

Excitation-assisted inelastic processes in trapped Bose-Einstein condensates

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We find that inelastic collisional processes in Bose-Einstein condensates induce local variations of the mean-field interparticle interaction, and are accompanied by the creation and annihilation of elementary excitations. The physical picture is demonstrated for the case of three-body recombination in a trapped condensate. For a high trap barrier the production of high-energy trapped single-particle excitations results in a strong increase of the loss rate of atoms from the condensate.

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Since the discovery of Bose-Einstein condensation in trapped ultracold atomic gases [1], inelastic collisional processes in these systems have attracted a great deal of attention. Three-body recombination and two-body spin relaxation limit achievable densities of trapped clouds [2–7], and spin relaxation in atomic Cs even places severe limitations on achieving the regime of quantum degeneracy [7]. Theoretical studies of three-body recombination and two-body spin relaxation in ultracold gases provide valuable information on the mechanisms and rates of these processes (see Ref. [8] for a review, and Refs. [9,10] for the earlier work in hydrogen). These studies rely on a traditional approach of collisional physics in gases: The decay rates are found on the basis of the calculated probability of inelastic transition for a two- or three-body collision in a vacuum.

In this paper we find that inelastic collisional processes in Bose-condensed gases induce local variations of the mean-field interparticle interaction and can become “excitation assisted.” We demonstrate this phenomenon for the case of three-body recombination. The particles produced in the course of recombination have a high kinetic energy and immediately fly away from the point of recombination. Therefore, each recombination event leads to an instantaneous local change of the mean field, i.e., to a change of the field acting on the surrounding particles. In this respect the recombination leads to a “shaking” of the system: The time scale on which the Hamiltonian changes is so short that the wave function of the system remains unchanged and, hence, corresponds to a superposition of many eigenstates of a new Hamiltonian. For this reason the recombination event can be accompanied by the creation or annihilation of elementary excitations.

From a general point of view, the concept of “shaking” (sudden perturbations) was formulated by Migdal and Krainov and used for calculating the ionization of an atom in β decay (see, e.g., Ref. [11]). Sudden perturbations in condensed systems are accompanied by many-body collective effects. Dilute Bose-Einstein condensates are unique examples of gases, as the mean-field interparticle interaction

makes their behavior in many aspects similar to that of a solid. To some extent, the picture of inelastic processes in these gases resembles, for example, the absorption of light by impurity particles in solids, where the transfer of impurities to an excited state leads to a sudden local change of the polarization of the medium and, hence, to the creation and annihilation of phonons (see, e.g., Ref. [12]).

A distinct feature of “shaking” in a gaseous condensate concerns the back action of the shaking-created excitations on the condensate. This effect can be strongly pronounced in a trapped condensate. For a high trap barrier, created high-energy single-particle excitations are still trapped, and collide with condensate particles, removing them from the condensate. Every collision thus produces two energetic (trapped) atoms which again collide with the condensate, etc. As a result, one has a cascade production of noncondensed atoms out of the condensate. Despite a small probability of creating high-energy excitations in the recombination process, this mechanism can significantly increase the total loss rate of Bose-condensed atoms.

We will consider the most important channel of three-body recombination in a trapped Bose-condensed gas, that is, the recombination involving three condensate particles. For a single recombination event, the local change of the interparticle interaction, δH_i , can be obtained by considering the center of mass \mathbf{r}_i of the recombination-produced fast atom and molecule as a force center. Just before the event there are three atoms at the point \mathbf{r}_i , and each particle j of the sample interacts with this point via the potential $3U(\mathbf{r}_j - \mathbf{r}_i)$. After the event this interaction is equal to zero, since the fast atom and molecule fly away from the area of recombination. We will use a point approximation for the interaction potential: $U(\mathbf{r}) = \tilde{U} \delta(\mathbf{r})$, where $\tilde{U} = 4\pi\hbar^2 a/m$, with a being the s -wave scattering length and m the atom mass. Then, assuming that the condensate density n_0 in the Bose-Einstein condensates spatial region greatly exceeds the density of noncondensed atoms, we obtain

$$\begin{aligned} \delta \hat{H}_i &= -3 \int d^3 r_j \hat{\Psi}^\dagger(\mathbf{r}_j) U(\mathbf{r}_j - \mathbf{r}_i) \hat{\Psi}(\mathbf{r}_j) \\ &= -3 \tilde{U} \hat{\Psi}^\dagger(\mathbf{r}_i) \hat{\Psi}(\mathbf{r}_i). \end{aligned} \quad (1)$$

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The field operator of atoms, $\hat{\Psi} = \Psi_0 + \hat{\Psi}'$, where $\Psi_0 = n_0^{1/2}$ is the condensate wave function, and $\hat{\Psi}'$ is a noncondensed part of the operator. This part refers to quasiparticle excitations, and we linearize $\delta\hat{H}_i$ with respect to $\hat{\Psi}'$. Omitting the term $-3n_0\tilde{U}$, which is decoupled from the quasiparticle excitations, we obtain

$$\delta\hat{H}_i = -3\tilde{U}\Psi_0(\hat{\Psi}'^\dagger(\mathbf{r}_i) + \hat{\Psi}'(\mathbf{r}_i)). \quad (2)$$

The force center represents a sort of a ‘‘hole’’ in the coordinate space. It has a mass $3m$ and can undergo a translational motion. Therefore, the recombination-induced (sudden) change of the Hamiltonian \hat{H}_0 of the excitations can be written as

$$\delta\hat{H} = \int d\mathbf{r}_i \hat{\Phi}^\dagger(\mathbf{r}_i) \{ \delta\hat{H}_i - (\hbar^2/6m)\Delta \} \hat{\Phi}(\mathbf{r}_i), \quad (3)$$

where $\hat{\Phi}(\mathbf{r})$ is the field operator of the force center. The first term on the right-hand side of Eq. (3) is related to the interparticle interaction, and the second term to the motion of the force center.

We assume that the kinetic energy of fast particles produced in the recombination process greatly exceeds any other energy scale in the problem, and hence the creation or annihilation of excitations does not influence the energy conservation law for the recombination. Then, according to the general theory of sudden perturbations [11], in each recombination event the probability of transition of the excitation subsystem to a new state f , characterized by a different set of quantum numbers for the excitations, is given by $w_{if} = |\langle i|f\rangle|^2$. The symbol $\langle i|f\rangle$ stands for the overlap integral between the wave function of the initial state i , which is an eigenstate of the Hamiltonian \hat{H}_0 , and the wave function of the state f which is an eigenstate of the new Hamiltonian $\hat{H}_0 + \delta\hat{H}$. As $\sum_f w_{if} = 1$, the creation and annihilation of excitations does not change the total recombination rate.

In Thomas-Fermi condensates, most important is the creation of excitations with energies of the order of the chemical potential μ or larger (see below). These excitations are essentially quasiclassical, and their de Broglie wavelength is much smaller than the spatial size of the condensate. Hence the probability of recombination accompanied by the creation and annihilation of the excitations can be found in the local-density approximation. In other words, as well as the recombination without production of excitations, this process occurs locally at a given point \mathbf{r} characterized by local values of the chemical potential μ and condensate density n_0 . Hence one can use the Bogolyubov transformation for the spatially homogeneous case, and represent $\hat{\Psi}^\dagger$ and $\hat{\Psi}$ in terms of the creation and annihilation operators $\hat{b}_\mathbf{k}^\dagger$ and $\hat{b}_\mathbf{k}$ of excitations characterized by momentum \mathbf{k} :

$$\hat{\Psi}'^\dagger(\mathbf{r}_i) + \hat{\Psi}'(\mathbf{r}_i) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \left(\frac{E_{\mathbf{k}}}{\epsilon_{\mathbf{k}}} \right)^{1/2} (\hat{b}_\mathbf{k}^\dagger + \hat{b}_{-\mathbf{k}}) \exp(-i\mathbf{k}\mathbf{r}_i). \quad (4)$$

Here $E_{\mathbf{k}} = \hbar^2 k^2/2m$ is the energy of a free particle, $\epsilon_{\mathbf{k}} = (E_{\mathbf{k}}^2 + 2n_0\tilde{U}E_{\mathbf{k}})^{1/2}$ is the Bogolyubov energy of an excitation, and V is the normalization volume. The field operator of the force center can be represented in the form $\hat{\Phi}(\mathbf{r}) = (1/\sqrt{V}) \sum_{\mathbf{q}} \hat{a}_{\mathbf{q}} \exp(i\mathbf{q}\mathbf{r})$, where $\hat{a}_{\mathbf{q}}$ is the creation operator for the center. Then, using Eqs. (2) and (4), Eq. (3) is transformed to

$$\delta\hat{H} = -\frac{1}{\sqrt{V}} \sum_{\mathbf{k}, \mathbf{q}} h_{\mathbf{k}} \hat{b}_\mathbf{k}^\dagger + \hat{b}_{-\mathbf{k}} \hat{a}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{a}_{\mathbf{q}} + \sum_{\mathbf{q}} \frac{\hbar^2 q^2}{6m} \hat{a}_{\mathbf{q}}^\dagger \hat{a}_{\mathbf{q}}, \quad (5)$$

where

$$h_{\mathbf{k}} = 3\tilde{U}n_0^{1/2}(E_{\mathbf{k}}/\epsilon_{\mathbf{k}})^{1/2}. \quad (6)$$

The first term on the right-hand side of Eq. (5), originating from the interparticle interaction, couples the motion of the force center with the excitation subsystem and is responsible for creating and annihilating excitations in the recombination process. Considering this term as a small perturbation, we see that a single recombination event can be accompanied by the creation and annihilation of one excitation. Initially the momentum of the force center $\mathbf{q} = 0$, and after the creation of the excitation with momentum \mathbf{k} (annihilation of the excitation with momentum $-\mathbf{k}$) the center acquires the momentum $-\mathbf{k}$ and the kinetic energy $E_{\mathbf{k}}/3$. In a single recombination event, the probabilities of creating and annihilating the excitation with momentum \mathbf{k} are given by

$$w(N_{\mathbf{k}} \rightarrow N_{\mathbf{k}} + 1) = \frac{1}{V} \frac{|h_{\mathbf{k}}|^2 (1 + N_{\mathbf{k}})}{(\epsilon_{\mathbf{k}} + E_{\mathbf{k}}/3)^2}$$

$$w(N_{\mathbf{k}} \rightarrow N_{\mathbf{k}} - 1) = \frac{1}{V} \frac{|h_{\mathbf{k}}|^2 N_{\mathbf{k}}}{(\epsilon_{\mathbf{k}} - E_{\mathbf{k}}/3)^2},$$

where $N_{\mathbf{k}} = [\exp(\epsilon_{\mathbf{k}}/T) - 1]^{-1}$ are the equilibrium occupation numbers for the excitations at a given temperature T . Then, for the rate constant of recombination accompanied by the creation of excitations, we obtain

$$\alpha_{\text{ex}} = \alpha \int \frac{d^3k}{(2\pi)^3} |h_{\mathbf{k}}|^2 \left\{ \frac{1 + N_{\mathbf{k}}}{(\epsilon_{\mathbf{k}} + E_{\mathbf{k}}/3)^2} - \frac{N_{\mathbf{k}}}{(\epsilon_{\mathbf{k}} - E_{\mathbf{k}}/3)^2} \right\}, \quad (7)$$

with α being the total (event) rate constant of recombination. The first term on the right-hand side of Eq. (7) corresponds to spontaneous and stimulated creation of excitations, and the second term to their annihilation.

For $T \ll \mu$ the annihilation and stimulated emission of excitations can be omitted. One can put $N_{\mathbf{k}} = 0$, and Eq. (7) yields

$$\alpha_{\text{ex}} \approx 26\alpha(n_0 a^3)^{1/2}. \quad (8)$$

Even with a small value for the parameter $(n_0 a^3)$ the large numerical factor in front of expression (8) may imply that

the creation of excitations in the course of three-body recombination cannot be neglected [13]. At the highest densities $n_0 \approx 3 \times 10^{15} \text{ cm}^{-3}$ of the MIT sodium experiment [3] [Eq. (8)] gives $\alpha_{\text{ex}}/\alpha \approx 0.2$.

With increasing temperature, the role of annihilation of the excitations increases. However, our calculations from Eq. (7) show that even at $T \sim \mu$ the annihilation and stimulated emission of excitations give a small correction to Eq. (8). Only at $T > 10\mu$ does the annihilation dominates over the emission, and α_{ex} [Eq. (7)] becomes negative.

The most dramatic is the influence of created excitations on the loss rate of Bose-Einstein-condensed atoms, which we will discuss for temperatures $T \lesssim \mu$ [14]. For a high trap barrier single-particle excitations with energies $\epsilon_k \gg \mu$ are still trapped and collide with the condensate. In a spherical trap the fast atom penetrates the condensate once per half of the oscillation period π/ω [15]. A characteristic time which a fast atom with velocity v_k spends inside the condensate is $\sim R/v_k$, where $R = (2\mu/m\omega^2)^{1/2}$ is the Thomas-Fermi radius of the condensate. Hence the rate of elastic collisions of the fast atom with condensate atoms is $\sim n_0 \sigma v_k (\omega R/v_k) \sim n_0 \sigma c_s$, with $\sigma = 8\pi a^2$ being the elastic cross section, and $c_s = (\mu/m)^{1/2}$ the sound velocity. In each elastic collision the fast atom transfers on average a half of its energy to the collisional partner, and removes it from the condensate. One then has to deal with two energetic atoms, and so on. The time dependence of the energy of the fast atoms is governed by the equation $\dot{\epsilon} = -(\epsilon/2)n_0 \sigma c_s$ and, hence, is given by $\epsilon(t) = \epsilon_k \exp(-n_0 \sigma c_s t/2)$. This cascade process continues until the excitation energy becomes of the order of the chemical potential μ . Accordingly, the number of lost condensate atoms will be $\sim \epsilon_k/\mu$, and the characteristic time of the cascade process, $\tau \approx 2(n_0 \sigma c_s)^{-1} \ln(\epsilon_k/\mu)$. At realistic densities the time τ is much smaller than the characteristic recombination time $\tau_r \sim (an_0^2)^{-1}$.

The behavior of the excitations produced in the cascade process depends on the ratio T/μ . At $T \ll \mu$ their damping time strongly increases at energies well below μ (the decay rate is at least much smaller than $\mu(n_0 a^3)^{1/2} \sim n_0 \sigma c_s$ [16]) and is likely to exceed the recombination time τ_r . Therefore, these excitations mostly remain undamped and no longer influence the number of atoms in the (partially destroyed) condensate.

Thus one has a nonequilibrium ‘‘boiling’’ Bose-Einstein-condensed sample: High-energy single-particle excitations, created in the recombination process, initiate a significant destruction of the condensate and the formation of a nonequilibrium noncondensed cloud. The corresponding loss rate of condensate atoms, $\nu = \int L n_0^3 d^3 r$, is determined by the rate of recombination-induced production of excitations with energies $\epsilon_k \gg \mu$. As a single-particle excitation with energy ϵ_k generates ϵ_k/μ noncondensed atoms out of the condensate, the rate constant L is given by Eq. (7), with $N_k = 0$ and the integrand multiplied by $\sim \epsilon/\mu$,

$$L = \alpha \int \frac{d^3 k}{(2\pi)^3} \left| \frac{h_{\mathbf{k}}}{\epsilon_k + E_k/3} \right|^2 \frac{\epsilon_k}{\mu} \gamma, \quad (9)$$

where the numerical coefficient $\gamma \sim 1$. A precise value of γ depends on a detailed behavior of damping rates of the excitations and, hence, on the trapping geometry.

The integral in Eq. (9) is divergent at high energies, and one should put an upper bound $\epsilon_k = E_B$, where E_B is the trap barrier. The inequality $E_B \gg \mu$ justifies that Eq. (9) indeed gives the loss rate due to the production of high-energy excitations ($\epsilon_k \gg \mu$). From Eq. (9) we obtain

$$L = \alpha (n_0 a^3)^{1/2} \frac{81}{\sqrt{2\pi}} \left(\frac{E_B}{\mu} \right)^{1/2} \gamma. \quad (10)$$

As $\mu = n_{0m} \tilde{U}$, where n_{0m} is the maximum condensate density, the rate constant L is independent of the number of Bose-Einstein-condensed atoms.

The generated noncondensed cloud has energy $\sim \mu$ per particle, and occupies the volume which is of order the Thomas-Fermi volume of the condensate. Similarly to the condensate, this cloud decays due to three-body recombination and, in this respect, the quantity ν describes extra losses of (condensate) atoms from the sample.

The direct loss rate of Bose-Einstein-condensed atoms due to three-body recombination is $\nu_0 = \int 3\alpha n_0^3 d^3 r$, as three atoms disappear immediately in each recombination event. Then, using Eq. (10) and the Thomas-Fermi density profile $n_0(r) = n_{0m}(1 - r^2/R^2)$, we express the total loss rate of Bose-Einstein-condensed atoms, $\nu_t = \nu_0 + \int L n_0^3 d^3 r$, through ν_0 :

$$\nu_t = \nu_0 \left[1 + \frac{216}{11\sqrt{2\pi}} (n_{0m} a^3)^{1/2} \left(\frac{E_B}{\mu} \right)^{1/2} \gamma \right]. \quad (11)$$

The situation is the same at $T \sim \mu$, if the cascade production of excitations with energies $\epsilon \sim \mu$ makes the quasiparticle distribution strongly nonequilibrium and prevents the damping of these excitations caused by their interaction with each other and with the thermal cloud. The number of excitations produced in one cascade process is $\sim E_B/\mu$, and the number of thermal quasiparticles with $\epsilon \sim \mu$ is $N_\mu \sim (\mu/\hbar\omega)^3$. Thus, under the condition $E_B > \mu N_\mu$ one also has a nonequilibrium ‘‘boiling’’ Bose-Einstein-condensed sample, and the loss rate of condensate atoms will be determined by Eq. (11).

Our calculations assume the s -wave scattering limit $k|a| \ll 1$ [17], and hence the maximum trap barrier for which they are valid is $E_B \approx \hbar^2/2ma^2$. In the case of ^{87}Rb , the triplet scattering length is $a = 5.8 \text{ nm}$, and $E_B = 75 \mu\text{K}$. Assuming $\gamma \approx 1$, this gives $L_t \approx 3L_0$ and shows a qualitative significance of our mechanism: the loss rate of Bose-Einstein-condensed atoms is essentially magnified by the creation of high-energy excitations and their destructive influence on the condensate. To be more quantitative, one has to consider the kinetics of excitations produced in the sample by the initially high-energy (trapped) atom.

In ongoing Bose-Einstein condensation (BEC) experiments, a characteristic temperature of a Bose-Einstein-condensed sample is in the range from 100 nK to 1 μK and, hence, the above estimated magnification of the loss rate of

the condensate atoms (factor 3 for $E_B \approx 75 \mu\text{K}$) practically corresponds to switching off the evaporative cooling. With evaporative cooling on, the ratio E_B/μ for temperatures smaller than μ is in practice ranging from 2 to 5. Then, at typical densities $n_0 \sim 10^{14} \text{ cm}^{-3}$, Eq. (11) only gives a 10% increase of the total loss rate of Bose-Einstein-condensed atoms compared to L_0 . To some extent this explains the recent experiments [18], where a strong increase of the three-body losses in the condensate has been observed after switching off the evaporative cooling.

For $T \sim \mu$ one can also think of the situation, where the cascade production of excitations with energies of order μ does not significantly destroy the equilibrium distribution of quasiparticles in the sample. This should be the case if $E_B \ll \mu N_\mu$. Then the damping of these excitations comes into play, continuously decreasing their energy and partially refilling the condensate. This damping originates from (inelastic) scattering of a thermal excitation on a given excitation, which transfers them to the condensate particle and the thermal excitation with higher energy [16,19]. A characteristic damping rate is of order $\varepsilon(n_0 a^3)^{1/2}$, and even for the lowest excitations ($\varepsilon \sim \hbar \omega$) it can be larger than the rate of recombination.

Consequently, one can conclude that the energy of excitations produced in the recombination process is thermalized in the gas. The Bose-Einstein-condensed sample will be in quasiequilibrium, with a continuously increasing temperature. This provides extra losses of Bose-Einstein-condensed atoms. Due to refilling the condensate in the course of damping of the excitations, these losses will be smaller than the extra losses described by Eq. (10) in the case of a non-equilibrium ‘‘boiling’’ condensate.

The rate of energy transfer from the excitations, produced in the recombination process, to the thermal cloud deter-

mines the increase of the internal energy U of the gas. One can write it as $\dot{U} = \int W n_0^3 d^3 r$, where the quantity W is obtained in the same way as Eq. (9):

$$W = \alpha \int \frac{d^3 k}{(2\pi)^3} \left| \frac{\hbar \mathbf{k}}{\epsilon_k + E_k/3} \right|^2 \epsilon_k. \quad (12)$$

Relying on Eq. (12) and the known expressions for U and the number of Bose-Einstein-condensed atoms N_0 as functions of T and the total number of particles (see Ref. [16]), we have calculated the extra losses of condensate atoms $|\partial N_0 / \partial T| \dot{T}$, related to the increase of temperature. At initial density $n_0 \sim 10^{14} \text{ cm}^{-3}$ they do not exceed 10%.

In conclusion, we have found that inelastic collisional processes in Bose-Einstein condensates can be accompanied by the creation of elementary excitations. It is worth mentioning that this phenomenon is not related to BEC in and of itself. It originates from the presence of the mean-field interparticle interaction, and will also occur in a noncondensed ultracold gas, as soon as the parameter na^3 is not extremely small. We have revealed the influence of the production of high-energy excitations in the course of three-body recombination on the loss rate of atoms from a trapped condensate. This effect is especially pronounced for a high trap barrier E_B , and it would be valuable to perform a systematic experimental investigation of the loss rate of condensed atoms as a function of E_B .

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