Continuous-probe solutions for self-similar pulses in four-level systems

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(Received 9 August 1999; published 10 December 1999)

We propose the use of a copropagating probe pulse in experimental tests of correlated pulse theory. We present formulas for amplitudes and velocities of a self-similar pulse trio propagating in a medium with energy levels in the configuration of the letter N. Remarkably, in contrast to previous theoretical studies, we find that the inclusion of the probe pulse lifts a rigid restriction on the oscillator strengths of the transitions.

PACS number(s): 42.50.Md, 42.50.Hz, 42.65.Tg

Since the discovery of simulton propagation [1] the variety of resonant phenomena employing correlated multiplets of short optical pulses has expanded significantly (*simultons* are simultaneous different-wavelenth solitons). Experimental progress has been reviewed recently by Harris [2]. Theoretical work has clarified a number of issues, and applications have been proposed [3–8]. In particular, the behavior of pulses in quantum systems of Λ, double-Λ, and V type is now much better understood. The effects of the finite lifetime of the excited state of the Λ system on pulse propagation are well described [9,10]. A new factorization procedure provides exact inclusion of inhomogeneous broadening [11] and Rahman has proposed a two-pulse ''area theorem'' [12]. A family of exact solutions describing self-similar pulses in five-level ''M'' systems has been described by Hioe and Grobe [13]. A new type of adiabaticity condition was discovered [14] and recently shown to persist even in the absence of a second pulse if quantum decay interference occurs [15]. Pulse shaping and cloning in a coherently prepared atomic system (phaseonium or an ensemble of phase-coherent atoms [16]) have been elucidated [17,18]. Some of this progress has been made numerically and some by the use of rigorous analytical techniques based on Bäcklund transformations [19] or inverse scattering methods [3,20–22].

However, in one important respect all self-induced transparency (SIT)-like extensions of the McCall-Hahn discovery of the first self-similar optical pulse [23,24] are constrained by an assumption of limited validity. The existence of self-similar pulses has been established only under the condition that the oscillator strengths f of all transitions are equal. While not forbidden to occur by any selection rule, such an equality would usually require special arrangements to be realized experimentally. We have discovered what appears to be the first exception to this rule, and in this paper we present an example of self-similar short-pulse propagation in which this restriction can be lifted, potentially opening the way for extensive experimental testing of predicted long-range multipulse correlation effects.

We will demonstrate exact three-correlated-pulse solutions appropriate to the ''N system'' of four coupled quantum states shown in Fig. 1. To obtain a solution for an odd number of self-similar correlated pulses is new itself. Much more important, however, appears to be the discovery that the addition of the third pulse (here referred to as the probe) permits lifting of the rigid equal-f restriction. Given the presence of the probe, we show that the positive-definite character of pulse energy imposes inequalities rather than equalities among the f numbers of the three transitions.

Let us consider the N system shown in Fig. 1, which can be realized in most popular optical absorbers. Our model's two lower levels can be realized, for example, in the two hyperfine components of an alkali-metal ground state ($F=1$ and $F=2$ states of the ground state of sodium, say) and the two upper levels can be the fine-structure components of the P state ($3P_{1/2}$ and $3P_{3/2}$ again in sodium). The fields have pulse envelopes $\vec{\mathcal{E}}_c$, $\vec{\mathcal{E}}_v$, and $\vec{\mathcal{E}}_f$, where $\vec{E}_\alpha(z,t)=\vec{\mathcal{E}}_\alpha(z,t)e^{i(k_\alpha z-\omega_\alpha t)}+\text{c.c.}$ The pulses act, respectively, on the transitions ei, vi, and ef, as shown. We are assuming that the rotating-wave approximation holds so that each pulse interacts with only one transition [25]. The envelopes $\vec{\mathcal{E}}_\alpha$ and hence the Rabi frequencies $\Omega_\alpha=2\vec{d}_\alpha\cdot\vec{\mathcal{E}}_\alpha/\hbar$ are assumed slowly varying in space and time on the scale of the k's and ω's. Here $\vec{d}_\alpha$ is the dipole moment matrix element of the αth transition. For simplicity we temporarily assume that all the fields are nominally resonant with their respective transitions. The iv transition can be thought of as a running probe of the Λ system made of the fe and ei transitions.

In the slowly varying envelope approximation, the resonant coupled Maxwell-Schrödinger equations are

$$i\frac{\partial}{\partial\tau}C_e=-\frac{\Omega_f}{2}C_f-\frac{\Omega_e}{2}C_i,$$

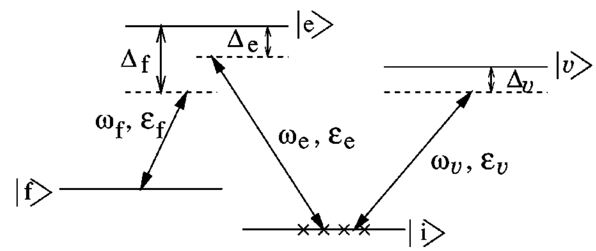

FIG. 1. Schematic diagram of the energy levels and the transitions among them. The pulse envelopes and field frequencies are denoted by $\mathcal{E}$'s and ω's and the detunings are given by Δ's. In the absence of the fields, the atomic population is in the state $|i\rangle$. Note that the transitions sketched are similar to those responsible for the D lines in alkali-metal atoms, if levels f and i are interpreted as the two hyperfine components of the ground state.

$$i\frac{\partial}{\partial\tau}C_v=-\frac{\Omega_v}{2}C_i, \tag{1}$$

$$i\frac{\partial}{\partial\tau}C_i=-\frac{\Omega_e^*}{2}C_e-\frac{\Omega_v^*}{2}C_v,$$

$$i\frac{\partial}{\partial\tau}C_f=-\frac{1}{2}\Omega_f^*C_e,$$

$$\frac{\partial}{\partial\zeta}\Omega_f=i\mu_f C_e C_f^*,$$

$$\frac{\partial}{\partial\zeta}\Omega_e=i\mu_e C_e C_i^*, \tag{2}$$

$$\frac{\partial}{\partial\zeta}\Omega_v=i\mu_v C_v C_i^*.$$

Here C_α $(\alpha=e,v,i,f)$ represents the probability amplitude of finding the atom in the state $|\alpha\rangle$. We follow the standard practice of excluding any relaxation terms in Eq. (1), i.e., we work under the conditions that the pulse widths are smaller than the relaxation times. Many of such approximations are discussed in text books (see, for example, Ref. [24], p. 81). The coordinates τ and ζ measure time relative to pulse center. They are related to space-time coordinates z and t via $\tau=t-(z/c)$, $\zeta=z$. The parameter μ is given by

$$\mu_\alpha\equiv\frac{4\pi\mathcal{N}|d_\alpha|^2\omega_\alpha}{\hbar c}=\frac{2\pi e^2\mathcal{N}f_\alpha}{mc},$$

where $\mathcal{N}$ is the density of the atoms and f_α is the oscillator strength for the transition of frequency ω_α. Note that μ has the dimensions frequency $(\text{distance})^{-1}$.

We will search for self-similar (shape-preserving) solutions, i.e., solutions which are functions of only one variable $X=(q\zeta-\Gamma\tau)$. The parameter Γ^{-1} will give the pulse duration and q and Γ will be related via the pulse velocity in the medium. We will assume that all the atomic population is in the state $|i\rangle$ before the interaction with the pulses. Note that the self-similar solutions have to be consistent with the boundary conditions and the conservation laws. Guided by analogy to three-level Λ and V cases [1,7,22], we make the following ansatz:

$$\Omega_e=A_e\operatorname{sech}X,\quad \Omega_v=A_v\operatorname{sech}X,\quad \Omega_f=A_f\tanh X,$$

$$C_i=\tanh X,\quad C_v=b_v\operatorname{sech}X, \tag{3}$$

$$C_f=b_f\operatorname{sech}X,\quad C_e=b_e\operatorname{sech}X.$$

Using the ansatz (3) we can find consistency conditions on all the parameters and amplitudes. We *summarize* the results here:

$$2q\Gamma=\mu_v, \tag{4}$$

$$|A_v|^2=4\Gamma^2\left(1-\frac{\mu_e}{\mu_f}\right),\quad |A_f|^2=4\Gamma^2\left(\frac{\mu_e}{\mu_v}-1\right),$$

$$|A_e|^2=4\Gamma^2\frac{\mu_e^2}{\mu_v\mu_f}, \tag{5}$$

$$b_e=\frac{iqA_e}{\mu_e},\quad b_v=\frac{iqA_v}{\mu_v},\quad b_f=-\frac{A_f^*\mu_e}{A_e^*\mu_f}. \tag{6}$$

These new solutions have a feature not seen in previous coherent multipulse studies—they permit a range of values for the μ coefficients (i.e., of the oscillator strengths). While not fixed as previously, the f values are still mildly constrained. It is clear by inspection that the positive definite character of the pulse intensities requires the inequalities $\mu_f\geqslant\mu_e\geqslant\mu_v$, or

$$f_f\geqslant f_e\geqslant f_v. \tag{7}$$

We can make the following observations to help clarify the solution formulas.

(a) If $\mu_e=\mu_v=\mu_f$, then $A_v=A_f=0$, $b_v=b_f=0$, and we recover the standard sech solution [23,24] for the two-level ei system.

(b) If $\mu_e=\mu_v$, then $A_f=0, b_f=0$,

$$|A_v|^2=4\Gamma^2\left(1-\frac{\mu_e}{\mu_f}\right),\ |A_e|^2=4\Gamma^2\frac{\mu_e}{\mu_f}. \tag{8}$$

Here μ_f acts like a free parameter such that $[(\mu_e/\mu_f)<1]$ and it determines the relative magnitude of the two pulses Ω_v and Ω_e. We recover the previously derived results for the V system [11].

(c) If $\mu_e=\mu_f$, then $A_v=0$, $b_v=0$, $|A_f|^2=4\Gamma^2[(\mu_e/\mu_v-1]$, and $|A_e|^2=4\Gamma^2(\mu_e/\mu_v)$. Now we have to impose the condition on the free parameter μ_v: $\mu_e/\mu_v>1$. In this case we recover the solutions [11] for Λ systems.

In addition to pulse intensity, the uniform group velocity v is also determined by our solutions. From Eq. (4) we can derive the group velocity as $X=q\zeta-\Gamma\tau=qz-\Gamma[t-(z/c)]=z[q+(\Gamma/c)]-\Gamma t\equiv\Gamma[(z/v)-t]$, where

$$\frac{1}{v}=\frac{1}{c}+\frac{q}{\Gamma}, \tag{9}$$

which upon using Eq. (4) becomes

$$\frac{1}{v}-\frac{1}{c}=\frac{\mu_v}{2\Gamma^2}. \tag{10}$$

As the pulses become shorter (Γ gets larger), the group velocity approaches the background medium velocity, as expected.

We will next examine the physically more realistic situation in which detunings are nonzero and again ask if shape-preserving solutions are possible. We consider the interesting partially resonant case when the fields Ω_e and Ω_f are detuned by the same amount, i.e., $\Delta_e=\Delta_f$, but the detuning Δ_v is left arbitrary:

$$\Delta_e=\omega_{ei}-\omega_e=\omega_{ef}-\omega_f,\qquad \Delta_v=\omega_{vi}-\omega_v. \tag{11}$$

Again, the iv transition will be regarded as a running probe of the fe-ei Λ system, but now we have Δ_v available as an adjustable independent probe parameter.

We find that the first two equations in Eqs. (1) are modified to

$$i\frac{\partial}{\partial\tau}C_e=-\frac{\Omega_f}{2}C_f-\frac{\Omega_e}{2}C_i+\Delta_eC_e,\qquad i\frac{\partial}{\partial\tau}C_v=-\frac{\Omega_v}{2}C_i+\Delta_vC_v, \tag{12}$$

and the following modified ansatz is now appropriate [11]:

$$\Omega_e=A_e(\operatorname{sech}X)e^{iQ\zeta},\quad \Omega_v=A_v(\operatorname{sech}X)e^{iQ\zeta},\quad \Omega_f=A_f\tanh X,\quad C_i=b_i\tanh X+\alpha,\quad C_f=b_f(\operatorname{sech}X)e^{iQ\zeta},\quad C_v=b_v(\operatorname{sech}X)e^{iQ\zeta},\quad C_e=b_e(\operatorname{sech}X)e^{iQ\zeta}. \tag{13}$$

The new phase $Q\zeta$ signals a modification of the index of refraction that will depend on the detunings, and $\alpha\equiv 1-b_i$.

Using this second ansatz in the Schrödinger-Maxwell equations (1), (2), and (12), we can still find a set of conditions under which shape-preserving pulses are possible:

$$\Delta_v/\Delta_e=\mu_v/\mu_e, \tag{14}$$

$$b_i=\frac{\Gamma}{\Gamma+i\Delta_v},\quad \alpha=i\frac{\Delta_v}{\Gamma+i\Delta_v}, \tag{15}$$

$$q=\frac{\mu_v}{2\Gamma}|b_i|^2=\frac{\mu_v}{2}\frac{\Gamma}{\Delta_v^2+\Gamma^2}, \tag{16}$$

$$Q=q\frac{\Delta_v}{\Gamma}=\frac{\mu_v}{2}\frac{\Delta_v}{\Delta_v^2+\Gamma^2}. \tag{17}$$

Clearly q and Q appear as absorptive and dispersive parts of b_i, and we will give their effect on the group and phase velocities below.

The amplitudes A_e, A_v, and A_f are still given by Eqs. (5), but now the levels' probability amplitudes depend on probe detuning factors and are given in terms of field amplitudes as follows:

$$b_e=\frac{iA_e\mu_v}{2\Gamma\mu_e}\frac{1}{1+i\Delta_v/\Gamma},\quad b_v=\frac{iA_v}{2\Gamma}\frac{1}{1+i\Delta_v/\Gamma},\quad b_f=-\left(\frac{A_f}{A_e}\right)^*\frac{\mu_e}{\mu_f}\frac{1}{1+i\Delta_v/\Gamma}. \tag{18}$$

The shape-preserving solutions (13) are valid under condition (14) on the two detunings and condition (7) on the f's. It can be checked that $\Sigma_\alpha|C_\alpha|^2\equiv 1$, as required. The phase and group velocities are determined by q and Q. We find

$$\frac{1}{v}-\frac{1}{c}=\frac{\mu_v/2}{\Delta_v^2+\Gamma^2}, \tag{19}$$

and $Q=\delta k=\delta n(\omega/c)$, where the modification of the index of refraction is therefore

$$\delta n=\frac{\mu_v c}{2\omega}\frac{\Delta_v}{\Delta_v^2+\Gamma^2}. \tag{20}$$

In summary, we have found that a near-resonant ''N'' medium permits shape-preserving solutions for an odd number of copropagating correlated pulses, apparently the first such example since the McCall-Hahn discovery of SIT. Potentially much more important, however, is the discovery that the presence of the probe pulse significantly mitigates the formerly rigid equal-oscillator-strength condition. We have focused on the case of a continuously probed two-photon-resonant Λ system, and have given pulse amplitudes, atomic probablity amplitudes, and group and phase velocities in terms of the probe detuning Δ_v and inverse pulse length Γ. We emphasize that the relaxation of the restriction to equal oscillator strengths in all transitions opens self-similar correlated-pulse propagation theory to flexible experimental test.

The authors thank Ashiqur Rahman for several conversations. NSF Grants No. INT97-12760 and No. PHY97-22024 provided financial support for this collaboration.

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[25] We add that the question of pulse propagation in a system where the pulse can interact with several transitions at the same time is a very interesting one and is being numerically studied. Note that for this situation there are no analytical solutions even in the context of a simple Λ system.