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Raman-induced photon correlations in optical fiber solitons

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Recent experimental results on edge-filtered (also referred to as low- and high-pass filters) subpicosecond optical solitons have raised a number of theoretical questions concerning the origin of the reduced photon-number fluctuations. One of the main experimental results was that for long fibers, on the order of 100 soliton periods, the measured photon-number squeezing *increased* with propagation distance in contrast with theoretical calculations for picosecond solitons at that time. Here I present theoretical results on photon statistics of edge-filtered subpicosecond optical solitons. The numerical simulations include coupling to phonons via a Raman interaction, in addition to the electronic Kerr nonlinearity, and are in good agreement with the experimental data over the range of propagation distances investigated so far. The theoretical results clearly show that the phenomena observed are not related to long fiber propagation but to the Raman self-frequency shift and should be more clearly observable using cooled fiber. [S1050-2947(99)51508-4]

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Spectral filtering in one form or another has proved to be an extremely useful and simple method for the manipulation of optical fields. For nonlinear pulse propagation it is a natural method for engineering pulse characteristics and has proved useful in noise reduction in propagating optical solitons [1], such as reducing Gordon-Haus timing jitter [2]. A method for reducing photon-number fluctuations of optical solitons below the shot-noise level using spectral filtering was demonstrated several years ago [3,4]. It took advantage of the internal quantum noise structure [5] of soliton pulses to reduce the noise below shot noise.

It was shown in the first theoretical prediction [3] that the method did not simply remove excess noise nor did photon-number squeezing arise from only sub-shot-noise variances of individual frequency components. In fact, it was stated in [3] that intermode correlations were important for the following reason. The optimal bandpass filter corresponded to only a partial removal of outer noisy frequencies. It was found that for bandpass filtering an ideal coherent fundamental soliton the optimal cutoff frequency corresponded to the peak of the electronic nonlinearity induced spectral intensity noise sidebands. The only way that can occur is for some remaining frequencies to be correlated with other unfiltered spectral components to reduce the integrated spectral noise (i.e., anticorrelated). The fact that the photon-number noise increased when the filter was further narrowed, even though those frequencies that were additionally removed had variances that were above shot noise, was proof that intermode correlations were reducing the photon-number variance of the filtered pulse. This was made obvious by describing both the noise reduction variation with filter bandwidth along with the intensity spectral noise variance [3,5]. Such correlations were later experimentally measured [6]. Of course, in a nonlinear process frequency correlations are trivial, but what is interesting in this case is the way they are correlated.

The Raman effect changes the balance between correlated spectral components in two ways. First, there is a Raman amplification process that converts photons at frequency ω to $\omega \pm \Omega$ (predominantly ω to $\omega - \Omega$ at or below room temperature), whereby a phonon at frequency Ω is involved. This is a three-wave-mixing process, but now only two of the bosons are detected. Essentially there is a filtering process where the spectral filter is given by the response function of the media. Second, there is a colored noise associated with this frequency-dependent response function. For the fast electronic nonlinearity all participating photons are in principle detectable. This is one reason why self-interacting fields allow photon correlations to more easily produce optical fields with noise statistics measured substantially below shot noise. But there is a price to pay for ideal symmetrical noise generated by the electronic nonlinearity. When we consider initial propagation of a coherent fundamental soliton the noise is not in the far wings of the intensity spectrum, it is ''in-band'' noise that makes it difficult to remove without disturbing the signal. It would seem reasonable to argue that any physical process that conserves photon-number and moved noise within the main signal bandwidth out of band could allow larger squeezing than the ideal case. The new balance between removing correlated noise and leaving the anticorrelated noise would depend on the relative contributions from the electronic nonlinearity and the candidate physical process. In silica fiber a candidate process is the Raman effect, which can be effectively turned on simply by using subpicosecond pulses.

The experiments were performed using 130-fs (full width at half maximum) pulses [7] to investigate the effects of the Raman process and its associated quantum noise in optical fibers. High-pass filtered pulses showed greater noise reduction after propagating on the order of 100 soliton periods compared to a few soliton periods. Low-pass filtered pulses showed at best shot-noise level fluctuations for long fibers and similar noise levels (although smaller squeezing) than high-pass filtered pulses for short fibers less than about three soliton periods. Qualitatively the experimental results are consistent with the Raman downshifted spectral components exhibiting excess noise with reduced anticorrelation, while the higher-frequency components remain anticorrelated. The noise correlation balance must depend on the particular Raman contribution to the Kerr nonlinearity of silica fiber, which here is taken to be approximately 18%. An interesting question is whether a medium that exhibits an almost purely Raman interaction and little instantaneous Kerr nonlinearity would exhibit larger squeezing with an asymmetric filter than a purely electronic nonlinearity with a symmetric filter. Previous work on the generation of single-cycle classical radiation using Raman scattering of femtosecond pulses [8] suggests that a quantum theory for media with strong Raman interactions on femtosecond time scales will take on a different form than the one presented here. A more general quantum theory for femtosecond time scales has been developed and will be described elsewhere [9].

By comparing the numerical results for solitons experiencing no Raman effect with those that do in the subpicosecond pulse regime used in the experiments, it is shown in this paper that the Raman effect can indeed *increase* the squeezing relative to the non-Raman case using an edge filter for certain propagation distances, even after propagating only a few soliton periods. This is in contrast to previous theoretical results for picosecond pulses where for short propagation distances there was no evidence of Raman effects increasing the squeezing. The predicted form of the evolution of the photon-number variance is consistent with the experimental observations. Moreover, at shorter propagation distances, where the experimental data published so far shows a definite oscillatory structure, good quantitative agreement is found. An asymptotic soliton perturbation theory [10] is much better suited to answer questions about the long fiber results of order 100 soliton periods. However, the results presented here indicate that new insight into the optimal noise reduction from Raman effects in soliton photon-number squeezing is interesting, not only for long fibers, but also for other physical systems for which the ratio of the electronic to the Raman contribution of the nonlinearity is smaller.

The Raman-modified stochastic nonlinear Schrödinger equation for the normalized photon flux field $\phi(\xi,\tau)$ is given by [11]

$$\frac{\partial\phi}{\partial\xi}=-\frac{i}{2}\left[1\pm\frac{\partial^2}{\partial\tau^2}\right]\phi+if\phi^\dagger\phi^2+\sqrt{i}\phi\Gamma_e + i\phi\left[\int_{-\infty}^{\tau}d\tau' h(\tau-\tau')\phi^\dagger(\tau')\phi(\tau')+\Gamma_v\right], \quad (1)$$

where the length and time variables (ξ,τ) in the comoving frame at speed ω' (group velocity at the carrier frequency) in the laboratory frame (x,t) are $\tau=(t-x/\omega')/t_0$, $\xi=x/x_0$, $x_0=t_0^2/|k''|$. The characteristic time scale t_0 will be chosen to be the pulse width later, and the soliton period is $\pi/2$ times longer than the dispersion length x_0 determined by t_0 and the second-order dispersion k''. The Raman noise correlations are given by the autocorrelation

$$\langle\Gamma_v(\xi,\omega)\Gamma_v(\xi',\omega')\rangle=\frac{1}{\bar{n}}\delta(\xi-\xi')\delta(\omega+\omega')\{-i\sqrt{2\pi}h(\omega) + [n_{th}(|\omega|)+\theta(-\omega)]\alpha(|\omega|)\}, \quad (2)$$

where n_{th} is the thermal phonon occupation number, $\theta(\omega)$ is a step function with value unity for positive arguments,

$$\sqrt{2\pi}h(\omega)=\int_0^\infty d\Omega\frac{\alpha(\Omega)\Omega}{\pi(\Omega^2-\omega^2)}+i\frac{\mathrm{sgn}(\omega)}{2}\alpha(|\omega|), \quad (3)$$

and the cross-correlation

$$\langle\Gamma_v(\xi,\omega)\Gamma_v^\dagger(\xi',\omega')\rangle=\frac{1}{\bar{n}}\delta(\xi-\xi')\delta(\omega+\omega') \times[n_{th}(|\omega|)+\theta(-\omega)]\alpha(|\omega|). \quad (4)$$

The quantum noise from the electronic nonlinearity Γ_e is a real δ-correlated Gaussian noise with variance given by the product of the electronic fraction f and inverse photon-number scale $1/\bar{n}=\eta_0 t_0/k''\omega'^2$, where η_0 is the nonlinearity over the time-scale of interest. The parameter $\alpha(\omega)$ is the Raman gain that has a peak near 13 THz for silica fiber. Raman inhomogeneous model parameters used here correspond to the Raman gain curve in Ref. [11]. The techniques to study the propagation of the quantum field equations are discussed elsewhere [12]. Error bars in the plots represent the estimated combined sampling and step-size error. By using a normally ordered representation, no additional noise sources need be added to compensate for the effect of the spectral filter $f(\omega)$ on vacuum fluctuations. In the positive-P representation, the photon number is taken to be

$$\langle n\rangle=\bar{n}\int d\omega f^*(-\omega)f(\omega)\langle\phi^\dagger(-\omega)\phi(\omega)\rangle. \quad (5)$$

This paper considers the effect of the simplest asymmetric filters—high-pass and low-pass frequency filters.

The experimental parameters described in Ref. [7] that are relevant to the discussion here are $N=0.981$ and $t_0=73$ fs, using coherent input pulses with an $N\mathrm{sech}(\tau)$ amplitude profile containing $2\bar{n}=4.1\times10^8$ photons at $\lambda=1503.5$ nm. Using these parameters to set the time scale and photon-number scale of the system, the optimized photon-number squeezing versus the propagation distance is calculated by varying the cutoff frequency of an edge filter. The length scale is determined by the anomalous group-velocity dispersion at λ. Any corrections due to higher-order effects, such as third-order dispersion or frequency-dependent electronic nonlinearity, have not been included here for simplicity. The results given in Fig. 1 correspond to inclusion of Raman effects at $T=4.2$ K and with no Raman effects. An important effect shown in Fig. 1 is that after the first minimum in the oscillation the Raman effect causes the next peak to be truncated $t=300$ K. This is clearly seen in the experimental data [7]. In the ideal case the first minimum is at about three soliton periods ($\xi=3\pi/2$) and the period of the oscillation is eight soliton periods ($\xi_{period}=4\pi$). The cooled Raman case clearly exhibits an *increase* in squeezing relative to the ideal case. The general trend after ten dispersion lengths is toward increased squeezing. The result for applying a low-pass filter is also given in Fig. 1. At ten dispersion lengths, the photon-number variances of the low-pass filtered pulse approached shot-noise level and displayed small departures below shot-noise thereafter, but were not statistically significant in these

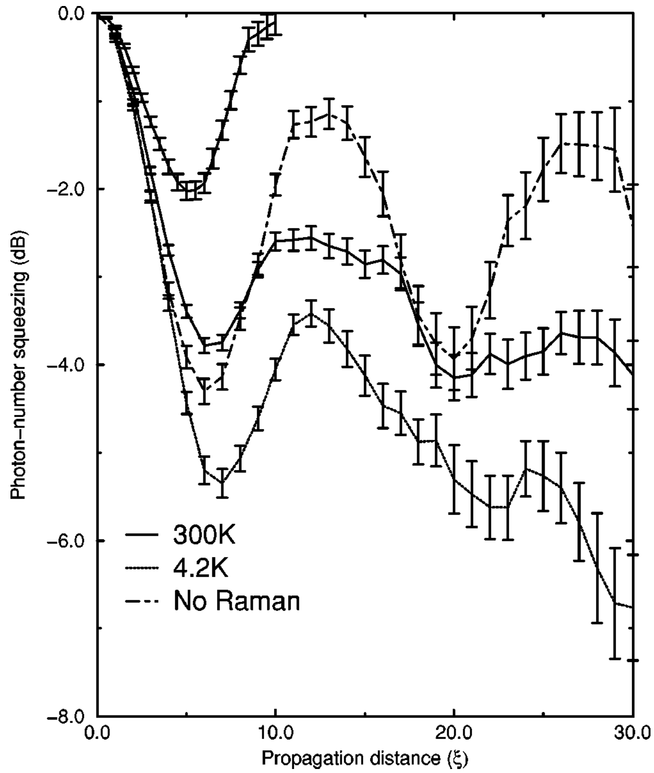

FIG. 1. Variation of the photon-number squeezing (dB) for optimal noise reduction using the high-frequency pass and low-frequency pass edge filters versus propagation distance in units of dispersion length with and without the Raman effect ($t_0 = 73$ fs, $T = 0$ K, 4.2 K, 300 K). Upper solid line is for a low-frequency pass edge filter at $T = 300$ K and for $\xi > 10$ remaining near the shot-noise level within error estimates. Lower solid line and other simulation data are for high-frequency pass edge filters.

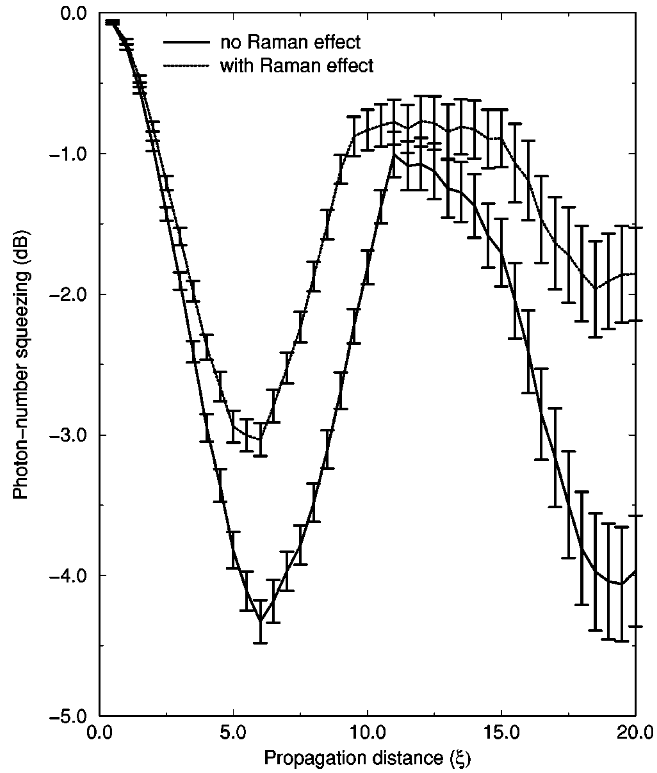

FIG. 2. Variation of the photon-number squeezing (dB) for optimal noise reduction using the high-frequency pass edge filter versus propagation distance in units of dispersion length with and without Raman effect ($t_0 = 1$ ps, $T = 0$ K, 300 K).

simulations. The results for the Raman affected pulse are remarkably similar to the experimental data of Spälter *et al.* [7].

A feature exhibited in Fig. 1 that should be noted is the squeezing after ten dispersion lengths for the non-Raman case. Upon inspection of the cutoff frequency for optimized noise reduction, one finds there is a sharp transition near this point towards a lower cutoff frequency. The non-Raman affected pulse requires a higher cutoff frequency than the Raman affected pulse after that point as the propagation distance increases further. One anticipates that the optimized cutoff frequency oscillates in the ideal case. However, the Raman affected pulse exhibits just the opposite trend in the optimized cutoff frequency as the propagation distance increases further. This is consistent with the Raman self-frequency shift; however, the frequency shift itself is still small after ten dispersion lengths, whereas the quantum noise shows a significant change in its spectral intensity variances, which is highly asymmetric, as expected [13] and measured [14]. As discussed earlier, one can show by the same reasoning as was done for a fundamental soliton not experiencing a self-frequency shift [3] that quantum intermode correlations are important for below shot-noise photon-number statistics, as has been discussed in [6]. It is even clearer in this case than in [3], since there is no spectral intensity variances below shot noise at room temperature after $\xi \sim 5.5$ and, for example, at the local minima in Fig. 1 at $\xi \sim 6$, the optimal cutoff frequency corresponds to the lower frequency peak in the spectral intensity variance. The auto-correlations and cross correlations in the spectral components will be presented elsewhere [9].

The Raman noise always appeared to decrease the squeezing in the picosecond pulse regime, as discussed earlier. In the theoretical studies that have included Raman effects carried out so far [3,5], the Raman self-frequency shift was negligible and the filters were symmetric about the initial pulse central frequency. Now, one is led to the interesting conclusion that either (i) Raman noise alone increases the photon-number noise while the Raman self-frequency shift allows this noise penalty to be overcome with the aid of the electronic nonlinearity or (ii) the Raman effect also allows an increase in the squeezing of picosecond pulses, provided the symmetric bandpass filter is replaced with edge filters.

We can now rule out (ii), since an explicit calculation has been carried out using edge filters for picosecond pulses. The optimized edge filter for propagation including the Raman effect always displays a smaller noise reduction with respect to the ideal case using $t_0 = 1$ ps at $T = 300$ K. Figure 2 shows the photon-number squeezing (dB) versus propagation distance for the latter case with and without the Raman effect. To decrease the Raman noise, one can cool the fiber, as was achieved in the original coherent quantum soliton experiments of Rosenbluh and Shelby [15]. Results comparing a cooled fiber at $T = 4.2$ K with a room-temperature fiber at $T = 300$ K and the non-Raman case with a purely electronic nonlinearity are shown in Fig. 1. They have a number of interesting features, some of which appear in the experimental data at room temperature. It is clear in Fig. 1 that the Raman noise causes the truncation of the oscillation near ten dispersion lengths, since the cooled fiber shows a clear oscillatory peak. At around 25 dispersion lengths the non-Raman case reaches a peak in its oscillatory photon-number variance, and at the same propagation distance the cooled fiber has an order-of-magnitude larger noise reduction. At 30 dispersion lengths the cooled fiber reaches an estimated squeezing of -6.8 ± 0.6 dB, which is comparable to the inferred squeezing found for long fibers in the experiments of -6.4 ± 0.8 dB [7].

All the qualitative features of the quantum noise properties of edge-filtered subpicosecond optical pulses found in the experiments of Spälter *et al.* [7] are consistent with the Raman-modified quantum nonlinear Schrödinger equation. There does not appear to be any need to modify the existing theoretical description to explain the experimental results. The results presented here indicate that the experimental data for fibers up to about 20 soliton periods are accurately described by the theory of Carter and Drummond for the Raman-modified quantum nonlinear Schrödinger equation [11]. For longer propagation distances the numerical results using the positive-P representation are currently not accurate enough to allow any comparison between theory and experiment. It is possible that introducing a stochastic differencing procedure [12] could enable longer propagation distances to be studied. However, a linearized quantum theory for this problem has been specified in the Heisenberg representation [16] and may be studied using other methods for long fibers [10].

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