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Measurement of Berry's phase using an atom interferometer

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We report on the demonstration of Berry's phase in an atomic state interacting with a laser field. We draw an analogy between this system and that of a spin interacting with a directionally varying magnetic field. This allows us to identify an effective magnetic quantum number for the atom-light system that governs the maximum Berry phase the atomic state can acquire. We realize two systems that have different effective magnetic quantum numbers, and use a recently developed atom interferometer to make measurements of Berry's phase. [S1050-2947(99)51309-7]

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In 1984 Berry [1] showed that, in addition to the familiar time-dependent dynamical phase, the time-dependent Schrödinger equation requires the existence of an additional phase that has *no* time dependence. This is a topological phase [2] that can be revealed by the adiabatic change [3] of a quantum system whose Hamiltonian traces out a cyclic path in its parameter space.

In this Rapid Communication we study one such system, an atom interacting with laser light. Here the Hamiltonian depends directly upon readily adjustable parameters such as the polarization and phase of the light field. This makes the system an ideal subject for an experimental study of Berry's phase.

The topological phase is generally small compared to the dynamical phase, so to observe it unambiguously the system should be in a state that has little dynamical phase evolution. A state which fulfils this criterion for the atom-light system is a so-called dark state, the zero-energy eigenstate of the atom-field Hamiltonian. If a state is initially dark, it remains dark if the Hamiltonian is adiabatically changed via the parameters of the light field. Thus it is possible to have a state that has no dynamical phase evolution, yet can trace out a path in the parameter space of the optical field. Another useful feature of the dark state is that its momentum can have a dependence on the Hamiltonian parameters. This makes dark states versatile tools for manipulating the momentum and position of atomic deBroglie waves.

This feature was used to create a Mach-Zehnder atom interferometer [4], in which each of the arms consisted of an atom in a dark state whose momentum was manipulated by making a cyclic evolution of the optical field. A comparison of the interferometer fringes when both arms experienced identical evolutions in the parameter space to those observed when the arms were subjected to different evolutions reveals the phase shift seen in Fig. 1. This phase shift has a geometric origin and was proposed by Ol'Shaniĭ as a means of realizing an achromatic ''atom waveplate'' for an atomic beam [5]. Although there have been realizations of Berry's phase in both neutrons [6] and atoms [7], our experiment shows that the flexibility afforded by an optical potential has been used to observe this effect. It has also allowed us to verify the spin dependence of Berry's phase through the use of two different atomic states.

The atom-light system we used to realize a dark state consisted of cesium atoms interacting with light resonant with the $F=3\rightarrow F'=3$ or $F=4\rightarrow F'=4$ transitions of the $D1$ $6\,^2S_{1/2}\rightarrow 6\,^2P_{1/2}$ line. It is a general feature of a $J\rightarrow J$ transition (where J is a generic angular momentum) that the dipole selection rules require that the $m=0$ state and $m=J$ state are dark when the field is π (linear) and σ^+ (circular) polarized, respectively. For a light field composed of two beams having these polarizations, the dark state is a superposition of all the magnetic substates between $m=0$ and $m=J$. If the light field configuration is changed adiabatically between the π and σ^+ beams, the atomic population is coherently transferred between $m=0$ and $m=J$ with a corresponding change in momentum. This process is known as adiabatic transfer (AT) [8] and is part of the beam splitter in our atom interferometer. In reality our state was not completely dark due to the nondegeneracy of the magnetic sub-

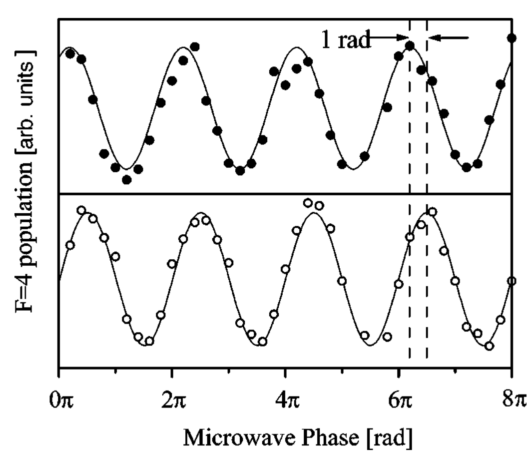

FIG. 1. Fringes from the adiabatic transfer (AT) atom interferometer. In the first set of data the interferometer used two identical AT pulses, while the second set shows data obtained when the phase difference between the AT beams is changed during the final pulse. The phase difference was altered in a way such that the starting and finishing values were the same. Note that the fringes in the two experiments are shifted with respect to each other by approximately 1 rad. This a Berry phase.

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states caused by their different kinetic energies. This was not an important factor since, in the interferometer used to measure the Berry phase, the states in each arm experienced the same dynamical phase shift.

According to Berry [1] the geometric phase imparted to the eigenstate $|\Psi(\mathbf{r})\rangle$ of a Hamiltonian with parameters $\mathbf{r}$ can be calculated from

$$\gamma = i \oint_c \langle \Psi(\mathbf{r}) | \nabla_{\mathbf{r}} \Psi(\mathbf{r}) \rangle \cdot d\mathbf{r}, \tag{1}$$

where c is a cyclic path in the parameter space. For the general $J \rightarrow J$ transition interacting with the field described above, the atom-light Hamiltonian is characterized by two parameters ζ and ϕ. The parameter ζ is determined by the ratio of the intensities of the two light beams and ϕ is the relative phase between the two laser fields. The dark state for this system takes on the form

$$|\Psi_{dark}(\zeta,\phi)\rangle = \sum_{k=0}^{J} f_k(\zeta) e^{ik\phi} |k\rangle, \tag{2}$$

where the sum is performed over the different magnetic substates which comprise the dark state. It is important to note that the overlap of the dark state with the magnetic substate $|k\rangle$ is separable in ζ and ϕ. The functions $f_k(\zeta)$ are related to the Clebsch-Gordan coefficients of the transition and can be calculated by finding the zero-energy eigenstate of the atom-light Hamiltonian. This dark state will have a corresponding Berry phase given by

$$\gamma = \oint_c \sum_{k=0}^{J} k |f_k(\zeta)|^2 d\phi. \tag{3}$$

Note that we have assumed that the norm of the state is conserved so that $\Sigma f_k(\zeta)\, df_k/d\zeta = 0$ and thus the integration over ζ has vanished.

An intuitive way to understand this system and its associated Berry's phase is via the geometric representation of the dark state on the surface of a sphere with ζ and ϕ related to the latitude and longitude, respectively. This is possible since ϕ is periodic and is uniquely defined only within an interval of 2π. For $\pi - \sigma^+$ AT, the poles of the sphere represent the states $m=0$ or $m=J$. Other positions on the surface describe superpositions of these and all intermediate magnetic substates. As we will show later, the Berry phase is related to the solid angle enclosed by the path traced out by the dark state on the sphere.

The phase shift of the fringes in Fig. 1 was obtained by transporting the dark state around the path's ''line'' and ''tear drop'' shown in Fig. 2. AT experiments that have been previously reported [8] involved transporting the dark state along geodesics of constant longitude, such as the ''line'' in Fig. 2, where only the relative intensities were changed. Since no area was enclosed by these paths, these earlier realizations of AT could *not* produce a topological phase. On the other hand, the ''tear drop'' path represents a transfer from $m=0$ to $m=J$ with a simultaneous change in ϕ. The phase difference ϕ then was discontinuously jumped back to

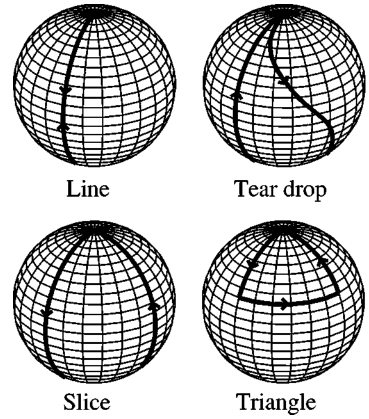

FIG. 2. The parameter space of the atom-light Hamiltonian and thus the dark state can be represented on the surface of a sphere. The latitude angle is related to the intensity ratio of the laser beams and the longitude to their difference in phase. Changing *only* the relative beam intensities produces the ''line'' path, the usual situation for adiabatic population transfer. The graph ''tear drop'' shows a more general trajectory where both relative beam intensity and phase are changed. This situation can produce closed loops with an enclosed area and thus reveal a Berry phase as shown in Fig. 1. When the ''slice'' and ''triangle'' type paths were realized, a Berry phase was produced since they too had an enclosed area.

its original value and, finally, the atomic population was transferred back to $m=0$ along a geodesic of constant longitude.

Originally Berry's phase was derived for the specific case of a spin in a magnetic field, where the direction of the field in polar coordinates $\mathbf{r}=(\theta,\phi)$ determined the Hamiltonian of the system. If the initial state is an eigenstate of the Hamiltonian, it will remain an eigenstate and alter its composition in the original basis as the Hamiltonian parameters are adiabatically changed.

For the example of a spin state $|\Psi_m^S(\theta,\boldsymbol{\phi})\rangle$, which has a total angular momentum S and magnetic quantum number m, we can use the basis states in the original direction of the magnetic field $|\Psi_{m'}^S(0,0)\rangle$ to write [9]

$$|\Psi_m^S(\theta,\boldsymbol{\phi})\rangle = \sum_{m'=-S}^{S} d_{mm'}^S(\theta) e^{i(m'-m)\phi} |\Psi_{m'}^S(0,0)\rangle, \tag{4}$$

where the $d_{mm'}^S$ coefficients depend only on θ and are elements of the rotation matrix $\mathbf{d}^S$. Using Eq. (1), the geometric phase for this state is found to be

$$\gamma = m \oint_c (1-\cos\theta) d\phi = m\Omega, \tag{5}$$

where Ω is the solid angle subtended by the cyclic path of the magnetic-field vector. This intuitive geometric picture allows a direct comparison of the Berry phase arising in different spin systems.

Providing $S=J/2$ and $m=J/2$, the rotated spin state encountered in Eq. (4) is a sum with $J+1$ terms and has a very similar form (separable in θ and ϕ) to Eq. (2). One important difference is that the Berry phase in the latter is not directly expressible as the solid angle subtended by the parameter vector (ζ,ϕ). This suggests that a mapping from ζ to θ of the form

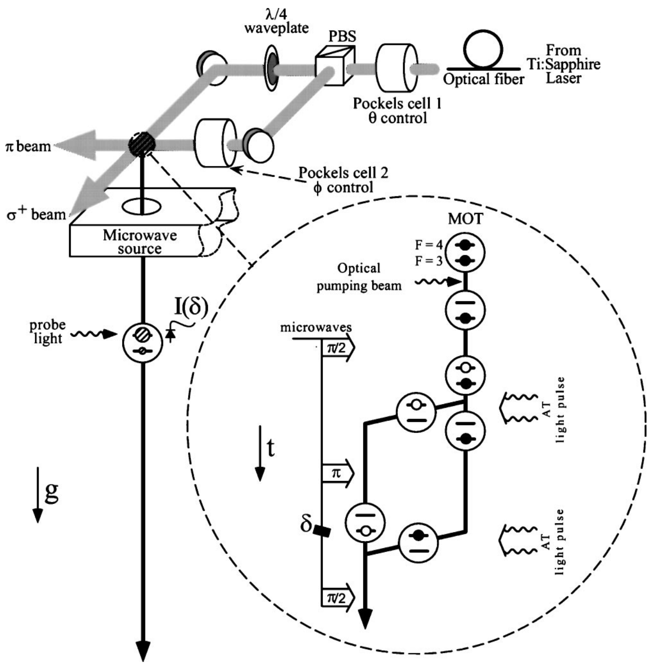

FIG. 3. Schematic of the experiment. The atoms were dropped from a magneto-optic trap and were immediately exposed to the sequence of microwave and AT pulses that constituted the interferometer. The AT was produced by linearly and circularly polarized laser beams propagating orthogonally to each other. The voltages on the two Pockels cells were the experimental handles on the dark state parameters: the phase difference and relative intensities of the beams. The inset shows the experiment with $F=4 \rightarrow F'=4$ AT beams.

$$\theta = \arccos\left[1 - \frac{2}{J}\sum_{k=0}^{J} k|f_k(\zeta)|^2\right] \tag{6}$$

must be applied to allow the identification of the Berry phase as the product of an effective magnetic quantum number $m = J/2$ and the solid angle enclosed by the path. It is noteworthy that even though both spin and mapped dark states produce Berry's phases, which can be identified as a solid angle, the states themselves are not equal. This follows from the fact that Eq. (3) cannot provide a unique solution for the constituent state amplitudes, $f_k(\zeta)$.

We carried out experiments to demonstrate the validity of the solid angle picture with a variety of other paths and with systems having different effective spins. We used a cesium atom interferometer which employed a combination of AT light pulses and microwaves to form the atom optical elements [4]. A $\pi/2$ pulse of the microwaves, resonant between the $F=3$ and $F=4$ ground hyperfine levels, was used to create a coherent superposition of the $F=3$, $m=0$ and $F=4$, $m=0$ states. Following this, an AT pulse formed by two orthogonal resonant laser beams of π and σ^+ polarization selectively manipulated the position of either the $F=3$ or $F=4$ part of the superposition. A state swapping microwave π pulse and a second AT pulse were then used to complete the closed loop. Finally, a $\pi/2$ microwave pulse allowed the internal states of the atom to be recombined and interfere. The phase of this pulse relative to the first, δ, was scanned to produce the fringes seen in Fig. 1. The full sequence of microwave and AT pulses is shown in Fig. 3. For the Berry phase experiments, the second AT pulse was configured so that the phase between the polarizations, ϕ, changed during the pulse. This caused the dark state to trace out a path which enclosed a nonzero area in the parameter space, while the effect on the displacement of the atomic wave function remained the same as the first pulse. The existence of this area manifested itself as a shift in the fringes when compared to an experiment with two identical ATs with no enclosed area.

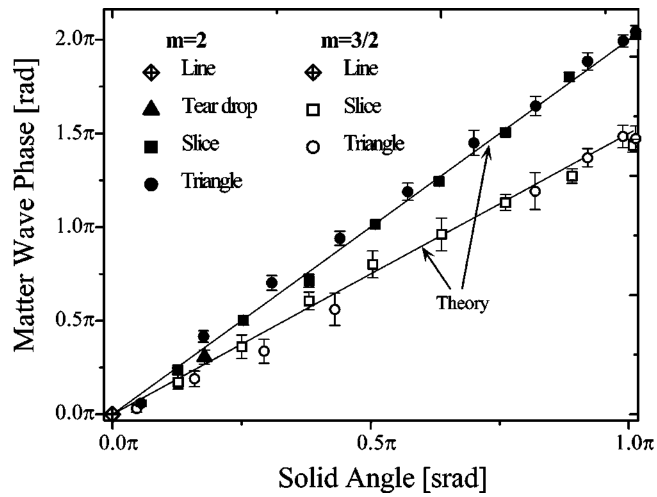

FIG. 4. Summary of the experimentally determined geometric phases as a function of the solid angle Ω. Systems with effective spins equal to 3/2 (open points) and 2 (solid points) are displayed for the four types of paths shown in Fig. 2. The solid lines are plots of the equation $\gamma = m\Omega$, where m is the corresponding maximum magnetic quantum number for the spin.

In order to examine the behavior of Berry's phase in systems with different effective spins, the interferometry was performed with two different types of AT, comprised of either $D1$ [10] $F=3 \rightarrow F'=3$ or $F=4 \rightarrow F'=4$ light. In the former case, after the atoms left the magneto-optic trap (MOT), they were pumped into the $F=4$ ground state with $F=3 \rightarrow F''=4$ light. After the sequence of microwave and light pulses of the interferometer, the population of the $F=4$ ground level was pushed away with a pulse of resonant $F=4 \rightarrow F''=5$ light. The $F=3$ population was then detected by measuring the absorption of an $F=4 \rightarrow F''=5$ probe beam, pulsed on in conjunction with a beam of $F=3 \rightarrow F''=4$ light. For the $F=4 \rightarrow F'=4$ AT the atoms were pumped into the $F=3$ level after leaving the MOT by a pulse of $F=4 \rightarrow F''=4$ light. After the interferometer sequence the population in $F=4$ was measured with an $F=4 \rightarrow F''=5$ probe beam.

The AT pulses were controlled with two Pockels cells. The first cell was aligned so a voltage across its electrodes caused a polarization change of the transmitted light. Thus the two output ports of a polarizing beam-splitting cube (PBS) placed after the cell had variable light intensities which depended on the applied voltage. This voltage V was our experimental handle on θ, the latitude angle on the dark state sphere, via Eq. (6) and with $\zeta = \pi V / V_{\lambda/2}$, where $V_{\lambda/2}$ was the half-wave voltage of the Pockels cell. The second Pockels cell was placed in one of the output beams of the PBS with its axis aligned so that a voltage across its electrodes produced a change in the phase of the light. Since the two outputs of the PBS eventually became the π and σ^+ beams of the AT, this phase change was a direct realization of the longitude angle ϕ. Typically the AT pulses had durations of approximately 50 μs and intensities of 40 mW/cm^2.

With control over the parameters θ and ϕ we were able to realize the different paths on the spheres shown in Fig. 2. Figure 4 shows the observed matter wave phase shift as a function of the solid angle subtended by the path. As can be seen, all of the data points for each particular spin lie on one straight line and the gradient of the line depends upon the effective magnetic quantum number. These lines are plots of the equation $\gamma = m\Omega$, with the effective magnetic quantum number m equal to either 3/2 or 2. The fact that the geometric phase has no time dependence was tested experimentally by changing the duration of the adiabatic interactions. No dependence was observed.

In this Rapid Communication we have reported on the measurement of Berry's phase in an atom that has undergone an internal evolution under the influence of an external field. The parameter space of the atomic state was described by the relative intensities and phase difference of the two laser beams that comprised the field. Through the realization of several types of closed paths in the parameter space we verified the geometric nature of Berry's phase. By utilizing the rich internal structure of the cesium atom we were able to realize a particle with either effective integer or half-integer spin. The ability to study Berry's phase with different spins in the same system is a unique feature of atoms, which is in contrast with other massive particles, such as electrons and neutrons, which have fixed spin. This, together with the possibility of altering dissipative interactions with the environment and creating nonadiabatic evolutions, may give further insight into the role of geometric phases in quantum mechanics.

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[10] We denote the 6 $^2P_{1/2}$ levels ($D1$ line) with a single prime and the 6 $^2P_{3/2}$ ($D2$ line) levels with a double prime.