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Decoherence in two Bose-Einstein condensates

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In this paper, decoherence in a system consisting of two Bose-Einstein condensates is investigated analytically. It is indicated that decoherence can be controlled by manipulating the interaction between the system and environment. The influence of the decoherence on quantum coherent atomic tunneling (AT) between two condensates with arbitrary initial states is studied in detail. Analytical expressions of the population difference (PD) and the AT current between two condensates are found. It is shown that the decoherence leads to the decay of the PD and the suppression of the AT current.

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I. INTRODUCTION

Recently, much attention has been paid to experimental investigations [1–7] and theoretical studies [8–15] of systems consisting of two and multiple Bose-Einstein condensates, since such systems give rise to a fascinating possibility of observing a rich set of new macroscopic quantum phenomena [16–20] which do not exist in a single condensate. Among important macroscopic quantum effects is the quantum coherent atomic tunneling (AT) between two trapped Bose-Einstein condensates [15–18]. Several authors [19,20] showed that AT can support macroscopic quantum self-trapping (MQST) due to the nonlinearity of atom-atom interactions in condensates. As is well known, no system can be completely isolated from its environment. In fact, in current experiments on trapped Bose-Einstein condensates of dilute alkali atomic gases, condensate atoms continuously interact with noncondensate atoms (environment). Interactions between a quantum system and an environment cause two types of irreversible effects: dissipation and decoherence [21,22]. Mathematically, the dissipation and decoherence can be understood in the following way. Let $\hat{H}_S$ and $\hat{H}_B$ be Hamiltonians of the system and environment (bath), respectively, and $\hat{H}_I$ be the interacting Hamiltonian between the system and environment. When $[\hat{H}_I,\hat{H}_S]\neq 0$, which implies that the energy of the system is not conservative, the interaction $\hat{H}_I$ describes the dissipation. When $[\hat{H}_I,\hat{H}_S]=0$, the energy of the system is conservative, so the interaction $\hat{H}_I$ describes the decoherence. The dissipation effect, which dissipates the energy of the quantum system into the environment, is characterized by the relaxation time scale τ_r. In contrast, the decoherence effect, which can be regarded as a mechanism for enforcing classical behaviors in the macroscopic realm, is much more insidious because the coherence information leaks out into the environment in another time scale τ_d, which is much shorter than τ_r, as the quantum system evolves with time. Since macroscopic quantum phenomena in Bose condensates depend mainly on τ_d rather than τ_r, the discussions in the present paper focus only on the decoherence problem rather than the dissipation effect. To study the decoherence in Bose-Einstein condensates is not only of theoretical interest but also of importance from a practical point of view, since decohering would always be present in any Bose-Einstein condensate experiments of trapped atoms.

On the aspect of modeling dissipation and decoherence in trapped Bose-Einstein condensates, some progress [9,23–26] has been made. In particular, Anglin [23] derived a master equation for a trapped Bose-Einstein condensate by considering a special model of a condensate confined in a deep but narrow spherical square-well potential. In his model the reservoir of non-condensate atoms consists of a continuum of unbound modes obtained by the scattering solutions of the potential well. Making use of Anglin's master equation, Ruostekoski and Walls [24] numerically simulated dissipative dynamics of a Bose-Einstein condensate in a double-well potential when the condensate is in the atomic coherent states, and showed that interactions between condensate and noncondensate atoms make the MQST decay. For a system consisting of two trapped weakly connected Bose-Einstein condensates, there is quantum coherent AT between two condensates. Questions that naturally arise are, what is the effect of decoherence on the quantum coherent AT? Does decoherence increase or decrease the AT current between them? In this paper, we analytically study the decoherence problem in two Bose-Einstein condensates, and investigate the influence of the decoherence on the quantum coherent AT between two trapped Bose-Einstein condensates in terms of an exactly solvable Hamiltonian. We will present analytic expressions of the population difference (PD) and the AT current between two Bose-Einstein condensates, and show that the decoherence leads to the PD decay and the suppression of the AT current.

This paper is organized as follows. In Sec. II, we present an approximate analytical solution of the system consisting of two Bose-Einstein condensates with a tunneling coupling without decoherence. In Sec. III, we introduce a decoherence model and apply it to the two-condensate system; we also discuss the influence of decoherence on the atomic tunneling. Concluding remarks are provided in Sec IV.

II. TWO BOSE-EINSTEIN CONDENSATES WITH TUNNELING COUPLING

Let us consider a system of two Bose-Einstein condensates with weak nonlinear interatomic interactions and

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Josephson-like coupling. Such a condensate system, in principle, can be produced in a double trap with two condensates coupled by quantum tunneling and ground collisions, or in a system with two different magnetic sublevels of an atom, in which case the two species of condensates correspond to the two electronic states involved. In the formalism of the second quantization, the Hamiltonian of such a system can be written as

$$\hat{H}=\hat{H}_1+\hat{H}_2+\hat{H}_{int}+\hat{H}_{Jos}, \tag{1}$$

$$\hat{H}_i=\int d\mathbf{x}\,\hat{\psi}_i^\dagger(\mathbf{x})\left[-\frac{\hbar^2}{2m}\nabla^2+V_i(\mathbf{x})+U_i\hat{\psi}_i^\dagger(\mathbf{x})\hat{\psi}_i(\mathbf{x})\right]\hat{\psi}_i(\mathbf{x}),\quad (i=1,2), \tag{2}$$

$$\hat{H}_{int}=U_{12}\int d\mathbf{x}\,\hat{\psi}_1^\dagger(\mathbf{x})\hat{\psi}_2^\dagger(\mathbf{x})\hat{\psi}_1(\mathbf{x})\hat{\psi}_2(\mathbf{x}), \tag{3}$$

$$\hat{H}_{Jos}=\Lambda\int d\mathbf{x}[\hat{\psi}_1^\dagger(\mathbf{x})\hat{\psi}_2(\mathbf{x})+\hat{\psi}_1(\mathbf{x})\hat{\psi}_2^\dagger(\mathbf{x})]. \tag{4}$$

Here $i=1$ and 2, and $\hat{\psi}_i(\mathbf{x})$ and $\hat{\psi}_i^\dagger(\mathbf{x})$ are the atomic field operators which annihilate and create atoms at position $\mathbf{x}$, respectively. They satisfy the commutation relation $[\hat{\psi}_i(\mathbf{x}),\hat{\psi}_j^\dagger(\mathbf{x}')]=\delta_{ij}\delta(\mathbf{x}-\mathbf{x}')$. $\hat{H}_1$ and $\hat{H}_2$ describe the evolution of each species in the absence of interspecies interactions. $\hat{H}_{int}$ describes interspecies collisions. $\hat{H}_{Jos}$ is the Josephson-like tunneling coupling term. Atoms are confined in harmonic potentials $V_i(\mathbf{x})$ ($i=1$ and 2). Interactions between atoms are described by a nolinear self-interaction term $U_i=4\pi\hbar^2a_i^{sc}/m$ and a term that corresponds to the nonlinear interaction between different species $U_{12}=4\pi\hbar^2a_{12}^{sc}/m$, where a_i^{sc} is the *s*-wave scattering length of species *i* and a_{12}^{sc} that between species 1 and 2. For simplicity, throughout this paper we set $\hbar=1$, and assume that $a_1^{sc}=a_2^{sc}=a^{sc}$ and $V_1(\mathbf{x})=V_2(\mathbf{x})$.

It has been well known that Hamiltonian (1) can reduce to a two-mode Hamiltonian [27] by the use of an approximation of the atomic field operators: $\hat{\psi}_i(\mathbf{x})=\hat{a}_i\phi_i(\mathbf{x})$, where $\hat{a}_i=\int d\mathbf{x}\,\phi_i(\mathbf{x})\hat{\psi}_i(\mathbf{x})$ are correspondent mode annihilation operators with real distribution functions $\phi_i(\mathbf{x})$ and $[\hat{a}_i,\hat{a}_i^\dagger]=1$. Then Hamiltonian (1) can be reduced to the two-mode Hamiltonian

$$\hat{H}=\omega_0(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2)+q(\hat{a}_1^{\dagger 2}\hat{a}_1^2+\hat{a}_2^{\dagger 2}\hat{a}_2^2)+g(\hat{a}_1^\dagger\hat{a}_2+\hat{a}_2^\dagger\hat{a}_1)+2\chi\hat{a}_1^\dagger\hat{a}_1\hat{a}_2^\dagger\hat{a}_2, \tag{5}$$

where q, χ, and g are coupling constants which characterize the strength of the interatomic interaction in each condensate, the interspecies interaction, and the Josephson-like coupling, respectively.

The valid conditions of the two-mode approximation were demonstrated in Ref. [19], which indicated that the two-mode approximation is valid for weak many-body interactions, i.e., for a small number of condensate atoms. As shown in Ref. [19], the two-mode approximation should be acceptable for the number of atoms $N\leqslant 2000$ if the scattering length is typically taken as $a=5$ nm for a large trap of size $L=10$ μm.

We note that the two-mode approximate Hamiltonian has the same form as that of a two-mode nonlinear optical directional coupler [28]. It cannot be solved exactly, but a closed analytical solution can be obtained under the rotating-wave approximation suggested by Alodjanc *et al.* [29]. The approximate analytic solution is valid for weak interactions between atoms, but it sheds considerable light on the AT under consideration.

In order to obtain an approximate analytic solution of Hamiltonian (5), we introduce a new pair of bosonic operators

$$\hat{A}_1=\frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2)e^{-igt},\quad \hat{A}_2=\frac{i}{\sqrt{2}}(\hat{a}_1-\hat{a}_2)e^{igt}, \tag{6}$$

which satisfy the usual bosonic commutation relation: $[\hat{A}_i,\hat{A}_j^\dagger]=\delta_{ij}$. Then Hamiltonian (5) reduces to the following form:

$$\hat{H}=\Omega\hat{N}+g(\hat{A}_1^\dagger\hat{A}_1-\hat{A}_2^\dagger\hat{A}_2)+\frac{1}{4}q[(3\hat{N}^2-2\hat{N})-(\hat{A}_1^\dagger\hat{A}_1-\hat{A}_2^\dagger\hat{A}_2)^2]+\frac{1}{2}\chi\hat{N}^2-\chi\hat{A}_1^\dagger\hat{A}_1\hat{A}_2^\dagger\hat{A}_2+\hat{H}', \tag{7}$$

where $\Omega=(\omega_0-\chi/2)$, the total number operator $\hat{N}=\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2=\hat{A}_1^\dagger\hat{A}_1+\hat{A}_2^\dagger\hat{A}_2$ is a conservative constant, and $\hat{H}'$ is a nonresonant term which oscillates at the frequency $4g$ in the sense of Alodjanc *et al.*'s proposal [29]. The account of the fast oscillating terms results only in some additional oscillations which play no essential role in the evolution of the measurable quantities specifying the macroscopic quantum phenomena of the two-condensate system, so that the nonresonant terms are fully negligible. This is the rotating-wave approximation (RWA) in the sense of Ref. [29]. After neglecting the nonresonant term H', we obtain the approximate Hamiltonian

$$\hat{H}_A=\Omega\hat{N}+g(\hat{A}_1^\dagger\hat{A}_1-\hat{A}_2^\dagger\hat{A}_2)+\frac{1}{4}q[(3\hat{N}^2-2\hat{N})-(\hat{A}_1^\dagger\hat{A}_1-\hat{A}_2^\dagger\hat{A}_2)^2]+\frac{1}{2}\chi\hat{N}^2-\chi\hat{A}_1^\dagger\hat{A}_1\hat{A}_2^\dagger\hat{A}_2. \tag{8}$$

It is worthwhile noting that the dynamics of the non-RWA Hamiltonian (5) is often chaotic. A detailed investigation of chaotic behaviors in the two-condensate system is beyond the scope of the present paper, and will be given elsewhere. Nevertheless, the RWA Hamiltonian (8) is an integrable Hamiltonian whose dynamics is regular, and which does not exhibit chaos. Hence the terms neglected in the RWA lead to chaos when they are kept in the two-condensate system. This is very analogous to the case of the

Jaynes-Cummings model (JCM) [30], which describes the interaction of a two-level atom with a single-mode electromagnetic field in quantum optics. It was well known that the RWA JCM is exactly solvable, but the non-RWA JCM [31] exhibits chaos. As shown in Ref. [31], the chaos is a consequence of inclusion of terms normally neglected in the RWA.

Obviously, the Hamiltonian $\hat{H}_A$ is diagonal in the Fock space of the $(\hat{A}_1,\hat{A}_2)$ representation defined by

$$|n,m)=\frac{1}{\sqrt{n!m!}}\hat{A}_1^{\dagger n}\hat{A}_2^{\dagger m}|0,0), \tag{9}$$

where n and m take non-negative integers. We have $\hat{H}_A|n,m)=E(n,m)|n,m)$, with the eigenvalues

$$E(n,m)=\left(\Omega-\frac{q}{2}\right)(n+m)+g(n-m)+\frac{1}{4}(3q+2\chi)(n+m)^2-\frac{q}{4}(n-m)^2-\chi nm. \tag{10}$$

For simplicity, all calculations below shall be carried out in the $(\hat{A}_1,\hat{A}_2)$ representation with the basis $\{|n,m),n,m=0,1,2,\dots\}$, which is related to the $(\hat{a}_1,\hat{a}_2)$ representation with a set of basis $\{|n,m\rangle=\hat{a}_1^{\dagger n}\hat{a}_2^{\dagger m}/\sqrt{n!m!}|0,0\rangle,n,m=0,1,2,\dots\}$ through the following relation:

$$|n,m\rangle=\sum_{r=0}^{n}\sum_{s=0}^{m}\frac{[n!m!(n-r+s)!(m-s+r)!]^{1/2}}{2^{(n+m)/2}(n-r)!(m-s)!}\times e^{-i(2r-3m)\pi/2}|n-r+s,m-s+r). \tag{11}$$

III. INFLUENCE OF DECOHERENCE ON ATOMIC TUNNELING

We now consider the effect of the decoherence. We use a reservoir consisting of an infinite set of harmonic oscillators to model the environment of condensate atoms in a trap, and we assume the total Hamiltonian to be

$$\hat{H}_T=\hat{H}_A+\sum_k\ \omega_k\hat{b}_k^\dagger\hat{b}_k+F(\{\hat{S}\})\sum_k\ c_k(\hat{b}_k^\dagger+\hat{b}_k)+F(\{\hat{S}\})^2\sum_k\ \frac{c_k^2}{\omega_k^2}, \tag{12}$$

where the second term is the Hamiltonian of the reservoir. The last term in Eq. (12) is a renormalization term [32]. The third term in Eq. (12) represents the interaction between the system and the reservior with a coupling constant c_k, where $\{\hat{S}\}$ is a set of linear operators of the system or their linear combinations in the same picture as that of $\hat{H}_A$, and $F(\{\hat{S}\})$ is an operator function of $\{\hat{S}\}$. In order to show that what the interaction between the system and environment describes is decoherence and not dissipation, we require that the linear operator $\hat{S}$ commutes with the Hamiltonian of the system $\hat{H}_A$. Then the interaction term in Eq. (12) commutes with the Hamiltonian of the system. This implies that there is no energy transfer between the system and its environment. Thus it does describe the decoherence. The concrete form of the function $F(\{\hat{S}\})$, which may be considered as an experimentally determined quantity, may be different for different environments. Therefore, the decohering interaction in Eq. (12) cannot only describe decoherence caused by the effect of elastic collisions between condensate and noncondensate atoms for a Bose-Einstein condensate system, but also simulate decoherence caused by other decoherencing sources by properly choosing the operator function of the system $F(\{\hat{S}\})$.

Hamiltonian (12) can be exactly solved by making use of the unitary transformation

$$\hat{U}=\exp\left[\hat{H}_A\sum_k\ \frac{c_k}{\omega_k}(\hat{b}_k^\dagger-\hat{b}_k)\right]. \tag{13}$$

Corresponding to Hamiltonian (12), the total density operator of the system plus reservoir can be expressed as

$$\hat{\rho}_T(t)=e^{-i\hat{H}_At}\hat{U}^{-1}e^{-it\Sigma_k\omega_k\hat{b}_k^\dagger\hat{b}_k}\hat{U}\hat{\rho}_T(0)\hat{U}^{-1}\times e^{it\Sigma_k\omega_k\hat{b}_k^\dagger\hat{b}_k}\hat{U}e^{i\hat{H}_At}. \tag{14}$$

In the derivation of the above solution, we have used $\hat{\rho}_T(t)=\hat{U}^{-1}\hat{\rho}_T'(t)\hat{U}$, where $\hat{\rho}_T'=e^{-i\hat{H}_T't}\hat{\rho}_T'(0)e^{i\hat{H}_T't}$ with $\hat{H}_T'=\hat{U}\hat{H}_T\hat{U}^{-1}$, and $\hat{\rho}_T'(0)=\hat{U}\hat{\rho}_T(0)\hat{U}^{-1}$, where $\hat{\rho}_T(0)$ is the initial total density operator.

We assume that the system and reservoir are initially in thermal equilibrium and uncorrelated, so that $\hat{\rho}_T(0)=\hat{\rho}(0)\otimes\hat{\rho}_R$, where $\hat{\rho}(0)$ is the initial density operator of the system, and $\hat{\rho}_R$ the density operator of the reservoir, which can be written as $\hat{\rho}_R=\Pi_k\hat{\rho}_k(0)$, where $\hat{\rho}_k(0)$ is the density operator of the kth harmonic oscillator in thermal equilibrium. After taking a trace over the reservoir, from Eq. (14) we can obtain the reduced density operator of the system, denoted by $\hat{\rho}(t)=tr_R\hat{\rho}_T(t)$; its matrix elements in the $(\hat{A}_1,\hat{A}_2)$ representation are explicitly written as

$$\rho_{(m',n')(m,n)}(t)=\rho_{(m',n')(m,n)}(0)R_{(m',n')(m,n)}(t)\times e^{-i[F(\{S(m',n')\})-F(\{S(m,n)\})]t}, \tag{15}$$

where $F(\{S(m,n)\})$ is an eigenvalue of the operator function $F(\{\hat{S}\})$ in an eigenstate of $\hat{H}_A$. $R_{(m',n')(m,n)}(t)$ is a reservoir-dependent quantity given by

$$R_{(m',n')(m,n)}(t)=e^{-i[F^2(\{S(m',n')\})-F^2(\{S(m,n)\})]Q_1(t)}\times e^{-[F(\{S(m',n')\})-F(\{S(m,n)\})]^2Q_2(t)}, \tag{16}$$

where the two reservoir-dependent functions are given by

$$Q_1(t)=\int_0^\infty d\omega\, J(\omega)\frac{c^2(\omega)}{\omega^2}\sin(\omega t), \tag{17}$$

$$Q_2(t)=2\int_0^\infty d\omega\, J(\omega)\frac{c^2(\omega)}{\omega^2}\sin^2\left(\frac{\omega t}{2}\right)\coth\left(\frac{\beta\omega}{2}\right). \tag{18}$$

Here we have taken the continuum limit of the reservoir modes, $\Sigma_k\rightarrow\int_0^\infty d\omega\, J(\omega)$, where $J(\omega)$ is the spectral density of the reservoir, $c(\omega)$ is the corresponding continuum expression for c_k, and $\beta=1/k_BT$, with k_B and T being the Boltzmann constant and the temperature, respectively.

It is well known that the decoherence corresponds to the decay of off-diagonal elements of the reduced density matrix of a quantum system. For the case under consideration, the degree of decoherence is determined by the decaying factor in Eq. (16). It is interesting to note that if we choose a proper operator function $F(\{\hat{S}\})$ to make $F(\{S(m',n')\})=F(\{S(m,n)\})$ for $(m',n')\neq(m,n)$, then we find that

$$\rho_{(m',n')(m,n)}(t)=\rho_{(m',n')(m,n)}(0), \tag{19}$$

which indicates that the quantum system maintains its initial quantum coherence, that is, the time evolution of decoherence-free of the quantum system is realized. Therefore, we conclude that one can control decoherence by manipulating the interaction function $F(\{\hat{S}\})$.

Equations (15) and (16) indicate that the interaction between the system and its environment induces a phase shift and a decaying factor in the reduced density operator of the system. We now consider the PD between the two condensates in the presence of the decoherence, defined by $p(t)\equiv N_1(t)-N_2(t)$ with $N_i=\langle\hat{a}_i^\dagger\hat{a}_i\rangle$. We find that

$$p(t)=-2\sum_r\sum_s\sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)|\times\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]e^{-(v_{rs}^-)^2Q_2(t)}, \tag{20}$$

where we have introduced the symbols

$$\rho_{(r+1,s-1)(r,s)}(0)=|\rho_{(r+1,s-1)(r,s)}(0)|e^{i\theta_{rs}}, \tag{21}$$

$$v_{rs}^{\pm}=F(\{S(r+1,s-1)\})\pm F(\{S(r,s)\}). \tag{22}$$

From Eq. (20) we see that if we do not take the influence of the decoherence into account, i.e., set $Q_1(t)=Q_2(t)=0$, then we obtain an expression of the PD between two condensates:

$$p(t)=-2\sum_r\sum_s\sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)|\times\sin(\theta_{rs}-v_{rs}^-t), \tag{23}$$

which implies that the time evolution of the PD is periodic. In particular, if we take $F(\{\hat{S}\}=\hat{H}_A$, we find that when $2g/(q-\chi)=K$ (being an integer), we have a nonzero time-average value of the PD:

$$\overline{p}=-2\sum_r\sqrt{(r-K+1)(r+1)}|\rho_{(r+1,r-K)(r,r-K+1)}(0)|\times\sin(\theta_{rr-K+1}), \tag{24}$$

which means that the two-condensate system under consideration exhibits the MQST [18,19] when decoherence is absent.

The coherent AT current between the two condensates, defined by $I(t)\equiv\dot{N}_1(t)-\dot{N}_2(t)$, is given by

$$\begin{aligned}I(t)=2\sum_r\sum_s\sqrt{s(r+1)}v_{rs}^-|\rho_{(r+1,s-1)(r,s)}(0)|\\ \times\{(1-v_{rs}^+\dot{Q}_1(t))\cos[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]\\ +v_{rs}^-\dot{Q}_2(t)\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]\}e^{-(v_{rs}^-)^2Q_2(t)}.\end{aligned} \tag{25}$$

From Eqs. (18), (20), and (25), we can immediately draw one important qualitative conclusion: since $Q_2(t)$ is positive definite, the existence of the decoherence is always to tend to suppress the PD and the AT current between the two condensates. This answers the question ''Does the decoherence increase or decrease the AT?.''

From Eqs. (17), (18), (20), and (25), we see that all necessary information about the effects of the environment on the PD and the AT current is contained in the spectral density of the reservoir. To proceed further let us now specialize to the Ohmic case [33] with the spectral distribution $J(\omega)=[\eta\omega/c^2(\omega)]e^{-\omega/\omega_c}$, where ω_c is the high-frequency cutoff, and η is a positive characteristic parameter of the reservoir. With this choice, at low temperature the functions $Q_1(t)$ and $Q_2(t)$ are given by the following expressions:

$$Q_1(t)=\eta\tan^{-1}(\omega_ct), \tag{26}$$

$$Q_2(t)=\eta\left\{\frac{1}{2}\ln[1+(\omega_ct)^2]+\ln\left[\frac{\beta}{\pi t}\sinh\left(\frac{\pi t}{\beta}\right)\right]\right\}. \tag{27}$$

Recent experiments [1,7] on two condensates have established a typical time scale at which the two condensates preserve coherence. The value of the typical time scale is $t\doteq100$ ms. In the meaningful domain of time $\omega_ct\gg1$ which requires $\omega_c\gg10$ Hz, which can be easily satisfied for a typical reservoir [34,32,33], at zero temperature we have $\dot{Q}_1(t)\doteq\eta/(\omega_ct^2)$, and $Q_2(t)\doteq\eta\ln(\omega_ct)$; then we find

$$p(t)=-2\sum_r\sum_s\sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)|\times\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))](\omega_ct)^{-\eta(v_{rs}^-)^2}, \tag{28}$$

$$I(t)=2\sum_r \sum_s \sqrt{s(r+1)}v_{rs}^-|\rho_{(r+1,s-1)(r,s)}(0)| \times\left\{\left(1-\frac{\eta v_{rs}^+}{\omega_c}t^{-2}\right)\cos[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))] + \eta v_{rs}^- t^{-1}\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]\right\}(\omega_c t)^{-\eta(v_{rs}^-)^2}, \tag{29}$$

which indicate that the PD and the AT current decay according to the ''power law,'' where we have noted that the decaying factors cannot be taken outside the summation on the right-hand sides of Eqs. (28) and (29) [34].

At finite temperature, we have $\dot{Q}_1(t)\doteq\eta/(\omega_c t^2)$, and $Q_2(t)\doteq\eta[\ln(\beta\omega_c/2\pi)+(\pi t/\beta)]$, so that

$$p(t)=-2\sum_r \sum_s \sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)| \times\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]\left(\frac{\beta\omega_c}{2\pi}\right)^{-\eta(v_{rs}^-)^2} \times\exp\left[-\frac{\eta(v_{rs}^-)^2\pi}{\beta}t\right], \tag{30}$$

$$I(t)=2\sum_r \sum_s \sqrt{s(r+1)}v_{rs}^-|\rho_{(r+1,s-1)(r,s)}(0)| \times\left\{\left(1-\frac{\eta v_{rs}^+}{\omega_c}t^{-2}\right)\cos[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))] + \frac{\eta\pi v_{rs}^-}{\beta}\sin[\theta_{rs}-v_{rs}^-(t-v_{rs}^+Q_1(t))]\right\} \times\left(\frac{\beta\omega_c}{2\pi}\right)^{-\eta(v_{rs}^-)^2}\exp\left[-\frac{\eta(v_{rs}^-)^2\pi}{\beta}t\right], \tag{31}$$

which indicate that at finite temperature the PD decays according to the ''exponential law,'' and the decay of the AT current becomes more complicated than that of the PD due to the factor $(\eta v_{rs}^+/\omega_c)t^{-2}$ before the cosine function in Eq. (31).

IV. CONCLUDING REMARKS

We have presented a decoherence model which is exactly solvable, and applied it to study decoherence in two Bose-Einstein condensates. We have indicated that one can control decoherence by manipulating the interaction between a quantum system and environment. We have investigated the influence of decoherence on quantum coherent AT between two trapped Bose-Einstein condensates with arbitrary initial states, and shown that the decoherence suppresses the PD and the AT current between two condensates. We have obtained analytical expressions for the PD and the AT current, and found that for the reservoir-spectral density of the Ohmic case the PD and the AT current decay due to the power law at zero temperature; at finite temperature, the PD decays due to the exponential law, while the decay of the AT current contains both exponential-law and power-law components. It is worthwhile to note that our results are obtained for arbitrary initial states of the two condensates, and our entire analysis is carried out without invoking the assumption of Bose-Einstein-broken symmetry which has recently been shown to be unnecessary for a Bose-Einstein condensate of trapped atoms [35]. Also, it should be pointed out that these results are obtained under the RWA in the sense of Alodjanic *et al.*'s proposal, and they are valid for interatomic weak nonlinear interactions in two condensates. The RWA essentially changes the two-condensate system into an integrable system; hence it suppresses the chaotic behaviors of the two-condensate system.

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[1] D.S. Hall, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **81**, 1543 (1998); M.R. Matthews, B.P. Anderson, P.C. Haljan, D.S. Hall, C.E. Wieman, and E.A. Cornell, preprint, cond-mat/9908209.

[2] H.-J. Miesner, D.M. Stamper-Kurn, J. Stenger, S. Inouye, A.P. Chikkatur, and W. Kettler, Phys. Rev. Lett. **82**, 2228 (1998); D.M. Stamper-Kurn, H.-J. Miesner, A.P. Chikkatur, S. Inouye, J. Stenger, and W. Ketteler, *ibid.* **83**, 661 (1999).

[3] L. Deng, E.W. Hagley, J. Wen, M. Trippenbach, Y. Band, P.S. Julienne, J.E. Simsarian, K. Helmerson, S.L. Rolston, and W.D. Phillips, Nature (London) **398**, 218 (1999).

[4] B.P. Anderson and M.A. Kasevich, Science **282**, 21686 (1998).

[5] D.M. Stamper-Kurn, M.R. Andrews, A.P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Kettler, Phys. Rev. Lett. **80**, 2027 (1998); M.-O. Mewes, M.R. Andrews, D.M. Kurn, D.S. Durfee, C.G. Townsend, and W. Ketteler, *ibid.* **78**, 582 (1997); M.R. Andrews, C.G. Townsend, H.-J. Miesner, D.S. Duefee, D.M. Kurn, and W. Ketterle, Science **275**, 637 (1997).

[6] C.J. Myatt, E.A. Burt, R.W. Ghrist, E.A. Cornell, and C.E. Wieman, Phys. Rev. Lett. **78**, 586 (1997).

[7] D.S. Hall, M.R. Matthews, J.R. Ensher, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **81**, 1539 (1998).

[8] A. Sinatra, P.O. Fedichev, Y. Castin, J. Dalibard, and G.V. Shlyapnikov, Phys. Rev. Lett. **82**, 251 (1999).

[9] E. Timmermans, Phys. Rev. Lett. **81**, 5718 (1998).

[10] C.K. Law, H. Pu, N.P. Bigelow, and J.H. Eberly, Phys. Rev. A

58, 531 (1998); C.K. Law, H. Pu, and N.P. Bigelow, Phys. Rev. Lett. **81**, 5257 (1998); E.V. Goldstein and P. Meystre, Phys. Rev. A **59**, 1509 (1999); M. Trippenbach, Y.B. Band, and P.S. Julienne, Opt. Express **3**, 530 (1998).

[11] J. Ruostekoski and D.F. Walls, Phys. Rev. A **59**, R2571 (1999).

[12] P. Villain and M. Lewenstein, Phys. Rev. A **59**, 2250 (1999).

[13] J.A. Dunningham and K. Burnett, Phys. Rev. Lett. **82**, 3729 (1999).

[14] P. Horak and S.M. Barnett, preprint, quant-ph/9903024.

[15] J. Williams, R. Walser, J. Cooper, E. Cornell, and M. Holland, Phys. Rev. A **59**, R31 (1999).

[16] R.J. Ballagh, K. Burnett, and T.F. Scott, Phys. Rev. Lett. **78**, 1607 (1997); J. Javanainen and S.M. Yoo, *ibid.* **76**, 161 (1996); J. Javanainen and M. Wilkens, *ibid.* **78**, 4675 (1997).

[17] J. Javanainen, Phys. Rev. Lett. **57**, 3164 (1986); S. Grassmann and M. Holthaus, Z. Naturforsch. A: Phys. Sci. **50**, 323 (1995); I. Zapata, F. Sols, and A.J. Leggett, Phys. Rev. A **57**, R28 (1998).

[18] A. Smerzi, S. Giovanazzi, and S.R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997); Le-Man Kuang and Zhao-Yang Zeng, Chin. Phys. Lett. **15**, 703 (1998).

[19] G.J. Milburn and J. Corney, Phys. Rev. A **55**, 4318 (1997); M.J. Steel and D.F. Walls, *ibid.* **57**, 3805 (1998); E.M. Wright, T. Wong, M.J. Collett, S.M. Tan, and D.F. Walls, *ibid.* **56**, 591 (1997).

[20] C.K. Law, H. Pu, N. P. Bigelow, and J. H. Eberly, Phys. Rev. Lett. **79**, 3105 (1997); Th. Busch, J.I. Cirac, V.M. Perez-Garcia, and P. Zoller, Phys. Rev. A **56**, 2978 (1997); H. Pu and N.P. Bigelow, Phys. Rev. Lett. **80**, 1130 (1998); **80**, 1134 (1998); R. Graham and D. Walls, Phys. Rev. A **57**, 484 (1998); B.D. Esry and C.H. Greene, *ibid.* **57**, 1265 (1998); B.D. Esry, C.H. Greene, J.P. Burke Jr., and J.L. Bohn, Phys. Rev. Lett. **78**, 3594 (1997).

[21] W.H. Zurek, Phys. Today **44**(10), 36 (1991); I.L. Chuang, R. Laflamme, P.W. Shor, and W.H. Zurek, Science **270**, 1633 (1991).

[22] C.P. Sun, H. Zhan, and X.F. Liu, Phys. Rev. A **58**, 1810 (1998); J. Shao, Mo-Lin Ge, and Hu Cheng, Phys. Rev. E **53**, 1243 (1996); Le-Man Kuang, Xin Chen, Guang-Hong Chen, and Mo-Lin Ge, Phys. Rev. A **56**, 3139 (1997).

[23] J. Anglin, Phys. Rev. Lett. **79**, 6 (1997).

[24] J. Ruostekoski and D.F. Walls, Phys. Rev. A **58**, R50 (1998); **56**, 2996 (1997).

[25] C.W. Gardiner and P. Zoller, Phys. Rev. A **58**, 536 (1998).

[26] R. Graham, Phys. Rev. Lett. **81**, 5262 (1998).

[27] J.I. Cirac, M. Lewenstein, K. Molmer, and P. Zoller, Phys. Rev. A **57**, 1208 (1998); M. Steel and M.J. Collett, *ibid.* **57**, 2920 (1998).

[28] N. Korolkova and J. Perina, Opt. Commun. **136**, 135 (1996).

[29] A.P. Alodjanc, S.M. Arakeljan, and A.S. Chirkin, Zh. Éksp. Teor. Fiz. **108**, 63 (1995) [JETP **81**, 34 (1995)].

[30] E.T. Jaynes and F.W. Cummings, Proc. IEEE **51**, 89 (1963); also see a topical review by B.W. Shore and P.L. Knight, J. Mod. Opt. **40**, 137 (1989).

[31] P.W. Milonni, J.R. Ackerhalt, and H.W. Galbraith, Phys. Rev. Lett. **50**, 966 (1983); **51**, 1108(E) (1983); R. Graham and M. Höhnerbach, Phys. Lett. A **101**, 61 (1984); R.F. Fox and J. Eidson, Phys. Rev. A **34**, 482 (1986).

[32] A.O. Caldeira and A.J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).

[33] S. Chakravarty and A.J. Leggett, Phys. Rev. Lett. **52**, 5 (1984); A.J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).

[34] C. Aslangul, N. Pottier, and D. Saint-James, J. Phys. (Paris) **46**, 2031 (1985).

[35] J. Javanainen and S.M. Yoo, Phys. Rev. Lett. **76**, 161 (1996); M. Naraschewski, H. Wallis, A. Schenzle, J.I. Cirac, and P. Zoller, Phys. Rev. A **54**, 2185 (1996).