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Transparency near a photonic band edge

E. Paspalakis, N. J. Kylstra, and P. L. Knight
Optics Section, Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom

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We study the absorption and dispersion properties of a Λ-type atom that decays spontaneously near the edge of a photonic band gap (PBG). Using an isotropic PBG model, we show that the atom can become transparent to a probe laser field, even when other dissipative channels are present. This transparency originates from the square-root singularity of the density of modes of the PBG material at threshold. [S1050-2947(99)50607-0]

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The study of quantum and nonlinear optical phenomena in atoms (impurities) embedded in photonic band gap (PBG) materials has attracted much attention recently. Many interesting effects have been predicted when radiative transitions of the atoms are near resonant with the edge of a PBG. As examples we mention the localization of light and the formation of ''photon-atom bound states'' [1–3], suppression, and even complete cancellation of spontaneous emission [4–7], population trapping in two-atom systems [7], phase-dependent behavior of the population dynamics [8], enhancement of spontaneous emission interference [9] and other phenomena [10,11]. In addition there is also current interest with regard to quantum nondemolition measurements in modified reservoirs, such as the PBG [12,13]. We note that there is a formal similarity between the models used in the above studies and those of near threshold photoionization and photodetachment [14,15].

In this Rapid Communication we study the probe absorption spectrum of a Λ-type system, similar to the one used in previous studies [8,10], with one of the atomic transitions decaying spontaneously near the edge of a PBG. We show that the atom becomes transparent to a probe laser field which couples to the second atomic transition. This transparency occurs even in the presence of the background decay of the upper atomic level. This effect is closely related to the phenomenon of electromagnetically induced transparency (EIT) which occurs, for example, in three level atoms driven by two laser fields [16,17] and with phenomena where intrinsic transparency occurs via decay interference [18–20].

The atomic system under consideration is shown in Fig. 1(a). It consists of three atomic levels, labeled $|n\rangle$, $(n=0,1,2)$, with $\omega_0<\omega_2<\omega_1$, where ω_n denotes the energy of each atomic state. The atom is assumed to be initially in state $|0\rangle$. The transition $|1\rangle\leftrightarrow|2\rangle$ is taken to be near resonant with a photonic band edge, while the transition $|0\rangle\leftrightarrow|1\rangle$ is assumed to be far away from the gap and can therefore be treated as occurring in free space. The Hamiltonian, which describes the dynamics of this system, in the interaction picture and the rotating wave approximation, is given by $(\hbar=1)$,

$$H=\left[\Omega e^{i\delta t}|0\rangle\langle 1|+\sum_{\mathbf{k},\lambda} g_{\mathbf{k},\lambda}e^{-i(\omega_{\mathbf{k}}-\omega_{12})t}|1\rangle\langle 2|\alpha_{\mathbf{k},\lambda}+\text{H.c.}\right] -i\frac{\gamma}{2}|1\rangle\langle 1|. \tag{1}$$

Here, $\Omega=-\boldsymbol{\mu}_{01}\cdot\boldsymbol{\epsilon}E$ is the Rabi frequency and $\delta=\omega-\omega_{10}$, with $\omega_{nm}=\omega_n-\omega_m$, is the laser detuning from resonance of the $|0\rangle\leftrightarrow|1\rangle$ transition. In addition, $g_{\mathbf{k},\lambda}=-i\sqrt{2\pi\omega_{\mathbf{k}}/V}\boldsymbol{\epsilon}_{\mathbf{k},\lambda}\cdot\boldsymbol{\mu}_{12}$ denotes the coupling of the atom with the modified vacuum modes. Both the Rabi frequency and the atom-vacuum coupling strength are taken to be real. The dipole matrix element of the $|n\rangle\leftrightarrow|m\rangle$ transition is denoted by $\boldsymbol{\mu}_{nm}$. Also, $\boldsymbol{\epsilon}$ and E are respectively the polarization unit vector and electric-field amplitude of the laser field, while $\boldsymbol{\epsilon}_{\mathbf{k},\lambda}$ is the polarization unit vector, $\alpha_{\mathbf{k},\lambda}$ is the photon annihilation operator, $\omega_{\mathbf{k}}$ is the angular frequency of the $\{\mathbf{k},\lambda\}$ mode of the quantized vacuum field, and V is the quan-

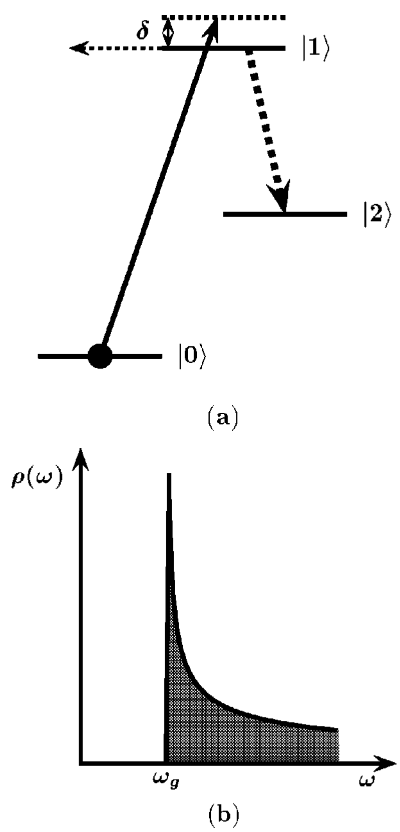

FIG. 1. Figure (a) displays a three level, Λ-type atomic system. The solid line denotes the probe laser coupling, the thick dashed line denotes the coupling to the modified reservoir (PBG) and, finally, the thin dashed line denotes the background decay. Figure (b) shows the density of modes for the case of the isotropic PBG model.

tization volume. Finally, γ denotes the background decay rate from state $|1\rangle$ to all other states of the atom. It is assumed that these states are situated far from the gap so that such background decay can be treated as a Markovian process. The radiative shift associated with this decay has been omitted. We note that, as long as the laser field is sufficiently weak, γ can also account for the radiative decay of state $|1\rangle$ to state $|0\rangle$.

We proceed by expanding the wave function of the system, at a specific time t, in terms of the "bare" state vectors, such that

$$|\psi(t)\rangle = a_0(t)|0,\{0\}\rangle + a_1(t)e^{-i\delta t}|1,\{0\}\rangle + \sum_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda}(t)|2,\{\mathbf{k},\lambda\}\rangle. \tag{2}$$

Substituting Eqs. (1) and (2) into the time-dependent Schrödinger equation and eliminating the vacuum amplitude $a_{\mathbf{k},\lambda}(t)$, we obtain

$$i\dot{a}_0(t) = \Omega a_1(t), \tag{3}$$

$$i\dot{a}_1(t) = \Omega a_0(t) - \left(\delta + i\frac{\gamma}{2}\right)a_1(t) - i\int_0^t dt' K(t-t')a_1(t'), \tag{4}$$

with the kernel

$$K(t-t') = \sum_{\mathbf{k},\lambda} g_{\mathbf{k},\lambda}^2 e^{-i(\omega_{\mathbf{k}}-\omega_{12}-\delta)(t-t')}. \tag{5}$$

For the case of a Markovian reservoir, $K(t-t') = (\gamma_1/2)\delta(t-t')$ with γ_1 being the decay rate to the state $|2\rangle$. However, for the case of an *isotropic* model of the PBG which we consider here, an effective mass dispersion relation [5,7,11] $\omega_{\mathbf{k}} = \omega_g + A(|\mathbf{k}| - |\mathbf{k}_0|)^2$, with $A \approx \omega_g/|\mathbf{k}_0|^2$ is used, so that one obtains for the kernel

$$K(t-t') = \frac{\beta^{3/2} e^{-i[\pi/4 + (\delta_g - \delta)(t-t')]}}{\sqrt{\pi(t-t')}}, \quad t > t', \tag{6}$$

with $\beta^{3/2} = 2\omega_{12}^{7/2}|\boldsymbol{\mu}_{12}|^2/(3c^3)$ and $\delta_g = \omega_g - \omega_{12}$. The isotropic dispersion relation leads to an inverse square root density of modes for the modified reservoir $\rho(\omega) \sim \Theta(\omega - \omega_g)/\sqrt{\omega - \omega_g}$, with Θ being the Heaviside step function [see Fig. 1(b)]. We note that a similar density of modes is also found in waveguides [21] and in microcavities [22], so that our results apply to these cases as well.

The aim here is to investigate the absorption and dispersion properties of our system for a *weak* probe laser field. The equation of motion for the electric-field amplitude $E(z,t)$ is given by [23]

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t}\right)E(z,t) = -i\frac{\omega}{2c}\chi(\delta)E(z,t), \tag{7}$$

where $\chi(\delta)$ is the steady-state linear susceptibility of the medium and $v_g = c/[1 + (\omega/2)(\partial\,\mathrm{Re}(\chi)/\partial\omega)]$ is the group velocity of the laser pulse with the derivative of the real part of the susceptibility being evaluated at the carrier frequency.

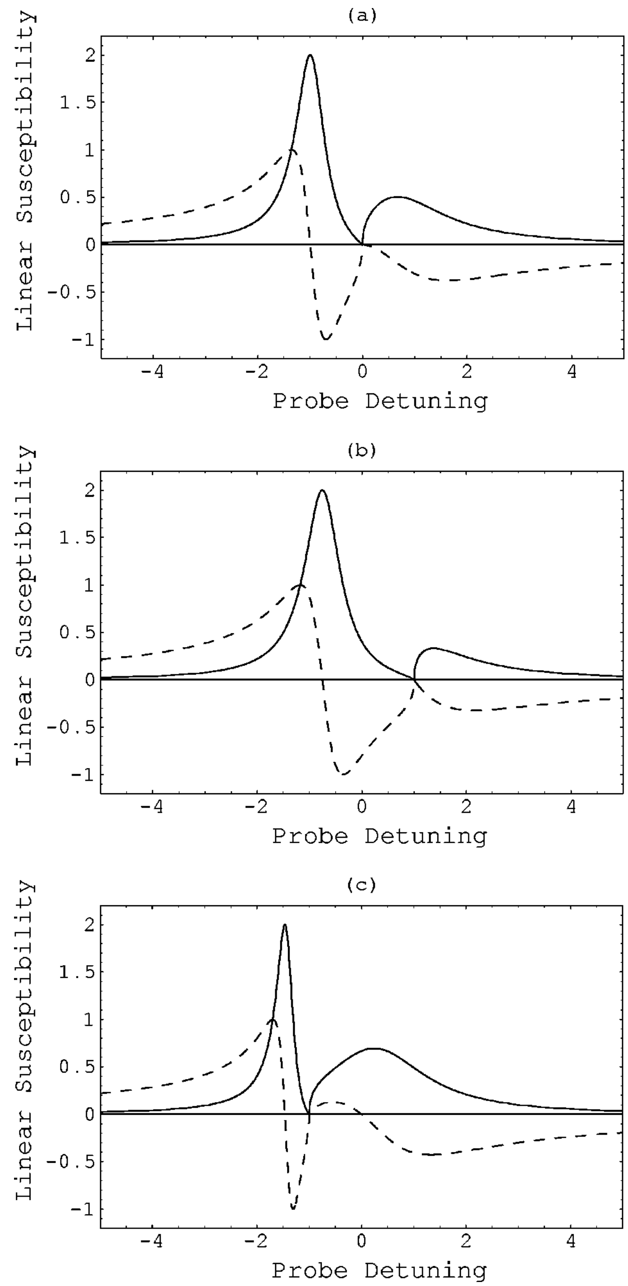

FIG. 2. The absorption and dispersion spectra (in arbitrary units) of our system for parameters $\gamma = 1$, and (a) $\delta_g = 0$; (b) $\delta_g = 1$; (c) $\delta_g = -1$. All parameters are in units of β. The solid curve is the absorption profile ($-\mathrm{Im}[\chi(\delta)]$), while the dashed curve the dispersion profile ($\mathrm{Re}[\chi(\delta)]$).

Since the transition $|0\rangle \leftrightarrow |1\rangle$ is treated as occurring in free space, the steady-state linear susceptibility is given by [17]

$$\chi(\delta) = -\frac{4\pi\mathcal{N}|\boldsymbol{\mu}_{01}|^2}{\Omega} a_0(t\to\infty)a_1^*(t\to\infty), \tag{8}$$

with $\mathcal{N}$ being the atomic density. The solution of Eqs. (3) and (4) is obtained by means of perturbation theory [16–18]. We assume that the laser-atom interaction is very weak ($\Omega \ll \beta, \gamma$), so that $a_0(t) \approx 1$ for all times. With the use of the Laplace transform, we obtain from Eq. (4),

$$A_1(s)=\frac{\Omega}{s[\delta+i\gamma/2+i\tilde{K}(s)+is]},\tag{9}$$

where $A_1(s)=\mathcal{L}[a_1(t)]$, $\tilde{K}(s)=\mathcal{L}[K(t)]$, and s is the Laplace variable. The inversion $a_1(t)=\mathcal{L}^{-1}[A_1(s)]$ is cumbersome and will not be presented here. If $\gamma\neq 0$, then the terms inside the brackets of Eq. (9) have only complex, not purely imaginary, roots. Therefore, we can easily obtain, using the final value theorem, the long-time behavior of the probability amplitude,

$$a_1(t\to\infty)=\lim_{s\to 0}[sA_1(s)]=\frac{\Omega}{\delta+i\gamma/2+i\tilde{K}(0)}.\tag{10}$$

For the isotropic PBG model, using Eq. (6),

$$\tilde{K}(s)=\frac{\beta^{3/2}e^{-i\pi/4}}{\sqrt{s+i(\delta_g-\delta)}},\tag{11}$$

and the linear susceptibility reads

$$\chi(\delta)\sim\begin{cases}-\dfrac{\sqrt{\delta_g-\delta}}{(\delta-i\gamma/2)\sqrt{\delta_g-\delta}+\beta^{3/2}} & \text{for}\quad \delta\leqslant\delta_g\\[2ex] -\dfrac{\sqrt{\delta-\delta_g}}{(\delta-i\gamma/2)\sqrt{\delta-\delta_g}-i\beta^{3/2}} & \text{for}\quad \delta>\delta_g.\end{cases}\tag{12}$$

We see that if $\delta=\delta_g$, then $\chi(\delta_g)=0$ and the system becomes transparent to the laser field. In the case that the threshold frequency of the band edge is equal to the $|1\rangle\leftrightarrow|2\rangle$ transition frequency ($\delta_g=0$), transparency occurs when the laser is on resonance, i.e., at $\delta=0$. This result is in contrast with the case in which the transition $|1\rangle\leftrightarrow|2\rangle$ occurs in free space, where the well-known Lorentzian absorption profile [17] is obtained. In Fig. 2 we plot the linear absorption and dispersion spectrum of our system for different values of the detuning of the atomic transition $|1\rangle\leftrightarrow|2\rangle$ from the band-edge threshold. Both the absorption and dispersion spectra are asymmetric, and their shape depends critically on this detuning.

The group velocity of the pulse can also exhibit interesting properties due to the steepness of the dispersion curve. Unusually small group velocities in atomic vapors have been predicted [23] and recently observed by several groups [24–26] in the phenomenon of EIT. In our system the derivative of the real part of the susceptibility diverges as the transparency condition $\delta=\delta_g$ is approached from below, leading to extremely slow group velocities, $v_g\to 0$.

We note that the transparency condition $\delta=\delta_g$ is similar to the two-photon resonance condition that leads to EIT in Λ-type atoms [16,17]. However, EIT occurs through the application of two laser fields: one strong, coupling laser field and one weak, probe laser field. Here transparency is intrinsic to the system as it occurs due to the presence of a square-root singularity at the density of modes threshold.

Up to now, we have discussed the case of an isotropic model for the PBG. We can also investigate an *anisotropic* model of the PBG, where the dispersion relation is given by [3,5,11] $\omega_{\mathbf{k}}=\omega_g+A(\mathbf{k}-\mathbf{k}_0)^2$. In this case the associated density of modes near the edge of the PBG has a square-root threshold behavior, $\rho(\omega)\sim\Theta(\omega-\omega_g)\sqrt{\omega-\omega_g}$. The kernel of Eq. (5) for the anisotropic PBG model is given by [3,5,11],

$$K_a(t-t')\approx\frac{\beta_a^{1/2}e^{i[\pi/4-(\delta_g-\delta)(t-t')]}}{\sqrt{\pi}(t-t')^{3/2}},\quad\text{for}\quad \omega_g(t-t')\gg 1,\tag{13}$$

with $\beta_a^{1/2}=\omega_{12}^2|\boldsymbol{\mu}_{12}|^2/(2\omega_g A^{3/2})$. Therefore, $\tilde{K}_a(s)\sim\sqrt{s+i(\delta_g-\delta)}$ so the linear susceptibility does not go to zero for any value of the probe detuning and transparency does not occur in the anisotropic PBG model.

In summary, we have shown that a Λ-type atom, in which one transition spontaneously decays near the edge of an isotropic PBG, can become transparent to a weak laser field. Studies of quantum optical processes occurring in atoms embedded in PBG materials have, to date, concentrated on the spontaneous emission dynamics [3–12]. Our results suggest that the absorption and dispersion dynamics of such atoms could reveal many surprising effects, in particular in connection with other quantum coherence and interference phenomena such as, for example, lasing without inversion and nonlinear processes involving transparency [16,17].

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