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Pointer states via decoherence in a quantum measurement

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We consider the interaction of a quantum system (spin-$\frac{1}{2}$) with a macroscopic quantum apparatus (harmonic oscillator) which in turn is coupled to a bath of harmonic oscillators. Exact solutions of the Markovian master equation show that the reduced density matrix of the system-apparatus combination decoheres to a statistical mixture where up and down spins eventually correlate with pointer states of the apparatus (harmonic oscillator), with associated probabilities in accordance with quantum principles. For the zero-temperature bath these pointer states turn out to be *coherent states* of the harmonic oscillator (apparatus) for *arbitrary initial states* of the apparatus. Further, we see that the decoherence time is inversely proportional to the square of the separation between the two coherent states with which the spins correlate. For a high-temperature bath, pointer states no longer remain coherent states but are Gaussian distributions (*generalized coherent states*). Spin up and down states of the system now correlate with *nearly diagonal distributions in position* of these generalized coherent states. The diagonalization in position increases with the temperature of the bath. The off-diagonal elements in spin space decohere over a time scale which goes inversely as the square of the separation between the peaks of the two position distributions that correlate with the spin states. Zurek's earlier approximate result for the decoherence time is consistent with our exact results. Our analysis brings out the importance of looking at a measurementlike scenario where definite correlations are established between the system and apparatus to determine the nature of the pointer basis of the apparatus. Further, our exact results demonstrate in an unambiguous way that the pointer states in this measurement model emerge independent of the initial state of the apparatus.

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I. INTRODUCTION

In a typical quantum measurement, the coupling between a microscopic system and a macroscopic measuring apparatus results in an entangled state which seems to allow the read out of the apparatus (‘‘meter’’) to exist in a coherent superposition of macroscopically distinct states, a situation which is difficult to reconcile with classical intuition and perceptions. For a measurement to be classically interpretable and meaningful, one expects the system-apparatus correlations to appear as a statistical mixture. von Neumann [1] postulated that an irreversible reduction process takes such a quantum superposition (entanglement) into a statistical mixture in a measurement process. However, the apparent nonunitary nature of such a reduction raises several questions about the validity of quantum mechanics and its connection with the emergence of classicality.

In recent years, decoherence [2,3] has been widely discussed and accepted as the mechanism responsible for the emergence of classicality in a quantum measurement and the absence, in the real world, of Schrödinger-cat-like states [4]. Decoherence results from the irreversible coupling of the apparatus to an environment. The appearance of classical behavior via decoherence in a quantum measurementlike scenario is marked by the dynamical transition of the reduced density matrix of the system apparatus combination from a pure entangled state to a statistical mixture with appropriate correlations. This line of approach to the quantum measurement problem was initiated by Zeh [2] and later followed up by Zurek [3] and several others. Most studies relating to decoherence in the literature deal with an environment modeled by a collection of harmonic oscillators with which the system of interest interacts via a coordinate-coordinate coupling. The dynamics of the reduced density matrix (after tracing over the degrees of freedom of the environment) is then described by the Markovian master equation derived separately by Caldeira and Leggett [5], Agarwal [6], Dekker [7], and others [8] in the context of quantum Brownian motion. Using the Markovian master equation with some approximations, Zurek has argued that the density matrix for a free particle in an initial coherent superposition of two Gaussian wave packets separated by Δx decoheres (i.e., the off-diagonal elements of the density matrix decay) over a time scale which goes inversely as the square of the separation (Δx^2) between the two parts of the superposition [3]. For classical systems and standard macroscopic separations, Δx, this ‘‘decoherence time’’ is shown to be almost 10^{-40} times smaller than the thermal relaxation time of the system. Thus macroscopic superpositions are almost instantaneously reduced to a statistical mixture [3]. Savage and Walls [9] have solved the master equation for a harmonic oscillator in an initial superposition of coherent states and seen the decay of the off-diagonal elements of the density matrix at zero temperature and finite temperatures. The master equation approach has been used by Venugopalan *et al.* [10] to study a Stern-Gerlach-type measurement model where a spin-$\frac{1}{2}$ particle interacts with a quantum apparatus (represented by the position and momentum degrees of freedom of the particle), which in turn is coupled to a bath of oscillators through its

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position. They solve this equation exactly and show that the reduced density matrix for the system and apparatus is driven to a diagonal form as a consequence of decoherence and the spin components correlate with momentum distributions [10].

Recently, interest in the understanding of decoherence has been heightened by advances on the experimental front. Brune *et al.* [11] experimentally created a mesoscopic superposition of quantum states involving radiation fields with classically distinct phases and observed its progressive decoherence to a statistical mixture through two-atom correlation measurements. Schrodinger-cat-like states were recently created in an ion trap experiment [12] using a single beryllium ion and a combination of static and oscillating electric fields. Though only a limited number of models have been approximately studied so far, it is generally accepted now that the two main signatures of the decoherence mechanism are as follows: (a) In the classical regime decoherence takes place over a time scale that is much smaller than the thermal relaxation time of the system, and (b) the decoherence time goes inversely as the square of the separation between the two parts of the superposition [3]. These features have been observed in the experiment of Brune *et al.* [11] and thus confirm the theoretical predictions [3]. Recently there have been several proposals to exploit purely quantum-mechanical features such as the linear superposition principle and quantum entanglements [13] to build high speed quantum computers [14] and also to experimentally implement other ideas from quantum information such as quantum cryptography [15] and quantum teleportation [16]. Since environmental influence is often unavoidable, decoherence can ruin the functioning of such systems which rely heavily on the maintenance of quantum coherence. A clearer understanding of the behavior of quantum coherences in dissipative environments is, thus, of fundamental importance. Our experience of the classical world suggests that unlike quantum systems, which are allowed to exist in all possible states, classical systems only exist in a few select states which are singled out by the environment from a larger quantum menu [3]. These special states are the ‘‘preferred basis,’’ also referred to as the ‘‘pointer states’’ in a quantum-measurement-like scenario [3]. In spite of the progress in the theoretical and experimental understanding of decoherence, the models studied so far do not answer the question concerning the nature of the preferred basis satisfactorily. For simplified models where the self-Hamiltonian of the system has either been ignored or considered codiagonal with the interaction Hamiltonian, the ‘‘pointer’’ variable has been shown to be the one which commutes with the interaction Hamiltonian [3]. However, in more general situations where all terms are included and the various parts of the Hamiltonian may not commute, it is not obvious what decides the preferred basis. For the coordinate-coordinate coupling model, the position basis is intuitively expected to emerge as the preferred basis. However, this is contrary to the conclusion of Venugopalan *et al.* [10] in their analysis of the Stern-Gerlach measurement model where the spin components eventually correlate with distributions which are completely diagonal in the momentum basis and only approximately diagonal in the position basis [10]. In the literature, the preferred basis has been variously described as the one in which the final-state density matrix becomes diagonal or that set of basis states which are characterized by maximum stability or a minimum increase in linear or statistical entropy, decided by a ‘‘predictability sieve’’ [17]. In a measurementlike scenario, the pointer basis should be understood as those states of the apparatus in which correlations with the system states are eventually established *irrespective of the initial states of the apparatus*. Using the Markovian master equation for a harmonic oscillator coupled to a heat bath and the criterion of the ‘‘predictability sieve,’’ Zurek argues that coherent states emerge as the preferred basis. In a recent paper, Paz and Zurek [18] investigate decoherence in the limit of weak interaction with the environment and show that the eigenstates of energy emerge as pointer states. Roy and Venugopalan have recently obtained the exact solutions of the Markovian master equation for a harmonic oscillator and a free particle in a compact factorizable form and have shown that the density matrix diagonalizes in the energy basis which is number states for the oscillator and momentum states for the free particle for arbitrary initial conditions [19]. It is intuitive that the pointer states should naturally be a consequence of the interplay between the various components of the total Hamiltonian and one should also expect them to be *independent of the initial state of the system/apparatus*. The limited number of examples studied so far do not bring out this feature clearly.

In this paper we seek to analyze a measurementlike scenario where a spin-$\frac{1}{2}$ particle is coupled to a harmonic oscillator through its coordinate and the oscillator in turn is coupled to a bath of oscillators via coordinate-coordinate coupling. The dynamics of the system-apparatus combination is studied via the Markovian master equation for zero-temperature and high-temperature cases. Exact solutions for arbitrary initial states of the apparatus clearly show that the spin components eventually correlate with *coherent states* of the apparatus at zero temperature. This brings out the role of coherent states as the pointer basis in an unambiguous way. At high temperatures the pointer states are Gaussian distributions (generalized coherent states) and up and down spins correlate with *nearly diagonal position distributions* of these generalized coherent states. We also see the two main signatures of decoherence in the measurement, i.e., the decoherence time is much shorter than the thermal relaxation time in the classical regime and it goes inversely as the square of the separation between the ‘‘pointer states’’ with which the spin components correlate for zero temperature as well as for the high-temperature case. The model we have considered is equivalent to the spin-boson Hamiltonian and also corresponds to the physical example of a two-level atom coupled to a single mode of a radiation field—a simple model that describes many interesting physical situations in quantum optics [21], which could be used to produce a system-apparatus entangled state where decoherence can be experimentally monitored. For example, Brune *et al.* [11] have used a Ramsey-type experiment using two-level Rydberg atoms and microwave cavities to produce superposition states of the electromagnetic field as well as atom-field entangled states which interact with a bath. Further, they have monitored the progressive decoherence of these pure states to a statistical mixture. Meekhof *et al.* [20] have created thermal, Fock, coherent, and squeezed states of motion of a harmonically bound, cooled, and trapped beryllium ion where the coupling between its motion and internal states can be described by the Jaynes-Cummings-type interaction [21]. This, again, involves a two-level atom radiatively coupled to the single mode radiation field. It is possible that similar systems, with suitable modifications, could be used to physically implement the system-apparatus entangled state of the model analyzed in this paper and to subsequently monitor its decoherence mechanism. The rest of the paper is organized as follows. In Sec. II we introduce our model for the measurement and the equivalent Markovian master equation and solve it for the reduced density matrix of the system-apparatus combination. In Sec. III we analyze our results and observations and discuss the pointer basis. Finally, in Sec. IV we summarize the main results of this paper.

II. REDUCED DENSITY MATRIX FOR THE SYSTEM APPARATUS

Consider our model for the measurement of spin. A spin-$\frac{1}{2}$ represents the system. A harmonic oscillator represents the apparatus which is coupled to the system via its coordinate. The harmonic oscillator can be considered as a (macroscopic) measuring apparatus in the sense that it can measure the state of the system (spin) via its position/momentum degrees of freedom, which have well-understood classical distributions. Alternately, this apparatus (harmonic oscillator) could also be thought of as corresponding to a single mode of a radiation field whose quantum state correlates with the state of the system (spin) and hence can affect a measurement of the spin. The apparatus is in turn coupled via a coordinate-coordinate coupling to a collection of oscillators which represent the environment. The environmental influence via this bath of oscillators brings about the decoherence of the entangled system-apparatus pure state to a statistical mixture. This arrangement represents *a general model for quantum measurement* and the total Hamiltonian for such a system can be written as

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda\sigma_Z+\epsilon x\sigma_Z +\sum_j \frac{P_j^2}{2M_j}+\frac{M_j\Omega_j^2}{2}\left(X_j-\frac{C_jx}{M_j\Omega_j^2}\right)^2. \qquad (1)$$

Here x and p denote the position and momentum of the harmonic oscillator (apparatus) of mass m and frequency ω. $\lambda\sigma_Z$ is the Hamiltonian of the system and ϵ is the strength of the system-apparatus coupling. The last term represents the Hamiltonian for the bath of oscillators (environment) and the apparatus-environment interaction. X_j and P_j are the position and momentum coordinates of the jth harmonic oscillator of the bath, C_j's are the coupling strengths, and Ω_j's are the frequencies of the oscillators comprising the bath [22]. For our analysis we deal directly with the Markovian master equation for the reduced density matrix for the system-apparatus combination in the $|s,x\rangle$ representation, where the environmental degrees of freedom have been traced out [10]:

$$\frac{\partial\rho_{ss'}(x,x',t)}{\partial t}=\Bigg[-\frac{\hbar}{2im}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)-\gamma(x-x')\times\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)-\frac{D}{4\hbar^2}(x-x')^2-\frac{m\omega^2}{2i\hbar}(x^2-x'^2)+\frac{i\epsilon(xs-x's')}{\hbar}+\frac{i\lambda(s-s')}{\hbar}\Bigg]\rho_{ss'}(x,x',t), \qquad (2)$$

where $s,s'=+1$ (or ↑) or -1 (or ↓). Thus Eq. (2) represents four equations in the coordinate representation, each corresponding to one of the four elements in spin space $(\uparrow\uparrow,\uparrow\downarrow,\downarrow\uparrow,\downarrow\downarrow)$ of the reduced density matrix for the system-apparatus combination [10]. Here γ is the relaxation rate and

$$D=8m\gamma\hbar\omega(\overline{n}+\tfrac{1}{2}), \qquad (3)$$

where

$$\overline{n}=[\exp(\hbar\omega/k_BT)-1]^{-1} \qquad (4)$$

is the expected number of quanta in a harmonic oscillator of frequency ω in equilibrium at temperature T and k_B is the Boltzmann coefficient [6]. At $T=0$, $D=4m\gamma\omega\hbar$ and for a high-temperature bath, $D=8m\gamma k_BT$. Consider Eq. (2) in the changed coordinates:

$$R=\frac{x+x'}{2}, \quad r=x-x'. \qquad (5)$$

The spin diagonal density matrix, ρ_d, and the spin off-diagonal density matrix, ρ_{od}, obey the equations

$$\frac{\partial\rho_d(R,r,t)}{\partial t}=-\left[\frac{\hbar}{im}\frac{\partial^2}{\partial r\partial R}+2\gamma\frac{\partial}{\partial r}+\frac{Dr^2}{4\hbar^2}+\frac{m\omega^2rR}{i\hbar}\mp\frac{i\epsilon r}{\hbar}\right]\rho_d(R,r,t), \qquad (6)$$

where the "+" sign in the last term corresponds to $\rho_{\uparrow\uparrow}$ and "−" to $\rho_{\downarrow\downarrow}$, and

$$\frac{\partial\rho_{\mathrm{od}}(R,r,t)}{\partial t}=-\left[\frac{\hbar}{im}\frac{\partial^2}{\partial r\partial R}+2\gamma\frac{\partial}{\partial r}+\frac{Dr^2}{4\hbar^2}+\frac{m\omega^2rR}{i\hbar}\mp\frac{2i\epsilon R}{\hbar}\mp\frac{2i\lambda}{\hbar}\right]\rho_{\mathrm{od}}(R,r,t), \qquad (7)$$

where the upper signs in the last two terms correspond to $\rho_{\uparrow\downarrow}$ and the lower ones to $\rho_{\downarrow\uparrow}$. To solve these equations, it is convenient to take a partial Fourier transform in the variable R:

$$\rho(Q,r,t)=\int_{-\infty}^{\infty} e^{iQR}\rho(R,r,t)dR. \tag{8}$$

Equations (6) and (7) then simplify to a pair of first-order partial differential equations:

$$\frac{\partial\rho_d(Q,r,t)}{\partial t}=\left(\frac{\hbar Q}{m}-2\gamma r\right)\frac{\partial\rho_d}{\partial r}-\frac{m\omega^2 r}{\hbar}\frac{\partial\rho_d}{\partial Q}-\frac{Dr^2}{4\hbar^2}\rho_d \pm\frac{i\epsilon r}{\hbar}\rho_d, \tag{9}$$

$$\frac{\partial\rho_{\text{od}}(Q,r,t)}{\partial t}=\left(\frac{\hbar Q}{m}-2\gamma r\right)\frac{\partial\rho_{\text{od}}}{\partial r}-\frac{m\omega^2 r}{\hbar}\frac{\partial\rho_{\text{od}}}{\partial Q}-\frac{Dr^2}{4\hbar^2}\rho_{\text{od}} \pm\frac{2\epsilon}{\hbar}\frac{\partial\rho_{\text{od}}}{\partial Q}\pm\frac{2i\lambda}{\hbar}\rho_{\text{od}}. \tag{10}$$

Equations (9) and (10) can be solved by the method of characteristics [10,19,23]. Let the initial state of the system-apparatus combination be a product of any arbitrary state of the apparatus (oscillator) and a general superposition state for the spin-$\frac{1}{2}$ system of the form

$$\psi(x,s,0)=\phi(x)\otimes[a|\uparrow\rangle+b|\downarrow\rangle], \tag{11}$$

where $\phi(x)$ is *any* initial state of the harmonic oscillator (apparatus). The time-evolved density matrix would then appear as

$$\rho=|a|^2|\uparrow\rangle\langle\uparrow|\rho_{\uparrow\uparrow}(x,x',t)+|b|^2|\downarrow\rangle\langle\downarrow|\rho_{\downarrow\downarrow}(x,x',t)+ab^*|\uparrow\rangle \times\langle\downarrow|\rho_{\uparrow\downarrow}(x,x',t)+a^*b|\downarrow\rangle\langle\uparrow|\rho_{\downarrow\uparrow}(x,x',t). \tag{12}$$

The solution for the spin off-diagonal elements of the density matrix (corresponding to $\rho_{\uparrow\downarrow}$ and $\rho_{\uparrow\downarrow}$) in the partial Fourier transform representation is

$$\rho_{\text{od}}(Q,r,t)=\rho_{\text{od}}(Q',r',0)\exp\left\{-\frac{\epsilon^2 tD}{\hbar^2 m^2\omega^4}\mp\frac{\epsilon D}{m\omega^2\hbar^2}Z_1 -\frac{D}{4\hbar^2}Z_2\pm\frac{2i\lambda t}{\hbar}\right\}, \tag{13}$$

where

$$\begin{aligned}Z_1={}&\frac{m\lambda_+\Gamma}{\hbar}\left[Q-\frac{r}{\lambda_+}\pm\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_+}\right)\right]\\&\times(1-e^{-\hbar t/m\lambda_+})+\frac{m\lambda_-\Gamma}{\hbar}\left[Q-\frac{r}{\lambda_-}\mp\frac{2\epsilon}{\hbar\omega^2}\right.\\&\left.\times\left(2\gamma-\frac{\hbar}{m\lambda_-}\right)\right](1-e^{-\hbar t/m\lambda_-}),\\Z_2={}&\frac{m\Gamma^2\lambda_+}{2\hbar}\left[Q-\frac{r}{\lambda_+}\pm\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_+}\right)\right]^2\\&\times(1-e^{-2\hbar t/m\lambda_+})+\frac{m\Gamma^2\lambda_-}{2\hbar}\left[Q-\frac{r}{\lambda_-}\mp\frac{2\epsilon}{\hbar\omega^2}\right.\\&\left.\times\left(2\gamma-\frac{\hbar}{m\lambda_-}\right)\right]^2(1-e^{-2\hbar t/m\lambda_-})-\frac{\Gamma^2}{\gamma}\left[Q-\frac{r}{\lambda_+}\right.\\&\left.\pm\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_+}\right)\right]\left[Q-\frac{r}{\lambda_-}\mp\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_-}\right)\right]\\&\times(1-e^{-2\gamma t}),\end{aligned} \tag{14}$$

and

$$\begin{aligned}\Gamma&=\frac{\lambda_+\lambda_-}{\lambda_+-\lambda_-},\\\lambda_\pm&=\frac{\hbar}{m\omega^2}(\gamma\pm\sqrt{\gamma^2-\omega^2}).\end{aligned} \tag{15}$$

Q' and r' are defined as

$$\begin{aligned}Q'&=\frac{c_+\lambda_+-c_-\lambda_-}{\lambda_+-\lambda_-}\pm\frac{4\epsilon\gamma}{\hbar\omega^2},\\r'&=\Gamma(c_+-c_-)\pm\frac{2\epsilon}{m\omega^2},\end{aligned} \tag{16}$$

where the coefficients $c_\pm$ are

$$\begin{aligned}c_+&=\left[Q-\frac{r}{\lambda_+}\mp\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_+}\right)\right]e^{-\hbar t/m\lambda_+},\\c_-&=\left[Q-\frac{r}{\lambda_-}\mp\frac{2\epsilon}{\hbar\omega^2}\left(2\gamma-\frac{\hbar}{m\lambda_-}\right)\right]e^{-\hbar t/m\lambda_-}.\end{aligned} \tag{17}$$

It is clear from Eq. (13) that the leading-order decay term for the spin off-diagonal elements of the reduced density matrix goes as $e^{-\alpha t}$, which would drive the entire expression to zero with time, independent of all other arguments. This, essentially, is the decoherence of the pure state density matrix and happens over a time scale,

$$\tau_D=\frac{\hbar^2 m^2\omega^4}{D\epsilon^2}. \tag{18}$$

We will discuss the features of this decoherence in greater detail in the next section. Consider now the solution for the spin diagonal elements of the reduced density matrix for the system-apparatus:

$$\rho_d(Q,r,t)=\rho(Q'',r'',0)\exp\left(-\frac{D}{4\hbar^2}Z_3\pm\frac{i\epsilon}{\hbar}Z_4\right), \tag{19}$$

where

$$\begin{aligned}Z_3=&\frac{m\Gamma^2\lambda_+}{2\hbar}\left(Q-\frac{r}{\lambda_+}\right)^2(1-e^{-2\hbar t/m\lambda_+})+\frac{m\Gamma^2\lambda_-}{2\hbar}\\&\times\left(Q-\frac{r}{\lambda_-}\right)^2(1-e^{-2\hbar t/m\lambda_-})-\frac{\Gamma^2}{\gamma}\left(Q-\frac{r}{\lambda_+}\right)\\&\times\left(Q-\frac{r}{\lambda_-}\right)(1-e^{-2\gamma t}),\end{aligned} \tag{20}$$

$$\begin{aligned}Z_4=&\frac{m\Gamma\lambda_+}{\hbar}\left(Q-\frac{r}{\lambda_+}\right)(1-e^{\hbar t/m\lambda_+})-\frac{m\Gamma\lambda_-}{\hbar}\left(Q-\frac{r}{\lambda_-}\right)\\&\times(1-e^{\hbar t/m\lambda_-}),\end{aligned}$$

and Q'' and r'' are defined as

$$\begin{aligned}Q''&=\frac{c'_+\lambda_+-c'_-\lambda_-}{\lambda_+-\lambda_-},\\r''&=\Gamma(c'_+-c'_-).\end{aligned} \tag{21}$$

The coefficients $c'_\pm$ are

$$c'_\pm=\left(Q-\frac{r}{\lambda_-}\right)e^{-\hbar t/m\lambda_\pm}. \tag{22}$$

Equations (13) and (19) are the exact solutions corresponding to the two diagonal and two off-diagonal elements in spin space of the reduced density matrix of the system apparatus in the Q,r representation. In the next section we analyze these solutions and discuss the decoherence mechanism and the emergence of the pointer basis in this measurement model.

III. DECOHERENCE AND PREFERRED BASIS

In the preceding section we have seen [from Eq. (13)] that the spin off-diagonal elements of the reduced density matrix of the system apparatus *decay to zero* with time irrespective of the initial state of the apparatus. Thus the entangled system-apparatus pure state (12) eventually diagonalizes over a time scale given by Eq. (18) to a *mixed density matrix* with definite system-apparatus correlations:

$$\rho=|a|^2|\uparrow\rangle\langle\uparrow|\rho_{\uparrow\uparrow}(x,x',t)+|b|^2|\downarrow\rangle\langle\downarrow|\rho_{\downarrow\downarrow}(x,x',t). \tag{23}$$

The environmental influence manifested via the Markovian master equation for the reduced density matrix has, thus, clearly destroyed the off-diagonal elements in spin space and affected a measurement of the spin. Let us now examine the nature of the pointer states, $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$, which correlate with up and down spin states in Eq. (23). In the Q,r representation this is given by Eq. (19). Consider the long time limit ($t\to\infty$) of Eq. (19) ($\gamma>\omega$). It can be seen that at long times Eq. (19) takes the form

$$\begin{aligned}\rho_d(Q,r,t)&=\rho(0,0,0)\exp\left\{-\frac{D}{16m^2\omega^2\gamma}\left(Q^2+\frac{m^2\omega^2r^2}{\hbar^2}\right)\right.\\&\qquad\left.\pm\frac{i\epsilon Q}{m\omega^2}\right\}\\&=\frac{1}{2\pi}\exp\left\{-\frac{D}{16m^2\omega^2\gamma}\right.\\&\qquad\left.\times\left(Q^2+\frac{m^2\omega^2r^2}{\hbar^2}\right)\pm\frac{i\epsilon Q}{m\omega^2}\right\}.\end{aligned} \tag{24}$$

The Fourier transform of Eq. (24) in the position representation (x,x') is

$$\begin{aligned}\rho_d(R,r,t)&=2m\omega\sqrt{\frac{\gamma}{\pi D}}\exp\left\{-\frac{4m^2\omega^2\gamma}{D}\left(R\pm\frac{\epsilon}{m\omega^2}\right)^2\right.\\&\qquad\left.-\frac{Dr^2}{16\hbar^2\gamma}\right\}\\&=2m\omega\sqrt{\frac{\gamma}{\pi D}}\exp\left\{-\frac{4m^2\omega^2\gamma}{D}\right.\\&\qquad\left.\times\left(\frac{x+x'}{2}\pm\frac{\epsilon}{m\omega^2}\right)^2-\frac{D(x-x')^2}{16\hbar^2\gamma}\right\}.\end{aligned} \tag{25}$$

Equation (25) is the final form of the pointer states, which eventually correlate with up and down spins in the mixed density matrix (23).

A. Zero temperature

For the zero-temperature Markovian bath, $D=4m\omega\gamma\hbar$. Substituting for D in Eq. (25) gives

$$\rho_d(R,r,t)=\sqrt{\frac{m\omega}{\pi\hbar}}\exp\left\{-\frac{m\omega}{\hbar}\left(R\pm\frac{\epsilon}{m\omega^2}\right)^2-\frac{m\omega r^2}{4\hbar}\right\}. \tag{26}$$

This is nothing but the density matrix corresponding to a *coherent state*, $|\alpha\rangle$, of a harmonic oscillator with zero mean momentum, mean positions $=\pm\epsilon/m\omega^2$, and

$$|\alpha|^2=\frac{m\omega}{2\hbar}\left(\frac{\epsilon}{m\omega^2}\right)^2=\frac{\epsilon^2}{2m\omega^3\hbar}. \tag{27}$$

Thus up and down spins correlate with *coherent states*, which clearly establishes the coherent states as the pointer basis or the preferred states here. Zurek *et al.* [17] have earlier derived an approximate expression for the ''predictability sieve,'' which is the measure of the increase in entropy, $S = \mathrm{Tr}(\rho - \rho^2)$, for a harmonic oscillator coupled to a heat bath whose dynamics is described by the Markovian master equation. If Δx and Δp are the initial dispersions in x and p, in the limit of weak coupling and under the assumption that the initial state remains approximately pure, they show that

$$\frac{dS}{dt} \sim 4D\Delta x^2. \tag{28}$$

Further, they integrate Eq. (28) in the weak coupling limit over an oscillator period, after replacing the free Heisenberg equations for the oscillator operators, and show that

$$S(t) = 2D\left[\Delta x^2 + \frac{\Delta p^2}{m^2\omega^2}\right]. \tag{29}$$

The quantity (29) is minimum if $\Delta x \Delta p = \hbar/2$ and $\Delta x^2 = \hbar/2m\omega$. This corresponds to the spread in position of the ground state or of a coherent state of an oscillator. On this basis, Zurek *et al.* claim that the coherent states are the preferred basis for a harmonic oscillator [17]. Our analysis of the exact solutions for the full master equation here shows in a more rigorous way that coherent states emerge naturally as the apparatus states that eventually correlate with the system states. Moreover, this happens for *arbitrary initial states of the apparatus*, which firmly establishes the fact that coherent states are truly the preferred states for the apparatus in this measurement model. One can see that the decoherence time (18) is

$$\tau_D = \tau_R\left(\frac{m\omega^3\hbar}{\epsilon^2}\right) = \frac{\tau_R}{2|\alpha|^2}, \tag{30}$$

where $\tau_R = \gamma^{-1}$ is the thermal relaxation time. When $|\alpha|^2 \gg 1$, it is clear that $\tau_D \ll \tau_R$. From Eq. (27) one can see that this would be the case when one has conditions expected in the classical limit, i.e., when Planck's constant, $\hbar$, is small relative to the actions involved. It is in such a regime that one would expect a fast decoherence of the superposition of ''macroscopically distinct'' pointer positions to a statistical mixture. The main signatures of a quantum measurement via the decoherence mechanism are thus clearly seen here, namely (a) in the classical regime there is a fast decoherence of the off-diagonal elements of the spin density matrix over a time scale τ_D given by Eq. (30) which is *much smaller than* τ_R, the relaxation time of the bath, (b) there is a *one-to-one correlation between the spin states and the pointer states of the apparatus* which are *coherent states*, and (c) τ_D is inversely proportional to the square of the separation $|\alpha|^2$ between the two pointer states. The strength of this model is that the system-apparatus correlations established at long times are permanent and the emergent pointer basis is independent of the initial state of the apparatus.

B. High temperature

For the high-temperature thermal bath, $D = 8m\gamma k_BT$. Substituting this in Eq. (25) gives

$$\rho_d(R,r,t) = \sqrt{\frac{m\omega^2}{2\pi k_BT}}\exp\left\{-\frac{m\omega^2}{2k_BT}\left(R \pm \frac{\epsilon}{m\omega^2}\right)^2 - \frac{mk_BTr^2}{2\hbar^2}\right\}. \tag{31}$$

Equation (31) is no longer a coherent state but a Gaussian distribution which is also referred to as a *generalized coherent state*. Tegmark and Shapiro [24] have earlier shown that generalized coherent states tend to be produced naturally when one looks at the reduced Wigner distribution of infinite systems of coupled harmonic oscillators at $t \to \infty$. Our results are in tune with their predictions. For high temperatures, one can see that the variance corresponding to the off-diagonal elements in the position basis, r, is small and decreases with increasing temperature. Thus for a high-temperature bath, this generalized coherent state (31) is nearly diagonal in the position representation. Spin-up and -down states are clearly then correlated with these approximately diagonal position distributions which are centered around $\pm\epsilon/m\omega^2$:

$$\rho \sim |a|^2|\uparrow\rangle\langle\uparrow| \otimes \sqrt{\frac{m\omega^2}{2\pi k_BT}}\exp\left\{-\frac{m\omega^2}{2k_BT}\left(x + \frac{\epsilon}{m\omega^2}\right)^2\right\} + |b|^2|\downarrow\rangle\langle\downarrow| \otimes \sqrt{\frac{m\omega^2}{2\pi k_BT}}\exp\left\{-\frac{m\omega^2}{2k_BT}\left(x - \frac{\epsilon}{m\omega^2}\right)^2\right\}. \tag{32}$$

It can be checked that spin-apparatus correlations do not exist in the diagonal elements of the momentum basis and hence *position* is quite obviously ''preferred'' by the environment. This contrasts with the Stern-Gerlach model analyzed by Venugopalan *et al.* [10], in which it is the momentum distributions with which the spin-apparatus correlations ultimately get established. The decoherence time (18) over which the spin off-diagonal elements of the reduced density matrix of the system apparatus combination decay for the high-temperature bath is now given by

$$\tau'_D = \tau_R\left(\frac{m\hbar^2\omega^4}{8\epsilon^2k_BT}\right) = \tau_R\left(\frac{\lambda_d}{\Delta^2}\right)^2, \tag{33}$$

where $\lambda_d = \hbar/\sqrt{2mk_BT}$ is the thermal de Broglie wavelength of the particle and $\Delta = 2\epsilon/m\omega^2$ is the separation between the peaks of the two pointer distributions in Eq. (32). It is obvious that whenever $\Delta \gg \lambda_d$, there is a fast decoherence of the entangled system-apparatus pure state to a statistical mixture. Such a condition would correspond to a regime expected in the classical limit. Thus, once again, we can clearly see the main signatures of the decoherence mechanism here as in the zero-temperature case discussed above, namely, the off-diagonal elements in spin space decohere to zero completely in a time scale which is much smaller than γ^{-1} and which goes inversely as the square of the spatial separation between the ‘‘pointers,’’ eventually leading to a mixed density matrix (32) with appropriate system-apparatus correlations. The decoherence time (33) was earlier obtained by Zurek [3] from the high-temperature Markovian master equation for a free particle in an initial coherent superposition of two Gaussian wave packets separated by Δ under the approximation that the only dominant term is $D/4\hbar^2(x-x')^2$, in Eq. (2). We obtain the same result for the decoherence time from *exact solutions of the full master equation* where no terms are neglected and no approximations are made. Moreover, this behavior of τ_D is seen for all times, unlike previous estimates of decoherence times in which decoherence was looked for at short times (in the limit of negligible friction). Our results are also consistent with the solutions obtained by Savage and Walls [9] for a harmonic oscillator in an initial superposition of coherent states for the zero-temperature and high-temperature cases. For the measurement model analyzed in this paper, our exact solutions show that the final mixed state density matrix carrying system-apparatus correlations is independent of the initial state of the apparatus. From Eqs. (30) and (33) it is clear that the decoherence time for a high-temperature heat bath is much shorter than that for the zero-temperature bath ($\tau'_D/\tau_D \sim \hbar\omega/K_BT$) and decreases with an increase in temperature of the bath. Similar features have been seen by Kim and Buzek in their study of the influence of a heat bath on superposition states of light in a microwave cavity [25].

It is interesting to note that the decoherence times (30) and (33) are directly proportional to the mass and frequency of the apparatus (oscillator) and are longer for heavier oscillators with higher frequencies. Of course, correspondingly, the ‘‘separation’’ between the two ‘‘pointers’’ positions, $2\epsilon/m\omega^2$, will be smaller and hence their superpositions would decohere slower. Thus, the ‘‘bigger’’ the cat state, the faster the decoherence. In the experiment of Brune *et al.* [11], an entangled state of the atom plus field (‘‘meter’’) is formed and its progressive decoherence to a statistical mixture is monitored. However, since the fields eventually relax towards vacuum, in their system the one-to-one correlation between the atom and meter states is eventually lost. The experimental study of decoherence is thus confined to extremely short-time scales which are much smaller than the cavity relaxation times [11]. It is interesting to note that in a physical realization of the measurement model discussed in this paper, the system-apparatus correlations will persist for all times for both the zero-temperature and high-temperature cases as is evident from Eqs. (26) and (32). The ‘‘permanence’’ of these system-apparatus correlations, therefore, makes this model very interesting to investigate experimentally from the point of view of quantum measurement and decoherence.

IV. CONCLUSIONS

In this paper, we have investigated a quantum measurement model comprised of a spin-$\frac{1}{2}$ (system), a harmonic oscillator (apparatus), and a bath of oscillators (environment). Our interest has been to look at the exact solutions for the dynamics of the reduced density matrix of the system and apparatus via the Markovian master equation which we have studied for the zero-temperature and high-temperature cases. We show that the coupling of the apparatus to the environment leads to the decoherence of the pure system-apparatus entangled state to a statistical mixture with definite system-apparatus correlations, thus affecting a measurement of the spin state. For both the zero-temperature and high-temperature cases, our exact solutions clearly demonstrate the two main signatures of the decoherence mechanism in a quantum measurement, namely (a) the decoherence time is much smaller than the thermal relaxation time, and (b) the decoherence time is inversely proportional to the square of the ‘‘separation’’ between the two ‘‘pointers’’ that correlate with the system states. Decoherence is much faster in a high-temperature bath compared to the zero-temperature bath. Our exact solutions also clearly show that the final apparatus states with which the system states eventually correlate at long times (the ‘‘pointer states’’) are coherent states for the zero-temperature bath and nearly diagonal position distributions of a generalized coherent state for the high-temperature heat bath. The strength of this model and analysis is that it clearly demonstrates that the emergent pointer basis in a measurement process is independent of the initial states of the apparatus. This fact was intuitively obvious, but has not been shown so far in the limited number of models studied in the literature. Our analysis also highlights the need to consider a measurementlike scenario to address the issue of the emergent pointer basis. The model considered here is fairly generic and our exact solutions make this model an interesting candidate to explore experimentally in the context of decoherence and quantum measurements.

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