1 = [\Delta_2 \pm \sqrt{(\Delta_2^2 + 4G^2)}]/2$. These are the two Autler-Townes components in the absorption spectrum. It can be further shown that $\text{Im}(\rho_{13}) > 0$.

We now consider the effects of VIC ($\theta \neq 90°$). The system of Eqs. (3) has been studied under a very wide range of conditions. We would now recall what has been done and in what ways our current work differs from the existing works. (a) We could first consider the case when pump is also replaced by the probe ($\vec{\varepsilon}_1 \equiv \vec{\varepsilon}_2,\ \omega_1 = \omega_2$). Here the effects of VIC manifest both in the emission [15,16] and absorption spectrum [17–19]. Zhou and Swain demonstrated the existence of ultranarrow spectral lines in emission [16]. Cardimona *et al.* showed a vanishing of absorption under certain conditions [17], whereas Zhou and Swain demonstrated the possibility of gain with no pump field present [18]. (b) Another case which is extensively studied by Knight and co-workers corresponds to degenerate pump and probe fields, i.e., $\vec{\varepsilon}_1 \neq \vec{\varepsilon}_2$, but $\omega_1 = \omega_2$ ($\delta = 0$). Here the pump can have arbitrary strength while the probe is kept relatively weak. Paspalakis *et al.* showed how VIC can lead to gain without inversion [19]. (c) In the present work we study the important case of nondegenerate pump and probe fields, $\vec{\varepsilon}_1 \neq \vec{\varepsilon}_2$, $\omega_1 \neq \omega_2$. We show how the VIC can invert the traditional Autler-Townes splittings in the absorption spectrum and produce gain features.

While we work with a three-level system, it should be noted that effects of VIC in the context of four- and five-level schemes have been very extensively investigated [20–28]. In particular, Zhu and co-workers discovered quenching of spontaneous emission [21,22]. An intuitive picture for spontaneous-emission suppression and enhancement was provided by Agarwal [23].

The nondegenerate case, which we treat, has a major complication due to explicit time dependence in the equations of motion (3). Since the time dependence in Eqs. (3) is periodic, we can solve these equations by Floquet analysis. The solution can be written as

$$\rho_{ij} = \sum_m \rho_{ij}^{(m)} e^{-im\delta t}. \tag{5}$$

Thus the absorption and emission spectra get modulated at various harmonics of δ. The dc component in the probe absorption spectrum is related to $\rho_{13}^{(+1)}$. The absorption coefficient α per unit length can be shown to be

$$\alpha = \frac{\alpha_0 \gamma_1}{g} \mathrm{Im}(\rho_{13}^{(+1)}), \tag{6}$$

where $\alpha_0 = 4\pi\mathcal{N}|d_{13}|^2 \omega_1 / \hbar\gamma_1 c$ and $\mathcal{N}$ denotes the atomic density. Note that in Eq. (6) only one term from the entire series (5) contributes. For the case of degenerate pump-probe ($\delta = 0$), all the terms in the series (5) are important.

III. NUMERICAL RESULTS

In order to obtain the probe absorption spectra, we solve Eqs. (3) numerically using the series solution (5) and the steady-state condition $\dot{\rho}_{ij}^{(m)} = 0$. The situation is much simpler for a weak probe when $\rho_{ij}^{(+1)}$ can be computed to first order in g, otherwise we use the Floquet method. In Fig. 2 we plot the probe absorption as a function of probe detuning. The dashed curves in Figs. 2(a) and 2(b) are the usual Autler-Townes components in the absence of VIC effects. The solid curves show the absorption spectra when VIC is included.

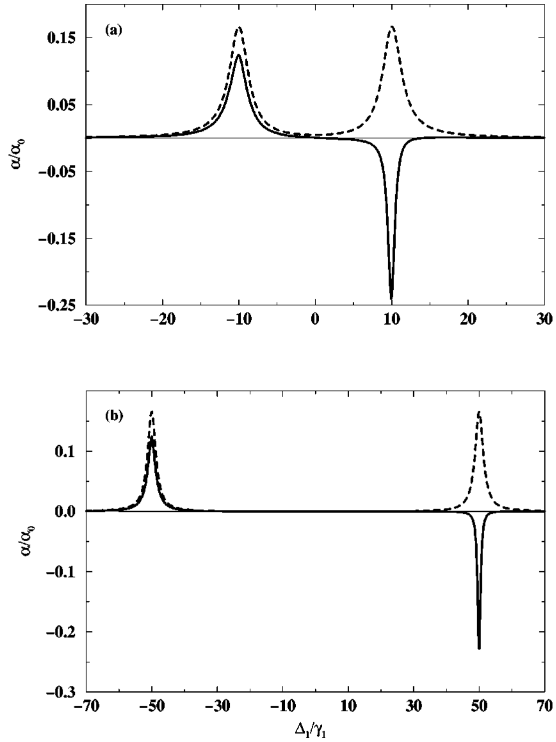

FIG. 2. Effect of interference between decay channels on probe absorption. For both of the frames, the dashed curves show the usual Autler-Townes components in the absence of VIC ($\theta = 90°$) and the solid curves are for $\theta = 15°$. The common parameters are $g = 0.01\gamma_1$, $\gamma_2 = \gamma_1$, and $\Delta_2 = 0$. Note that α will depend on W_{12} only when VIC is present and we take $W_{12} = -G$. In frame (a) we have kept $G = 10\gamma_1$ and in frame (b) we take $G = 50\gamma_1$. The solid curve in frame (b) shows that the effect of VIC is retained even for large W_{12}.

We observe that *one of the Autler-Townes components flips sign to give rise to significant gain*. This type of behavior is seen for any value of W_{12} provided the pump-field strength satisfies the condition $G = |W_{12}|$ for $\Delta_2 = 0$. For $W_{12} = G$ and $\Delta_2 = 0$, the gain appears at $\Delta_1 = -G$. The solid curve in Fig. 2(b) shows that the effect of VIC is observed even for large W_{12} compared to γ_1, γ_2. This is in contrast to the situation that exists in the absence of external fields, where one finds that VIC effects are important when the separation between the vacuum coupled levels is of the order of the natural linewidth. As can be seen from Fig. 2(b), for strong pump fields, such a restriction can be relaxed. Also note that one of the Autler-Townes components can be almost suppressed for a certain set of parameters (see, for example, the solid curve in Fig. 3). Thus the parameter θ (angle between the two transition dipole matrix elements) controls the spectra in the presence of VIC. The dot-dashed curve in Fig. 3 also shows the effect of unequal decays. For $\gamma_2 > 2\gamma_1$, both the Autler-Townes components flip. We analyze the origin of gain in the following sections. We note that the previous works [19(a)] on the *degenerate* pump and probe fields also reported gain, provided the energy separation between the two excited states can be scanned.

Finally, as mentioned in Sec. II, the observation of VIC-related effects requires the use of transitions with nonor-

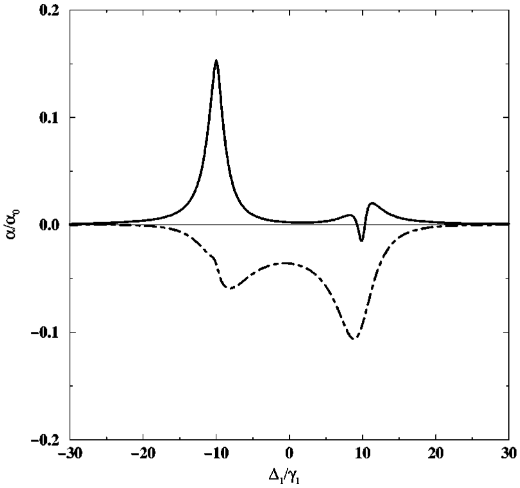

FIG. 3. Plots show the important role θ and unequal γ's play in the presence of VIC. The common parameters are $G=10\gamma_1$, $g=0.01\gamma_1$, $\Delta_2=0$, and $W_{12}=-G$. The solid curve presents the case when $\gamma_2=\gamma_1$ and $\theta=35°$ and the dot-dashed curve arises when $\gamma_2=6\gamma_1$ and $\theta=15°$.

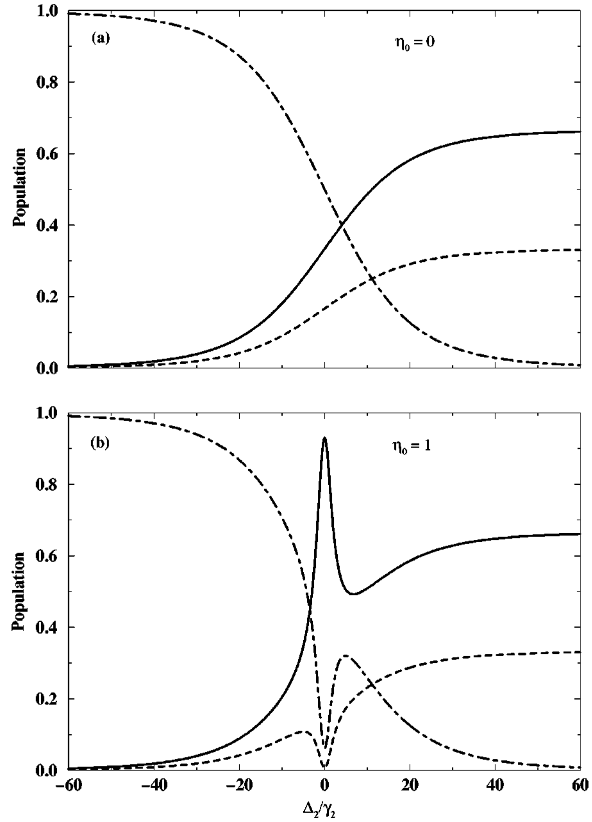

FIG. 4. The atomic population in the basis (10) as a function of pump detuning Δ_2/γ_2 the in presence [frame (b), $\theta=15°$] and in the absence [frame (a), $\theta=90°$] of VIC. The parameters are $G=20\gamma_2$, $W_{12}=-G$, and $\gamma_1=\gamma_2$. The solid curves denote σ^0_{ucuc}, the dashed curves are for σ^0_{cc}, and the dot-dashed curves denote σ^0_{++}.

thogonal dipole matrix elements [11–13,15–19,21–28]. The question of the production of a transition with nonorthogonal dipole matrix elements has been extensively discussed in the literature. This can be achieved by mixing the states using either internal fields [22] or external fields [27–30]. We may further note that the relaxation need not occur by spontaneous emission. For example, in problems involving intersubband transitions in semiconductors the relaxation can occur by emission of LO phonon [31]. In such cases the nonorthogonality of dipole matrix elements is not required.

IV. QUASI-TRAPPED-STATES FROM INTERFERENCE OF DECAY CHANNELS

For a very weak probe field ($g \ll \gamma_1,\gamma_2$) we can solve Eqs. (3) perturbatively with respect to the strength of the probe field. To the lowest order in g, the solution may be written as

$$\rho_{ij}=\sigma^0_{ij}+g\sigma^+_{ij}e^{-i\delta t}+g\sigma^-_{ij}e^{i\delta t}. \tag{7}$$

We first examine the behavior of the system in the presence of pump field alone ($g=0$). Note that in the absence of VIC effects the system reduces to the well known case of a coherently driven two-level atom. But the behavior is quite different in the presence of VIC effects, as we show in the following.

It is clear that field G creates a coherent mixing of states $|2\rangle$ and $|3\rangle$. The new eigenvalues will be

$$\lambda_{\pm}=\frac{\Delta_2\pm\sqrt{\Delta_2^2+4G^2}}{2}, \tag{8}$$

and the corresponding dressed energy states can be written as

$$|+\rangle=\cos\psi|2\rangle+\sin\psi|3\rangle, \tag{9}$$

$$|-\rangle=-\sin\psi|2\rangle+\cos\psi|3\rangle,$$

where $\tan\psi=-G/\lambda_+$. The crucial point to note is that the level $|1\rangle$ is coupled with $|\pm\rangle$ because of the presence of VIC. Thus the population in $|\pm\rangle$ also depends on the VIC parameter η. An important case arises when $|1\rangle$ is degenerate with either $|\pm\rangle$, i.e., when $W_{12}=\lambda_{\pm}$. The degenerate levels get strongly coupled via VIC, giving rise to trapping. When $|1\rangle$ and $|-\rangle$ are degenerate, we show that the dynamical behavior of the system can be best analyzed in the basis given below,

$$|+\rangle,|c\rangle=\frac{\sqrt{2\gamma_1}|1\rangle+\sqrt{\gamma_2}|-\rangle}{\sqrt{\gamma_2+2\gamma_1}},$$

$$|uc\rangle=\frac{\sqrt{\gamma_2}|1\rangle-\sqrt{2\gamma_1}|-\rangle}{\sqrt{\gamma_2+2\gamma_1}}. \tag{10}$$

Using the transformations (9), (10), and Eqs. (3), with $g=0$, we numerically compute the steady-state population in the states (10). In Fig. 4 we plot the population of these states as a function of pump detuning. Note that in the presence of VIC, σ^0_{ucuc} approaches unity at $\Delta_2=0$, i.e., when the states $|1\rangle$ and $|-\rangle$ are degenerate because $W_{12}=-G$. A similar kind of trapping will occur when $|1\rangle$ is degenerate with

$|+\rangle$. When $|W_{12}|\neq G$, the trapping will occur for an off-resonant pump field. Trapping also requires θ to be small. We show later that σ^0_{ucuc} cannot be unity, and for this reason we refer to it as a ''quasi-trapped-state'' (QTS). Figure 4 also shows that for $\Delta_2 \ll -G$, the entire population remains in $|+\rangle$. This is not an interference effect and happens irrespective of whether VIC is present or absent. For large negative pump detuning, $\lambda_+\to 0$ and thus $\sin\psi\to 1$ ($\cos\psi\to 0$) in Eqs. (9). Since the level $|3\rangle$ is the ground state, most of the population remains here if the pump is highly off-resonant. The QTS $|uc\rangle$ is a result of interference among decay channels of $|1\rangle$ and $|-\rangle$ levels. As a consequence, even if W_{12} is large in the bare basis, strong VIC effects can appear when dressed levels are degenerate with the bare excited levels unconnected by the pump field.

We next examine how the quasi-trapped-state is formed. For this purpose we transform the equations of motion (3) for the density-matrix elements in basis (10). For $\Delta_2=0$ and $W_{12}=-G$, a long calculation leads to

$$\dot{\sigma}^0_{ucuc}=-\frac{4\gamma_1\gamma_2(\gamma_1+\gamma_2)(1-\cos\theta)}{(2\gamma_1+\gamma_2)^2}\sigma^0_{ucuc}+\frac{\gamma_1(4\gamma_1^2+4\gamma_1\gamma_2\cos\theta+\gamma_2^2)}{(2\gamma_1+\gamma_2)^2}\sigma^0_{cc}+\frac{\gamma_1\gamma_2}{2\gamma_1+\gamma_2}\sigma^0_{++}-\frac{\gamma_2\sqrt{\gamma_1\gamma_2}(2\gamma_1-\gamma_2)(1-\cos\theta)}{\sqrt{2}(2\gamma_1+\gamma_2)^2}(\sigma^0_{ucc}+\sigma^0_{cuc}), \tag{11a}$$

$$\dot{\sigma}^0_{cc}=-\frac{(4\gamma_1+\gamma_2)(4\gamma_1^2+4\gamma_1\gamma_2\cos\theta+\gamma_2^2)}{2(2\gamma_1+\gamma_2)^2}\sigma^0_{cc}+\frac{2\gamma_1\gamma_2^2(1-\cos\theta)}{(2\gamma_1+\gamma_2)^2}\sigma^0_{ucuc}+\frac{\gamma_2^2}{2(2\gamma_1+\gamma_2)}\sigma^0_{++}-\frac{\gamma_1(2\gamma_1-\gamma_2)\sqrt{2\gamma_1\gamma_2}(1-\cos\theta)}{(2\gamma_1+\gamma_2)^2}(\sigma^0_{ucc}+\sigma^0_{cuc}), \tag{11b}$$

$$\dot{\sigma}^0_{++}=-\frac{\gamma_2}{2}\sigma^0_{++}+\frac{(4\gamma_1^2+4\gamma_1\gamma_2\cos\theta+\gamma_2^2)}{2(2\gamma_1+\gamma_2)}\sigma^0_{cc}+\frac{2\gamma_1\gamma_2(1-\cos\theta)}{(2\gamma_1+\gamma_2)}\sigma^0_{ucuc}+\frac{(2\gamma_1-\gamma_2)\sqrt{\gamma_1\gamma_2}(1-\cos\theta)}{\sqrt{2}(2\gamma_1+\gamma_2)}(\sigma^0_{ucc}+\sigma^0_{cuc}), \tag{11c}$$

$$\dot{\sigma}^0_{ucc}=-\left[\frac{4\gamma_1^2\gamma_2(1-\cos\theta)}{(2\gamma_1+\gamma_2)^2}+\frac{\gamma_1\gamma_2\cos\theta+\gamma_2^2}{(2\gamma_1+\gamma_2)}+\gamma_1\right]\sigma^0_{ucc}-\frac{\gamma_1\gamma_2(1-\cos\theta)(2\gamma_1-\gamma_2)}{(2\gamma_1+\gamma_2)^2}\sigma^0_{cuc}-\frac{\sqrt{\gamma_1\gamma_2}(1-\cos\theta)}{\sqrt{2}}\sigma^0_{ucuc}-[8\gamma_1^2(1-\cos\theta)+(2\gamma_1+\gamma_2)^2\cos\theta]\times\frac{\sqrt{\gamma_1\gamma_2}}{\sqrt{2}(2\gamma_1+\gamma_2)^2}\sigma^0_{cc}-\frac{\gamma_2\sqrt{\gamma_1\gamma_2}}{\sqrt{2}(2\gamma_1+\gamma_2)}\sigma^0_{++}, \tag{11d}$$

$$\dot{\sigma}^0_{+uc}=-\frac{\gamma_2}{2(2\gamma_1+\gamma_2)}[\gamma_2+\gamma_1(1+6\sqrt{2}-\cos\theta)]\sigma^0_{+uc}-\frac{\sqrt{\gamma_1\gamma_2}}{\sqrt{2}(2\gamma_1+\gamma_2)}[2\gamma_1+\gamma_2\cos\theta-2\gamma_2]\sigma^0_{+c}, \tag{11e}$$

$$\dot{\sigma}^0_{+c}=-\frac{\sqrt{2\gamma_1\gamma_2}}{(2\gamma_1+\gamma_2)}[\gamma_1(1-\cos\theta)-\gamma_2]\sigma^0_{+uc}-[2\gamma_1\gamma_2(1+\cos\theta)+4\gamma_1^2+3\gamma_2^2]\frac{\sigma^0_{+c}}{2(2\gamma_1+\gamma_2)}. \tag{11f}$$

The above equations have been derived by neglecting terms rotating at $e^{\pm 2iGt}$ (secular approximation). Note that the above equations are not the usual rate equations because the diagonal elements are coupled with the off-diagonal elements as in Eqs. (1). It is this coupling which leads to quasitrapping even though σ^0_{ucuc} decays at a rate

$$\Gamma_{uc}=\frac{4\gamma_1\gamma_2(\gamma_1+\gamma_2)(1-\cos\theta)}{(2\gamma_1+\gamma_2)^2}. \tag{12}$$

Also note that for small nonzero θ the decay from state $|uc\rangle$ is very small, which makes it a highly ''stable'' state. We solve Eqs. (11) numerically with the initial condition $\sigma^0_{33}(0)=1$. In Fig. 5 we plot the time evolution of the population in states $|+\rangle$, $|c\rangle$, and $|uc\rangle$. Note that both $|+\rangle$ and $|c\rangle$ decay very rapidly, while population gets accumulated in $|uc\rangle$. Here complete trapping will occur $[\sigma^0_{ucuc}(\infty)=1]$ when $\Gamma_{uc}=0$. This is not possible for the geometry shown in Fig. 1(b). However, we have a quasi-trapped-state for small θ.

It should be noted that trapped states were shown to occur in the presence of VIC under several conditions. For example, a trapped state arises at a certain parameter regime when $|1\rangle$ and $|2\rangle$ are degenerate and when no external fields are applied, however the system is prepared in one of the excited states [11]. Trapping is also known to occur in the degenerate case ($\delta=0$) and when pump and probe have identical strengths ($\vec{\varepsilon}_2 \equiv \vec{\varepsilon}_1$) [16]. Recently, new trapping states in the presence of VIC have been found for four-level systems [25,26]. However, note that the QTS discussed above is due to a pump field G coupling $|2\rangle \leftrightarrow |3\rangle$ transition and thus is *different* from all the previous works.

V. EFFECTS OF QUASI-TRAPPED-STATE ON THE PUMP-FIELD LINE PROFILES

We now show the effects of the above trapping on the absorption and dispersion profiles of the pump field. The trapping leads to a very steep increase in the refractive index and reduces absorption drastically for the pump field. Various models in the past have demonstrated and discussed the importance of such a medium [32–34]. However, in the present case we show how VIC can be used to control the refractive index of a medium. It is known that a large population difference between the dressed states can result in large dispersion with vanishing absorption [33]. For our system the population of dressed states depends on η and hence in principle we can get a situation where a large population difference between dressed states can exist. Consider the case when $\Delta_2 = 0$ and $W_{12} = -G$. The coherence σ_{23}^0 can be evaluated using Eqs. (3) and (7). The optical coherence to all orders in the pump field is found to be

$$\sigma_{23}^0 = \frac{G^2\eta^2\{G^2(2\gamma_1+\gamma_2)\gamma_1 + (\eta^2-\gamma_1\gamma_2)(\gamma_1+\gamma_2)^2\} + iG(\gamma_1\gamma_2-\eta^2)\{A\gamma_2 - \eta^2\gamma_1(\gamma_1+\gamma_2)^2\}}{B}, \tag{13}$$

where

$$A = G^2\gamma_2^2 + 4G^2\gamma_1\gamma_2 + \gamma_1^2\gamma_2^2 + 4G^2\gamma_1^2 + 2\gamma_1^3\gamma_2 + \gamma_1^4,$$
$$B = (\gamma_1\gamma_2 - \eta^2)\{A(\gamma_2^2 + 2G^2) + \eta^2 G^2\gamma_1(\gamma_2 + 2\gamma_1)\} + \eta^2 G^4(\gamma_2 + 2\gamma_1)^2 + \eta^2(\gamma_1+\gamma_2)^2(3\gamma_1\gamma_2\eta^2 - 2\gamma_1^2\gamma_2^2 - \eta^4). \tag{14}$$

It is known that the $\mathrm{Re}(\sigma_{23}^0)$ corresponds to the dispersion and $\mathrm{Im}(\sigma_{23}^0)$ corresponds to absorption. When the alignment parameter θ is small, we have $\eta^2 \approx \gamma_1\gamma_2$ (for example, when $\theta = 15°$, $\eta^2 = 0.93\gamma_1\gamma_2$). Then we can approximate Eq. (13) by

$$\sigma_{23}^0 \approx \frac{\gamma_1}{\gamma_2 + 2\gamma_1} + i\frac{(\gamma_1\gamma_2 - \eta^2)\{A - \gamma_1^2(\gamma_1+\gamma_2)^2\}}{G^3\gamma_1(\gamma_2+2\gamma_1)^2}, \tag{15}$$

with the constraint that $G \neq 0$. Thus for $\gamma_1 > \gamma_2$ one can have

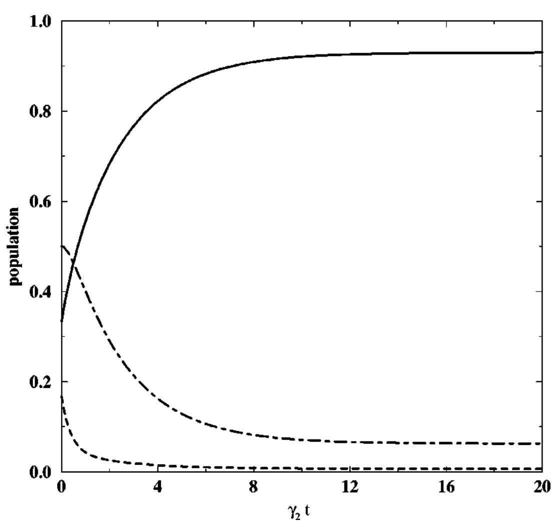

FIG. 5. Evolution of atomic population in the states $|+\rangle$, $|c\rangle$, and $|uc\rangle$ when $\Delta_2 = 0$, $W_{12} = -G$, and $\theta = 15°$. The other parameters are $G = 20\gamma_2$, $\gamma_1 = \gamma_2 = 1$, and $\theta = 15°$. The solid curve represents σ_{ucuc}^0, the dashed curve is for σ_{cc}^0, and the dot-dashed curve shows the evolution of σ_{++}^0.

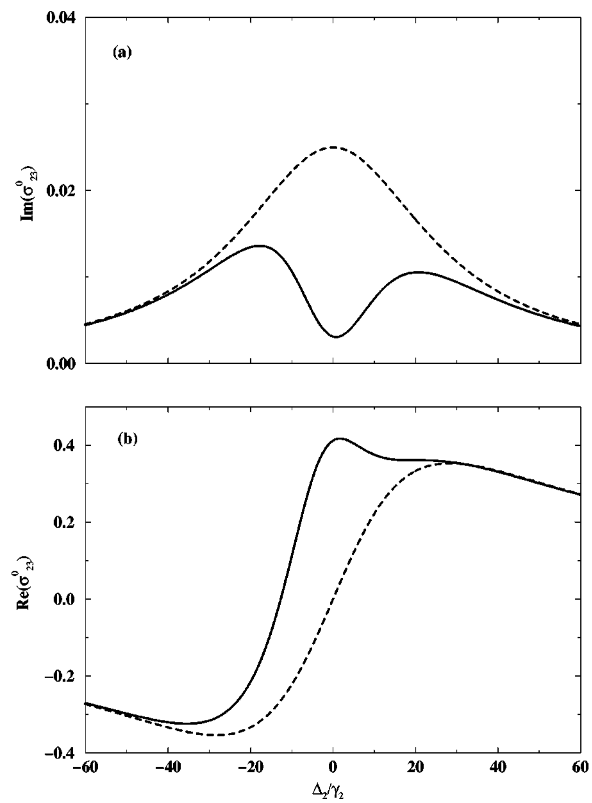

FIG. 6. Plots show the absorption and dispersion curves for the pump field in dimensionless units as a function of pump detuning Δ_2/γ_2. The solid curves show the effect of VIC, $\theta = 15°$, and the dashed curves are for $\theta = 90°$. The parameters are $G = 20\gamma_2$ and $\gamma_1 = 10\gamma_2$. For the solid curve, $W_{12} = -G$.

$\mathrm{Re}(\sigma_{23}^0)$ as high as 0.5 while the absorption remains low. It should be borne in mind that the absorption and dispersion have been computed to all orders in the pump-field strength. In Fig. 6 we plot the absorption and dispersion parts of σ_{23}^0 as a function of detuning Δ_2/γ_2. For comparison the dashed curves show the result in the absence of VIC. These curves are obtained from the steady-state numerical solutions of Eqs. (3) with $g=0$. Note that in the presence of VIC there is a dip in absorption and a peak in the dispersion curve. Due to trapping, most of the population tends to remain in states $|1\rangle$ and $|-\rangle$. For $W_{12}=-G$ and $\Delta_2=0$, the population in the three dressed states and the coherence σ_{+-}^0 was evaluated to be

$$\sigma_{11}^0=\frac{G^2\eta^2(\gamma_1+\gamma_2)^2(\eta^2-\gamma_1\gamma_2)-G^4\eta^2\gamma_2(\gamma_2+2\gamma_1)}{B}, \tag{16}$$

$$\sigma_{--}^0=[(\gamma_2^2+2G^2)A(\gamma_1\gamma_2-\eta^2)-\eta^2G^2(\gamma_1\gamma_2-\eta^2)\times(3\gamma_2^2+5\gamma_1\gamma_2+\gamma_1^2)+4G^4\eta^2\gamma_1\gamma_2+\eta^2\times(\gamma_2+\gamma_1)^2(3\eta^2\gamma_1\gamma_2-2\gamma_1^2\gamma_2^2-\eta^4)]/2B, \tag{17}$$

$$\sigma_{++}^0=\frac{(\gamma_1\gamma_2-\eta^2)\{A(\gamma_2^2+2G^2)+G^2\eta^2\gamma_1(\gamma_2+2\gamma_1)+\eta^2(\gamma_1+\gamma_2)^2(G^2-2\gamma_1\gamma_2+\eta^2)\}}{2B}. \tag{18}$$

$$\sigma_{+-}^0=\{(\eta^2-\gamma_1\gamma_2)[A\gamma_2^2-G^2(\gamma_2^2+\gamma_1\gamma_2-\gamma_1^2)\eta^2]-\eta^2(\gamma_1+\gamma_2)^2(3\gamma_1\gamma_2\eta^2-2\gamma_1^2\gamma_2^2-\eta^4)-2iG(\eta^2-\gamma_1\gamma_2)[A\gamma_2-\eta^2\gamma_1(\gamma_1+\gamma_2)^2]\}/2B, \tag{19}$$

which under the condition $\eta^2\approx\gamma_1\gamma_2$ reduce to

$$\sigma_{11}^0\approx\frac{\gamma_2}{\gamma_2+2\gamma_1},\quad \sigma_{--}^0\approx\frac{2\gamma_1}{\gamma_2+2\gamma_1}, \tag{20}$$

$$\sigma_{++}^0\approx\frac{(\gamma_1\gamma_2-\eta^2)\{A(\gamma_2^2+2G^2)+G^2\gamma_1^2\gamma_2(\gamma_2+2\gamma_1)+\gamma_1\gamma_2(\gamma_1+\gamma_2)^2(G^2-\gamma_1\gamma_2)\}}{2G^4\gamma_1\gamma_2(\gamma_2+2\gamma_1)^2}, \tag{21}$$

$$\mathrm{Im}(\sigma_{+-}^0)\approx\frac{(\gamma_1\gamma_2-\eta^2)\{A-\gamma_1^2(\gamma_1+\gamma_2)^2\}}{G^3\gamma_1(\gamma_2+2\gamma_1)^2}. \tag{22}$$

Note that σ_{++}^0 and $\mathrm{Im}(\sigma_{+-}^0)$ are very small compared to σ_{11}^0 and σ_{--}^0. One can equally write σ_{23}^0 as

$$\sigma_{23}^0=(\sigma_{--}^0-\sigma_{++}^0)/2+i\,\mathrm{Im}(\sigma_{+-}^0)\quad \text{at}\ \ \Delta_2=0. \tag{23}$$

Thus the *large difference in population between states* $|-\rangle$ *and* $|+\rangle$ *gives rise to the large dispersion*. Also note that state $|-\rangle$ lies below the ground state $|3\rangle$, which can also cause a large index of refraction with small absorption [33]. When θ=90°, we see from Eqs. (17), (18), and (19) that $\sigma_{--}^0=\sigma_{++}^0$ and $\mathrm{Im}(\sigma_{+-}^0)\neq0$, and hence dispersion at $\Delta_2=0$ is zero with substantial absorption, which is consistent with the well known power-broadened absorption and dispersion profiles for a two-level atom.

VI. ORIGIN OF GAIN THROUGH QUASI-TRAPPED-STATES

The origin of the Autler-Townes doublet in the absorption spectrum is well understood. The pump dresses the states $|2\rangle$ and $|3\rangle$. The population in the dressed states $|\pm\rangle$ absorbs a photon from the probe field leading to the Autler-Townes doublet. The situation changes drastically in the presence of VIC, which, as shown in Sec. V, for a suitable choice of parameters, leads to a quasi-trapped-state $|uc\rangle$. For $\Delta_2=0$, $W_{12}=-G$, $\gamma_1=\gamma_2$, and small values of θ the dressed state $|+\rangle$ is almost empty, whereas $\sigma_{--}^0>\sigma_{11}^0$ [Eq. (20)]. Thus the probe can be absorbed in the transition $|-\rangle\rightarrow|1\rangle$,whereas the probe will experience gain in the transition $|1\rangle\rightarrow|+\rangle$. We also note that in principle the coherence between two dressed states $|\pm\rangle$ can also contribute to the gain [35]. As discussed in Sec. V, the population in the states $|\pm\rangle$ and $|1\rangle$ depends on the angle θ between the two dipole matrix elements. For intermediate values of θ's, the population in $|1\rangle$ and $|+\rangle$ can be almost the same. This can suppress one of the Autler-Townes components as shown by the solid curve in Fig. 3. When $\gamma_2>2\gamma_1$, it is possible to have $\sigma_{11}^0>\sigma_{--}^0$ for small θ [see result (20)]. Thus both the Autler-Townes components will give rise to gain. This behavior is shown by the dot-dashed curve in Fig. 3.

VII. CONCLUSIONS

In summary, we have studied the *nondegenerate* pump-probe spectroscopy of *V* systems when the presence of interference in decay channels is significant. We have shown the possibility of gain components in the Autler-Townes doublet. We present a physical interpretation of this gain. We have also shown the possibility of new trapped states due to VIC, which we further show results in a very high refractive index with very low absorption.

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