Unconditional teleportation of continuous-variable entanglement

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We give a protocol and criteria for demonstrating unconditional teleportation of continuous-variable entanglement (i.e., entanglement swapping). The initial entangled states are produced with squeezed light and linear optics. We show that any nonzero entanglement (any nonzero squeezing) in both of two entanglement sources is sufficient for entanglement swapping to occur. In fact, realization of continuous-variable entanglement swapping is possible using only *two* single-mode squeezed states.

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Quantum teleportation enables reliable transportation of quantum information encoded in nonorthogonal quantum states. It is only possible with entanglement. Teleportation was originally proposed for discrete variables [1] and later also for continuous variables [2,3]. Discrete-variable teleportation has been performed experimentally for single-photon polarization states [4,5]. Continuous-variable teleportation has been realized for coherent states of electromagnetic field modes [6]. But coherent states, although nonorthogonal, are very close to classical states. A real challenge for quantum teleportation is the teleportation of truly nonclassical states like entangled states. This ''entanglement swapping'' was first introduced in the context of single-photon polarization states [7]. It means to entangle two quantum systems that have never directly interacted with each other. With single photons, it has already been demonstrated experimentally [8]. Practical uses of entanglement swapping have been suggested [9–12] and it has also been generalized for multiparticle systems [9]. All these investigations have only referred to discrete-variable systems, namely, two-level systems. We will demonstrate that entanglement swapping can also be realized in continuous-variable systems where the source of entanglement is two-mode squeezed light. In contrast to the scheme of Polkinghorne and Ralph [13] where polarization-entangled states of single photons are teleported using squeezed-state entanglement, in our scheme both entangled states are produced with squeezed light. This enables *unconditional* teleportation of entanglement *without* postselection of ''successful'' events by photon detections. Unconditional teleportation of continuous-variable entanglement has been independently investigated by Tan [14]. We will compare Tan's results with ours at the end.

Due to the finite degree of entanglement arising from squeezed states, the entanglement that emerges from entanglement swapping is never as good as the entanglement of the two initial entanglement sources. However, entanglement swapping as here proposed occurs every inverse bandwidth time and is very efficient (near unit efficiency). The fidelity criterion for coherent-state teleportation [15] will enable us to recognize the entanglement produced from entanglement swapping. The maximum average fidelity achievable using the output of entanglement swapping for teleportation indicates the quality of the entanglement from entanglement swapping. Applying this operational quality criterion for en-

tanglement gives us also a protocol for the experimental verification of entanglement swapping.

For our entanglement swapping scheme (Fig. 1), we need two entangled states of the electromagnetic field: a two-mode squeezed state of mode 1 and mode 2 and a two-mode squeezed state of mode 3 and mode 4. This can be described in the Heisenberg representation by

$$\hat{x}_1 = (e^{+r_1}\hat{x}_1^{(0)} + e^{-r_2}\hat{x}_2^{(0)})/\sqrt{2},$$
$$\hat{p}_1 = (e^{-r_1}\hat{p}_1^{(0)} + e^{+r_2}\hat{p}_2^{(0)})/\sqrt{2},$$
$$\hat{x}_2 = (e^{+r_1}\hat{x}_1^{(0)} - e^{-r_2}\hat{x}_2^{(0)})/\sqrt{2},$$
$$\hat{p}_2 = (e^{-r_1}\hat{p}_1^{(0)} - e^{+r_2}\hat{p}_2^{(0)})/\sqrt{2},$$
$$\hat{x}_3 = (e^{+s_1}\hat{x}_3^{(0)} + e^{-s_2}\hat{x}_4^{(0)})/\sqrt{2}, \quad (1)$$
$$\hat{p}_3 = (e^{-s_1}\hat{p}_3^{(0)} + e^{+s_2}\hat{p}_4^{(0)})/\sqrt{2},$$
$$\hat{x}_4 = (e^{+s_1}\hat{x}_3^{(0)} - e^{-s_2}\hat{x}_4^{(0)})/\sqrt{2},$$
$$\hat{p}_4 = (e^{-s_1}\hat{p}_3^{(0)} - e^{+s_2}\hat{p}_4^{(0)})/\sqrt{2},$$

where a superscript (0) denotes initial vacuum modes. The operators $\hat{x}$ and $\hat{p}$ represent the electric quadrature amplitudes (the real and imaginary parts of the mode's annihilation operator). These two-mode squeezed vacuum states can be generated either directly as the output of a nondegenerate
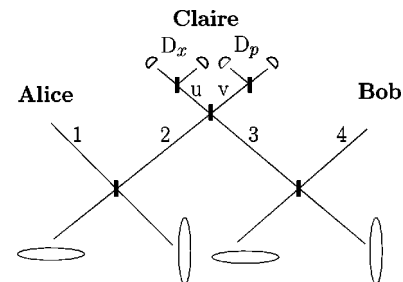

FIG. 1. Entanglement swapping using four squeezed vacuum states. Before the detections, mode 1 is entangled with mode 2 and mode 3 is entangled with mode 4.

optical parametric amplifier [16] or by combining two squeezed vacuum modes at a beam splitter (see Fig. 1). But note that the two independently squeezed single-mode states combined at a beam splitter to create entanglement need not be equally squeezed. In fact, even one single-mode squeezed state superimposed with vacuum yields an entangled two-mode state [17,18], enabling quantum teleportation [19]. Thus, we use four different squeezing parameters r_1, r_2, s_1, and s_2.

Let us introduce ''Alice,'' ''Bob,'' and ''Claire'' to illustrate the whole protocol with entanglement swapping and subsequent teleportation of a coherent state. Alice and Claire shall share the entangled state of mode 1 and 2 while Claire and Bob are sharing the other entangled state of mode 3 and 4 (Fig. 1). Thus, initially Alice and Bob do not share an entangled state. Now Alice wants to teleport an unknown coherent state to Bob and asks Claire for her assistance. Claire combines mode 2 and mode 3 at a beam splitter and detects the quadratures $\hat{x}_u=(\hat{x}_2-\hat{x}_3)/\sqrt{2}$, $\hat{p}_v=(\hat{p}_2+\hat{p}_3)/\sqrt{2}$. Let us write Bob's mode 4 as

$$\hat{x}_4=\hat{x}_2-(\hat{x}_3-\hat{x}_4)-\sqrt{2}\hat{x}_u,$$
$$\hat{p}_4=\hat{p}_2+(\hat{p}_3+\hat{p}_4)-\sqrt{2}\hat{p}_v, \quad (2)$$

and Alice's mode 1 as

$$\hat{x}_1=\hat{x}_3+(\hat{x}_1-\hat{x}_2)+\sqrt{2}\hat{x}_u,$$
$$\hat{p}_1=\hat{p}_3+(\hat{p}_1+\hat{p}_2)-\sqrt{2}\hat{p}_v. \quad (3)$$

Claire's detection yields classical results x_u and p_v. Bob's mode 4 in Eqs. (2) and Alice's mode 1 in Eqs. (3) then become

$$\hat{x}_4=\hat{x}_2-\sqrt{2}e^{-s_2}\hat{x}_4^{(0)}-\sqrt{2}x_u,$$
$$\hat{p}_4=\hat{p}_2+\sqrt{2}e^{-s_1}\hat{p}_3^{(0)}-\sqrt{2}p_v,$$
$$\hat{x}_1=\hat{x}_3+\sqrt{2}e^{-r_2}\hat{x}_2^{(0)}+\sqrt{2}x_u,$$
$$\hat{p}_1=\hat{p}_3+\sqrt{2}e^{-r_1}\hat{p}_1^{(0)}-\sqrt{2}p_v. \quad (4)$$

For $s_1=s_2=s\to\infty$, the quadrature operators of mode 4 become those of mode 2 up to a (random) classical phase-space displacement. In every single projection, mode 4 gets entangled with mode 1 as mode 2 has been before. For $r_1=r_2=r\to\infty$, the quadrature operators of mode 1 become those of mode 3 up to a (random) classical phase-space displacement. In every single projection, mode 1 gets entangled with mode 4 as mode 3 has been before. Mode 2 is perfectly teleported to mode 4 ($s\to\infty$) or mode 3 is perfectly teleported to mode 1 ($r\to\infty$) apart from local classical displacements. The entanglement of one of the initial two-mode squeezed states is completely preserved for infinite squeezing in the other two-mode squeezed state. But also for any nonzero squeezing in both initial entanglement sources, Claire's detection of mode 2 and 3 projects mode 1 and 4 on

inseparable states [20]. However, we will see that Alice and Bob cannot use mode 1 and 4 for subsequent quantum teleportation without information about Claire's measurement results. At least one of them, Alice or Bob, has to receive from Claire the information that the detection of mode 2 and 3 has been performed and its results. Finally, the entanglement of the single projections is ''unwrapped'' via appropriate displacements. Let us assume Bob obtains the classical results from Claire. Now Bob can displace mode 4 as $\hat{x}_4\to\hat{x}_4'=\hat{x}_4+g_{\text{swap}}\sqrt{2}x_u$, $\hat{p}_4\to\hat{p}_4'=\hat{p}_4+g_{\text{swap}}\sqrt{2}p_v$. The parameter g_{swap} describes a normalized gain. Bob's mode then becomes

$$\hat{x}_4'=\frac{g_{\text{swap}}}{\sqrt{2}}e^{+r_1}\hat{x}_1^{(0)}-\frac{g_{\text{swap}}}{\sqrt{2}}e^{-r_2}\hat{x}_2^{(0)}-\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+s_1}\hat{x}_3^{(0)}$$
$$-\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-s_2}\hat{x}_4^{(0)},$$

$$\hat{p}_4'=\frac{g_{\text{swap}}}{\sqrt{2}}e^{-r_1}\hat{p}_1^{(0)}-\frac{g_{\text{swap}}}{\sqrt{2}}e^{+r_2}\hat{p}_2^{(0)}+\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-s_1}\hat{p}_3^{(0)}$$
$$+\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+s_2}\hat{p}_4^{(0)}. \quad (5)$$

As in ''usual'' teleportation, Alice now couples the unknown input state she wants to teleport to Bob (described by $\hat{x}_{\text{in}}$, $\hat{p}_{\text{in}}$) with her mode 1 at a beam splitter and measures the superpositions $\hat{x}_u'=(\hat{x}_{\text{in}}-\hat{x}_1)/\sqrt{2}$, $\hat{p}_v'=(\hat{p}_{\text{in}}+\hat{p}_1)/\sqrt{2}$. Based on the classical results sent to him from Alice, Bob displaces his ''new'' mode 4', $\hat{x}_4'\to\hat{x}_{\text{tel}}=\hat{x}_4'+g\sqrt{2}x_u'$, $\hat{p}_4'\to\hat{p}_{\text{tel}}=\hat{p}_4'+g\sqrt{2}p_v'$, with the gain g. For $g=1$ and nonunit detector efficiencies, Bob's outgoing mode then becomes

$$\hat{x}_{\text{tel}}=\hat{x}_{\text{in}}+\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+r_1}\hat{x}_1^{(0)}-\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-r_2}\hat{x}_2^{(0)}$$
$$-\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+s_1}\hat{x}_3^{(0)}-\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-s_2}\hat{x}_4^{(0)}$$
$$+g_{\text{swap}}\sqrt{\eta_c^{-2}-1}(\hat{x}_d^{(0)}+\hat{x}_e^{(0)})$$
$$+\sqrt{\eta_a^{-2}-1}(\hat{x}_f^{(0)}+\hat{x}_g^{(0)}),$$

$$\hat{p}_{\text{tel}}=\hat{p}_{\text{in}}+\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-r_1}\hat{p}_1^{(0)}-\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+r_2}\hat{p}_2^{(0)}$$
$$+\frac{g_{\text{swap}}+1}{\sqrt{2}}e^{-s_1}\hat{p}_3^{(0)}+\frac{g_{\text{swap}}-1}{\sqrt{2}}e^{+s_2}\hat{p}_4^{(0)}$$
$$+g_{\text{swap}}\sqrt{\eta_c^{-2}-1}(\hat{p}_k^{(0)}+\hat{p}_l^{(0)})+\sqrt{\eta_a^{-2}-1}(\hat{p}_m^{(0)}+\hat{p}_n^{(0)}). \quad (6)$$

The parameters η_c and η_a describe detector efficiencies in Claire's and Alice's detections, respectively. Note that, for

$g_{swap} = 1$, Bob's teleported mode from Eqs. (6) is the same as if Alice teleports her input state to Claire with unit gain using the entangled state of mode 1 and 2, and Claire teleports the resulting output state to Bob with unit gain using the entangled state of mode 3 and 4.

The teleportation fidelity for a coherent state input α_{in}, defined as $F \equiv \langle \alpha_{in} | \hat{\rho}_{tel} | \alpha_{in} \rangle = \pi Q_{tel}(\alpha_{in})$ [15], is

$$F = \frac{1}{2\sqrt{\sigma_x \sigma_p}} \exp\left[-(1-g)^2\left(\frac{x_{in}^2}{2\sigma_x} + \frac{p_{in}^2}{2\sigma_p}\right)\right], \quad (7)$$

where σ_x and σ_p are the variances of the Q function of the teleported mode for the corresponding quadratures. Using Eqs. (6), the fidelity becomes for $g = 1$,

$$\begin{aligned}
F = [&1 + (g_{swap} - 1)^2(e^{+2r_1} + e^{+2s_1})/4 \\
&+ (g_{swap} + 1)^2(e^{-2r_2} + e^{-2s_2})/4 \\
&+ g_{swap}^2(\eta_c^{-2} - 1) + \eta_a^{-2} - 1]^{-1/2} \\
\times [&1 + (g_{swap} - 1)^2(e^{+2r_2} + e^{+2s_2})/4 \\
&+ (g_{swap} + 1)^2(e^{-2r_1} + e^{-2s_1})/4 \\
&+ g_{swap}^2(\eta_c^{-2} - 1) + \eta_a^{-2} - 1]^{-1/2}.
\end{aligned} \quad (8)$$

For unknown coherent input states, an (average) fidelity $F > \frac{1}{2}$ is only achievable using entanglement [15]. Thus, if for some g_{swap} (for some local operation on mode 4 by Bob based on Claire's results) $F > \frac{1}{2}$, entanglement swapping must have taken place. Otherwise Alice and Bob, who initially did not share entanglement, were not able to beat the classical fidelity limit using mode 1 and 4. The assumption $g = 1$ is the optimal choice for Bob's local operation based on Alice's results.

Let us first consider four equally squeezed states $r_1 = r_2 = s_1 = s_2 = r$. In this case with unit efficiency ($\eta_c = \eta_a = 1$), the fidelity is optimized for $g_{swap} = \tanh 2r$ ($g = 1$) and becomes $F_{opt} = (1 + 1/\cosh 2r)^{-1}$. For any $r > 0$, we obtain $F_{opt} > \frac{1}{2}$. For $\eta_c \neq 1$ and $\eta_a \neq 1$, the optimum gain is $g_{swap} = \sinh 2r/(\cosh 2r + \eta_c^{-2} - 1)$. For the more general case $r_1 = r_2 = r$ and $s_1 = s_2 = s$, we find the optimum gain

$$g_{swap} = \frac{\sinh 2r + \sinh 2s}{\cosh 2r + \cosh 2s + 2\eta_c^{-2} - 2}. \quad (9)$$

Using this gain we obtain the optimum fidelity with unit efficiency

$$F_{opt} = \left\{1 + \frac{\cosh[2(r-s)] + 1}{\cosh 2r + \cosh 2s}\right\}^{-1}. \quad (10)$$

This fidelity is equal to $\frac{1}{2}$ and never exceeds the classical limit if either $r = 0$ or $s = 0$. The reduced states of mode 1 and 4 after the detection of mode 2 and 3 are separable if either $r = 0$ or $s = 0$ [20]. *Both* initial two-mode states need to be squeezed and hence entangled for entanglement swapping to occur. Then, for any nonzero squeezing $r > 0$ *and s* > 0, we obtain $F_{opt} > \frac{1}{2}$, indicating that entanglement swap-

ping took place. The reduced states of mode 1 and 4 after detecting mode 2 and 3 are inseparable for any $r > 0$ *and s* > 0 [20].

Let us now consider the case where each of the two initial entangled states is generated with only one single-mode squeezed state. We set $r_1 = s_1 = r$ and $r_2 = s_2 = 0$. With unit efficiency, we find the optimum gain $g_{swap} = \tanh r$. The optimum fidelity then becomes

$$\begin{aligned}
F_{opt} = \{&[1 + 2e^{+2r}/(e^{+2r} + 1) + (\tanh r)^2(\eta_c^{-2} - 1) \\
&+ \eta_a^{-2} - 1][1 + 2e^{-2r}/(e^{-2r} + 1) \\
&+ (\tanh r)^2(\eta_c^{-2} - 1) + \eta_a^{-2} - 1]\}^{-1/2}.
\end{aligned} \quad (11)$$

Note that this fidelity can be optimized further for nonunit efficiency, as we have only used the optimum gain for unit efficiency. With unit efficiency ($\eta_c = \eta_a = 1$) this fidelity exceeds the classical limit $F_{opt} > \frac{1}{2}$ for any nonzero squeezing $r_1 = s_1 = r > 0$. Thus, entanglement swapping can be realized even with only two single-mode squeezed states, provided that two initial entangled states are produced. Indeed, the creation of entanglement is possible using only one single-mode squeezed state for any nonzero squeezing [17–19]. Therefore we can generally say that any nonzero entanglement in both of the two initial entanglement sources is sufficient for entanglement swapping to occur. In order to achieve perfect teleportation of arbitrary coherent states with fidelity $F = 1$, four infinitely squeezed states $r_1 = r_2 = s_1 = s_2 = r \to \infty$ are necessary and Bob has to perform a displacement with $g_{swap} = 1$ using Claire's results. It is impossible for Alice and Bob to achieve quantum teleportation of unknown coherent states with $F > \frac{1}{2}$ for $g_{swap} = 0$, i.e., without a local operation based on the results of Claire's detection. The optimum fidelity using mode 1 and 4 after entanglement swapping is worse than the optimum fidelity using the entanglement of the initial modes 1 and 2 or 3 and 4. This indicates that the degree of entanglement after entanglement swapping deteriorates compared to the initial entangled states. In Fig. 2 is shown the comparison between the optimum fidelities of coherent-state teleportation using entangled states produced from entanglement swapping and without swapping.

The maximum fidelity achievable using entanglement produced with one single-mode squeezed state is $F = 1/\sqrt{2}$ [19]. The maximum fidelity achievable using the output of entanglement swapping with two equally squeezed single-mode states is $F = 1/\sqrt{3}$. For 6-dB squeezing and detectors with efficiency $\eta^2 = 0.99$, the optimum fidelity using the output of entanglement swapping with two equally squeezed single-mode states becomes $F = 0.5201$. Squeezing of 10 dB and the same efficiency yields $F = 0.5425$. Here, the gain $g_{swap} = \tanh r$ has been chosen, that is, the optimum gain with two equally squeezed single-mode states for unit efficiency.

Tan proposes continuous-variable entanglement swapping as the teleportation of the signal beam of a parametric amplifier using the entanglement between signal and idler beam of another parametric amplifier [14]. The entanglement of the teleported signal beam with the idler beam in Tan's pro-
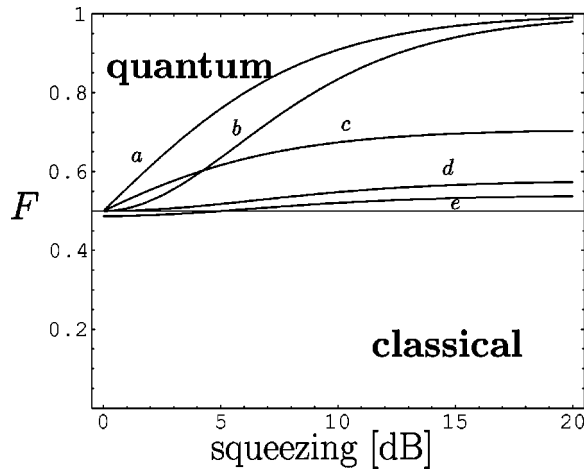
FIG. 2. Optimum fidelity for the teleportation of an arbitrary coherent state ($g=1$) using (a) entanglement produced with two equally squeezed single-mode states, (b) the output of entanglement swapping with four equally squeezed single-mode states, (c) entanglement produced with one single-mode squeezed state, (d) the output of entanglement swapping with two equally squeezed single-mode states, and (e) the state as in (d) with detector efficiencies $\eta_c^2 = \eta_a^2 = 0.95$.

tocol is confirmed by combining them at a beam splitter and detecting the $\hat{x}$ quadrature at one output and the $\hat{p}$ quadrature at the other output: $\hat{x}=(\hat{x}_1-\hat{x}_4)/\sqrt{2}$, $\hat{p}=(\hat{p}_1+\hat{p}_4)/\sqrt{2}$. A violation of the uncertainty relation $\langle\Delta\hat{x}^2\rangle\langle\Delta\hat{p}^2\rangle\geq1/16$ proves the entanglement of mode 1 and 4 [14]. A similar, sufficient inseparability criterion for continuous-variable systems has been given very recently [21] that in the context of our entanglement swapping scheme would require the violation of $\langle(\hat{x}_1-\hat{x}_4)^2\rangle+\langle(\hat{p}_1+\hat{p}_4)^2\rangle\geq1$.

It is obvious that the violation of these inequalities corresponds to a fidelity $F>\frac{1}{2}$ in our subsequent coherent-state teleportation. However, we have directly looked at the separability of the projected states of mode 1 and 4 after the detection of mode 2 and 3 [20]. Since these states are insepa-

rable for any nonzero entanglement in both initial sources, they should enable quantum teleportation, as has been shown. We have demonstrated, that in this case, the reduced state $\hat{\rho}_{14}$ can always be transformed by some local displacement to a state $\hat{\rho}'_{14}$ that provides $F>\frac{1}{2}$ in coherent-state teleportation and has quadratures violating $\langle(\hat{x}_1-\hat{x}'_4)^2\rangle+\langle(\hat{p}_1+\hat{p}'_4)^2\rangle\geq1$. We have given the optimum gain g_{swap} of this displacement for any verification, either by further teleportation or by simple detection [22].

In Tan's scheme [14], unit gain g_{swap} has been chosen. Therefore entanglement is only confirmed if the initial states exceed a certain degree of squeezing (e.g., for equally squeezed states 3 dB). By choosing the right gain, entanglement can be verified for any squeezing, in principle even if the initial entangled states are built from the minimal resource of two single-mode squeezed states.

Detecting the output and applying Tan's (or Duan *et al.*'s) inequality for verification is easy, but it requires bringing the entangled subsystems together and measuring states that are now local. This provides only an indirect confirmation that the entanglement is preserved through teleportation. Our verification leaves the subsystems separate and directly demonstrates the entanglement by exploiting it in a second round of teleportation.

In summary, we have given a protocol for continuous-variable entanglement swapping, i.e., for the unconditional teleportation of entanglement, using squeezed light and linear optics. Entanglement swapping occurs for any nonzero entanglement (any nonzero squeezing) in both of the two initial entanglement sources. *This can be realized even with only two single-mode squeezed states.* Our verification scheme would provide a compelling demonstration of unconditional teleportation of entanglement.

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