PHYSICAL REVIEW A VOLUME 60, NUMBER 3 SEPTEMBER 1999

Optical response of a superfluid state in dilute atomic Fermi-Dirac gases

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(Received 28 January 1999)

We theoretically study the propagation of light in a Fermi-Dirac gas in the presence of a superfluid state. BCS pairing between atoms in different spin levels increases the optical linewidth and line shift of a quantum degenerate Fermi-Dirac gas already at very low densities and introduces a collisional local-field shift that may dramatically dominate the Lorentz-Lorenz shift. These optical properties could possibly signal the presence of the superfluid state and determine the value of the BCS order parameter. [S1050-2947(99)50509-X]

PACS number(s): 03.75.Fi, 42.50.Vk, 05.30.Fk

After the first observations of atomic Bose-Einstein condensates [1] there has been an increasing interest in studies of Fermi-Dirac (FD) gases [2–10]. One especially fascinating property of FD gases is that with effectively attractive interaction between different particles the ground state of the system may become unstable with respect to the formation of bound pairs of quasiparticles or Cooper pairs [11]. This effect is analogous to the BCS transition in superconductors.

In this paper we study the optical response of a superfluid state in a zero-temperature FD gas, at low atom densities, and for low-intensity light. We show that the BCS pairing between atoms in different sublevels may dramatically *enhance* optical interactions and the scattering of light in FD gas.

One particularly promising candidate to undergo the BCS transition is spin-polarized atomic ^{6}Li. Atoms in two different internal levels can interact via *s*-wave scattering and the ^{6}Li atom has an anomalously large and negative *s*-wave scattering length $a \simeq -2160 a_0$, where a_0 is the Bohr radius. A nuclear spin mixture of ^{6}Li has been predicted to undergo a superfluid transition at 10^{-8} K with a density of 10^{12} cm^{-3} [3,4].

We study the propagation of light in the dipole approximation for atoms by performing the Power-Zienau-Woolley transformation [12,13]. FD gas is assumed to occupy two different sublevels $|g,\uparrow\rangle$ and $|g,\downarrow\rangle$ of the same atom with electronically excited levels $|e,\nu\rangle$. In the absence of the driving light field, atoms in the electronic ground state are described in second quantization by the Hamiltonian density $\mathcal{H}_1$ [11]:

$$\mathcal{H}_1 = \sum_\nu \psi^\dagger_{g\nu}(H^{g\nu}_{\text{c.m.}} - \mu_{g\nu})\psi_{g\nu} + \hbar u_g \psi^\dagger_{g\uparrow}\psi^\dagger_{g\downarrow}\psi_{g\downarrow}\psi_{g\uparrow}, \quad (1)$$

where $\psi_{g\nu}(\mathbf{r})$ is the atom field operator for level $|g,\nu\rangle$ in the Heisenberg picture, $\mu_{g\nu}$ is the corresponding chemical potential, and $H^{g\nu}_{\text{c.m.}}$ denotes the center-of-mass (c.m.) Hamiltonian. We have approximated the finite-range interparticle potential by a contact interaction with the strength given by $u_g = 4\pi a_g \hbar/m$, where a_g is the *s*-wave scattering length and m is the mass of the atom. The atoms in different sublevels can interact via *s*-wave scattering. On the other hand, there only is a very weak *p*-wave scattering between two atoms in the same sublevel, which is ignored in Eq. (1).

The driving light field introduces additional terms for the system Hamiltonian. The electric displacement $\mathbf{D}(\mathbf{r})$ interacts with the atomic polarization $\mathbf{P}(\mathbf{r})$,

$$\mathcal{H}_2 = -\mathbf{P}(\mathbf{r})\cdot\mathbf{D}(\mathbf{r})/\epsilon_0. \quad (2)$$

In the dipole approximation the positive frequency component of the polarization is given by

$$\mathbf{P}^+(\mathbf{r}) = \sum_{\nu,\eta} \mathbf{d}_{g\nu e\eta}\psi^\dagger_{g\nu}(\mathbf{r})\psi_{e\eta}(\mathbf{r}) \equiv \sum_{\nu,\eta}\mathbf{P}^+_{\nu\eta}(\mathbf{r}), \quad (3)$$

where $\mathbf{d}_{g\nu e\eta}$ denotes the dipole matrix element for the transition $|e,\eta\rangle \rightarrow |g,\nu\rangle$.

The polarization self-energy was shown in Ref. [14] to be inconsequential for dipole atoms. Moreover, we assume that to leading order all remaining interactions between the atoms and the light field, which cannot be accounted for when the atoms are modeled as point dipoles, are governed by the following interactions [13]:

$$\mathcal{H}_3 = \sum_\nu \psi^\dagger_{e\nu}(H^{e\nu}_{\text{c.m.}} + \hbar\omega_0 - \mu_{e\nu})\psi_{e\nu} + \hbar/2 \sum_{\text{spins}} u_e \psi^\dagger_e \psi^\dagger_e \psi_e \psi_e$$
$$+ \sum_{\nu,\sigma} \hbar u_{ge}\psi^\dagger_{g\nu}\psi^\dagger_{e\sigma}\psi_{e\sigma}\psi_{g\nu}. \quad (4)$$

Here $u_{ge} = 4\pi\hbar a_{ge}/m$ and $u_e = 4\pi\hbar a_e/m$ represent the strength of the two-body *s*-wave scattering between the atoms. For simplicity, the frequency of the optical transition ω_0 and the scattering length a_{ge} are assumed to be independent of the sublevel. For typical values of the optical linewidth the c.m. motion for the excited atoms may be omitted [2].

The electric field may be expressed in terms of the driving electric displacement, with the wave number k, and the dipole radiation field [15]

$$\epsilon_0 \mathbf{E}^+(\mathbf{r}) = \mathbf{D}^+_F(\mathbf{r}) + \int d^3r' \mathsf{G}(\mathbf{r}-\mathbf{r}')\mathbf{P}^+(\mathbf{r}'), \quad (5a)$$

$$\mathsf{G}_{ij}(\mathbf{r}) = (\partial r_i \partial r_j - \delta_{ij}\boldsymbol{\nabla}^2)\frac{e^{ikr}}{4\pi r} - \delta_{ij}\delta(\mathbf{r}). \quad (5b)$$

In the limit of low light intensity we have derived from the Heisenberg equations of motion a hierarchy of equations for

correlation functions involving atomic polarization and atom density [15,14]. In the case of the present system we may proceed similarly. As far as the optical response is concerned it is again assumed that we can concentrate on the dynamics of internal degrees of freedom for the atoms and the light. Hence, in the equation of motion for the atomic polarization the kinetic energy of the atoms is neglected.

We consider the low-intensity limit for the driving light. This is done by retaining only those products of operators that involve at most one excited-state field operator or the driving electric displacement [15]. Then, e.g., the term proportional to u_e in Eq. (4) makes no contribution to the equation of motion for $\mathbf{P}^+(\mathbf{r})$.

We introduce the projection operator $\mathsf{P}^{\nu\eta}_{\sigma\tau} \equiv \mathbf{d}_{g\nu e\eta}\mathbf{d}_{e\sigma g\tau}/\mathcal{D}^2$ to include the dependence of the scattered light on the polarizations and on the atomic level structure. Here $\mathcal{D}$ is the (real) reduced dipole matrix element that would pertain to a transition with the unit Clebsch-Gordan coefficient. We define the expectation value $\mathbf{P}_{1\nu\eta}\equiv\langle\mathbf{P}^+_{\nu\eta}\rangle$, with ν and η denoting the atomic sublevels. The steady-state solution of $\mathbf{P}_{1\nu\eta}$ is given by

$$\mathbf{P}_{1\nu\eta}(\mathbf{r}_1)=\alpha\rho_\nu\mathsf{P}^{\nu\eta}_{\eta\nu}\cdot\mathbf{D}^+_F(\mathbf{r}_1)+\sum_\sigma \mathcal{F}_{\sigma\nu}\mathbf{P}_2(\mathbf{r}_1\sigma\sigma;\mathbf{r}_1\nu\eta) + \alpha\sum_{\sigma\tau\xi}\int d^3r_2\mathsf{P}^{\nu\eta}_{\eta\sigma}\cdot\mathsf{G}'(\mathbf{r}_1-\mathbf{r}_2)\mathbf{P}_2(\mathbf{r}_1\nu\sigma;\mathbf{r}_2\tau\xi). \tag{6}$$

Here $\alpha=-\mathcal{D}^2/[\hbar\epsilon_0(\delta+i\gamma)]$ is the polarizability of an isolated atom, $\gamma=\mathcal{D}^2k^3/(6\pi\hbar\epsilon_0)$ the spontaneous linewidth, ρ_ν the atom density in level ν, and δ the atom-light detuning. We have also defined

$$\mathbf{P}_2(\mathbf{r}_1\nu\eta;\mathbf{r}_2\sigma\tau)\equiv\langle\psi^\dagger_{g\nu}(\mathbf{r}_1)\mathbf{P}^+_{\sigma\tau}(\mathbf{r}_2)\psi_{g\eta}(\mathbf{r}_1)\rangle, \tag{7}$$

$$\mathcal{F}_{\tau\nu}\equiv[u_{ge}-(1-\delta_{\tau\nu})u_g]/(\delta+i\gamma). \tag{8}$$

The normally ordered expectation value $\mathbf{P}_2(\mathbf{r}_1\nu\eta;\mathbf{r}_2\sigma\tau)$ describes correlations between an atomic dipole at $\mathbf{r}_2$ and a ground-state atom at $\mathbf{r}_1$. The tensor $\mathcal{F}_{\tau\nu}$ generates the collisionally induced level shifts.

Due to the hard-core interatomic potential we remove the contact dipole-dipole interactions between different atoms. In Eq. (6) this is done by introducing the propagator $\mathsf{G}'_{ij}(\mathbf{r})=\mathsf{G}_{ij}(\mathbf{r})+\delta_{ij}\delta(\mathbf{r})/3$. The purpose of this definition is to yield a vanishing integral for $\mathsf{G}'(\mathbf{r})$ over an infinitesimal volume enclosing the origin [14].

So far, we have obtained a steady-state solution for the atomic polarization (6) that acts as a source for the secondary radiation in Eq. (5a). Equation (6) involves unknown correlation function $\mathbf{P}_2$. Basically, we could continue the derivation and obtain the equations of motion for $\mathbf{P}_2$ and for the higher-order correlation functions. This would eventually result in an infinite hierarchy of equations analogous to the equations in Ref. [15]. However, even in the case of a simple level structure and in the absence of the *s*-wave interactions the solution for the whole system by stochastic simulations is demanding on computer time [9]. In the studies of the refractive index of a quantum degenerate Bose-Einstein gas Morice *et al.* [16,17] considered a density expansion in terms of the number of atoms repeatedly exchanging a photon by including the pair correlations between the ground-state atoms. Although the lowest-order density correction to the susceptibility of a $T=0$ FD gas may be obtained analytically [10], in the presence of nontrivial statistical position correlations a rigorous density expansion is in most cases a very challenging task. In this paper we consider low atom densities (in terms of ρ/k^3) and approximate Eq. (6) by the decoupling that is analogous to the lowest-order correction in Ref. [16],

$$\mathbf{P}_2(\mathbf{r}_1\nu\eta;\mathbf{r}_2\sigma\tau)\simeq\rho_2(\mathbf{r}_1\nu\eta,\mathbf{r}_2\sigma\sigma)\mathbf{P}_{1\sigma\tau}(\mathbf{r}_2)/\rho_\sigma, \tag{9}$$

where the ground-state pair correlation function ρ_2 is defined by

$$\rho_2(\mathbf{r}_1\nu\eta,\mathbf{r}_2\sigma\tau)=\langle\psi^\dagger_{g\nu}(\mathbf{r}_1)\psi^\dagger_{g\sigma}(\mathbf{r}_2)\psi_{g\tau}(\mathbf{r}_2)\psi_{g\eta}(\mathbf{r}_1)\rangle. \tag{10}$$

The decorrelation approximation (9) introduces the lowest-order correction to the optical response in terms the number of microscopic optical interaction processes between the atoms by ignoring the repeated scattering of a photon between the same atoms [17]. As shown in Ref. [10], in the absence of the BCS state, it also correctly generates the leading low-density correction. The predictions of the expansion by Ref. [16] were tested in Ref. [9] for a zero-temperature FD gas in one dimension. The agreement with the exact solution obtained by the numerical simulations was found to be semiquantitative and in the low-density limit excellent.

Before the light is switched on, the system is described by the Hamiltonian density $\mathcal{H}=\mathcal{H}_1$ [Eq. (1)]. The assumption that the driving light only weakly disturbs the system allows us to evaluate the pair correlation functions for the ground-state atoms [Eq. (10)] from $\mathcal{H}_1$, even in the presence of the driving light. We assume a homogeneous sample and introduce a plane-wave basis for the field operators: $\psi_{g\nu}(\mathbf{r})=V^{-1/2}\Sigma_{\mathbf{k}}b_{\mathbf{k}\nu}\exp(i\mathbf{k}\cdot\mathbf{r})$. The Hamiltonian (1) is diagonalized by the standard canonical transformation to the Bogoliubov quasiparticles [11]

$$\alpha_{\mathbf{k}}=u_{\mathbf{k}}b_{\mathbf{k}\downarrow}-v_{\mathbf{k}}b^\dagger_{-\mathbf{k}\uparrow}, \quad \beta_{-\mathbf{k}}=u_{\mathbf{k}}b_{-\mathbf{k}\uparrow}+v_{\mathbf{k}}b^\dagger_{\mathbf{k}\downarrow}, \tag{11}$$

where $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are real, depend only only on $|\mathbf{k}|$, and satisfy $u^2_{\mathbf{k}}+v^2_{\mathbf{k}}=1$. The requirement that $\mathcal{H}_1$ in Eq. (1) be diagonal in the quasiparticle representation sets an additional constraint, and we obtain

$$u^2_{\mathbf{k}}=\frac{1}{2}\left(1+\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right), \quad v^2_{\mathbf{k}}=\frac{1}{2}\left(1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right), \tag{12}$$

where $E_{\mathbf{k}}=\sqrt{\Delta^2+\xi^2_{\mathbf{k}}}$, $\xi_{\mathbf{k}}=\epsilon_{\mathbf{k}}-\bar{\mu}+\hbar u_g(\rho_\uparrow+\rho_\downarrow)/2$, and the energy gap $\Delta=-\hbar u_g V^{-1}\Sigma_{\mathbf{k}}u_{\mathbf{k}}v_{\mathbf{k}}(1-\bar{n}_{\alpha\mathbf{k}}-\bar{n}_{\beta\mathbf{k}})$. In equilibrium, the quasiparticle occupation numbers $\bar{n}_{\alpha\mathbf{k}}\equiv\langle\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}\rangle$ and $\bar{n}_{\beta\mathbf{k}}\equiv\langle\beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}}\rangle$ satisfy FD statistics with $\bar{n}_{\alpha\mathbf{k}}=\bar{n}_{\beta\mathbf{k}}=(e^{E_{\mathbf{k}}/k_BT}+1)^{-1}$. The dispersion relation for free particles is given by $\epsilon_{\mathbf{k}}=\hbar^2k^2/(2m)$ and the average of the chemical potentials is $\bar{\mu}=(\mu_\uparrow+\mu_\downarrow)/2$. For simplicity, we assume $\mu_\uparrow=\mu_\downarrow$.

In the superfluid phase transition the atoms in the different sublevels $\uparrow$ and $\downarrow$ form quasiparticle pairs resulting in a nonvanishing anomalous correlation $\langle\psi_\uparrow(\mathbf{r}_1)\psi_\downarrow(\mathbf{r}_2)\rangle$. The ef-

fect of this macroscopic two-particle coherence on the pair correlation function (10) can be clearly seen by considering the ground state of $\mathcal{H}_1$ [Eq. (1)] that is the vacuum of the Bogoliubov quasiparticles [Eq. (11)]. Then (for $\nu \neq \sigma$)

$$\rho_2(\mathbf{r}_1\nu\nu,\mathbf{r}_2\sigma\sigma)=\rho_\nu\rho_\sigma+|\langle\psi_{g\nu}(\mathbf{r}_1)\psi_{g\sigma}(\mathbf{r}_2)\rangle|^2, \quad (13a)$$

$$\rho_2(\mathbf{r}_1\nu\nu,\mathbf{r}_2\nu\nu)=\rho_\nu^2-|\langle\psi_{g\nu}^\dagger(\mathbf{r}_1)\psi_{g\nu}(\mathbf{r}_2)\rangle|^2. \quad (13b)$$

The optical response may now be evaluated by eliminating $\mathbf{D}_F^+$ and $\mathbf{P}_2$ from Eqs. (5a), (6), and (9). As an example we calculate the vector components of $\mathbf{P}_1$ for the $j=1/2 \rightarrow 3/2$ transition having the electronically excited sublevels $m_j=\pm 1/2, \pm 3/2$. The pair correlation function in Eq. (9) is nonvanishing only with $\nu=\eta$. Because we are dealing with a linear theory, the electric field and the polarization are related by the susceptibility as $\mathbf{P}^+=\epsilon_0\chi\mathbf{E}^+$. We consider a situation where FD gas fills the half-infinite space $z>0$. For simplicity, we assume equal and constant atom densities for the spin states $\rho\equiv\rho_\uparrow=\rho_\downarrow$. The incoming free field is written $\mathbf{D}_F(\mathbf{r})=D_F\hat{\mathbf{e}}\exp(ikz)$, and we assume that it is linearly polarized with $\hat{\mathbf{e}}$ parallel to $\mathbf{d}_{g1/2e1/2}$. Then $\mathbf{P}_{1\nu\eta}=0$ for $\nu\neq\eta$. With the ansatz $\mathbf{P}_{1\nu\nu}(\mathbf{r})=P\hat{\mathbf{e}}\exp(ik'z)$ for $\mathrm{Im}(k')>0$, by using Eq. (13), and by ignoring the effects of the surface of the atomic gas [9], we obtain a spatially constant susceptibility for the sample as

$$\chi=\frac{k'^2}{k^2}-1=\frac{2\mathcal{C}\alpha\rho}{1-2\mathcal{C}\alpha\rho/3+\Sigma_1+\Sigma_2}, \quad (14)$$

with

$$\Sigma_1=-\frac{\mathcal{C}\alpha}{\rho}\int d^3r\, e^{-ikz}\mathsf{G}'(\mathbf{r})[|\langle\psi_{g\uparrow}(\mathbf{r})\psi_{g\downarrow}(0)\rangle|^2 - |\langle\psi_{g\downarrow}^\dagger(\mathbf{r})\psi_{g\downarrow}(0)\rangle|^2], \quad (15)$$

$$\Sigma_2=-\frac{1}{\rho}\sum_\sigma \mathcal{F}_{\uparrow\sigma}\rho_2(\mathbf{r}\uparrow,\mathbf{r}\sigma). \quad (16)$$

Here we have used the obvious relation $\rho_2(\mathbf{r}_1\nu\nu,\mathbf{r}_2\sigma\sigma)=\rho_2(\mathbf{r}_1\sigma\sigma,\mathbf{r}_2\nu\nu)\equiv\rho_2(\mathbf{r}_1\sigma,\mathbf{r}_2\nu)$ and the Clebsch-Gordan coefficient $\mathcal{C}=|\langle 1/2|1/2,0\rangle|^2=2/3$.

In an uncorrelated atomic sample the atomic positions are statistically independent and the pair correlation function (10) satisfies $\rho_2(\mathbf{r}\nu,\mathbf{r}'\sigma)=\rho_\nu\rho_\sigma$. For uncorrelated atoms, and in the absence of the *s*-wave scattering, we would obtain Eq. (14) with $\Sigma_1=\Sigma_2=0$. This is the standard column density result stating that susceptibility equals polarizability of an atom times atom density. Equation (14) also contains the Lorentz-Lorenz local-field correction in the denominator.

The quantum-statistical corrections to the column density result are introduced by Σ_1. It describes the modifications of the optical interactions between neighboring atoms due to the position correlations. The second term in Eq. (15) represents the quantum-statistical contribution to the scattering process in which a photon emitted by an atom in spin level ν at position $\mathbf{r}$ is reabsorbed by another atom in spin level ν and located at the origin. According to FD statistics two fermions with the same quantum numbers repel each other and FD statistics forces a regular spacing between the atoms. The optical interactions are dominantly generated at small interatomic distances and the corrections to the susceptibility due to the second term in Eq. (15) correspond to *inhibited* light scattering. In the absence of a superfluid state FD gas exhibits a dramatic line narrowing [9,10].

The first term in Eq. (15) represents the quantum-statistical corrections to the reabsorption process between atoms in different spin levels due to the two-particle coherence. This term is nonzero only in the presence of a superfluid state. Because the total spin of an interacting atom pair in Eq. (1) is an integer, the pairs behave as bosons [11]. According to the Bose-Einstein statistics two bosons attract each other and the BCS pairing favors small interatomic spacing, hence *enhancing* the optical interactions and the light scattering.

The line shift induced by Σ_2 [Eq. (16)] is generated by the *s*-wave interactions. As far as they can be considered local on the scale of the optical wavelength in Eq. (4), the collisions induce a local-field shift [18] analogous to the Lorentz-Lorenz shift. The optical line shift of the atomic sample is obtained from Eq. (14),

$$\mathcal{S}/\gamma=4\pi\bar{\rho}\mathcal{C}+(\bar{u}_g-\bar{u}_{ge})\bar{\rho}_2(\uparrow,\downarrow)/\bar{\rho}-6\pi\,\mathrm{Re}(\Sigma_1/\bar{\alpha}), \quad (17)$$

where we have dropped the equal position coordinates in ρ_2, used $\rho_2(\mathbf{r}\nu,\mathbf{r}\nu)=0$, and defined the dimensionless variables $\bar{\rho}=\rho/k^3$, $\bar{\rho}_2=\rho_2/k^6$, $\bar{\delta}=\delta/\gamma$, $\bar{\alpha}=-6\pi/(\bar{\delta}+i)$, and $\bar{u}_\xi=u_\xi k^3/\gamma$. The first two terms form the local-field shift. For ^{6}Li the local-field shift due to the *s*-wave scattering in Eq. (17) is larger than the Lorentz-Lorenz shift, if $\gamma\lesssim 210[1+(\Delta/\hbar u_g\rho)^2](a_g-a_{ge})/(a_0\lambda^3)\ \mu m^3 s^{-1}$, where λ is the optical wavelength. Because $(\Delta/\hbar u_g\rho)^2$ is expected to be of the order of 1 [4], the local-field shift could strikingly depend on the BCS order parameter Δ.

If the the effective range r_u of the triplet *s*-wave potential in Eq. (4) is very short, $r_u \ll 1/k$, the resonant dipole-dipole interactions may suppress the effect of the *s*-wave scattering on the line shift just as they cancel the effect of the polarization self-energy [14]. However, for a metastable state, γ^{-1} may be large on the time scale of the atomic interactions. In that case the collisional shift could be observable even for very small r_u.

To calculate the nonlocal linewidth and line shift from integral (15) we need to evaluate the spatial correlation functions by using Eqs. (11) and (12). For instance, the anomalous expectation value reads

$$\langle\psi_\downarrow(\mathbf{r})\psi_\uparrow(0)\rangle=\frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\frac{\Delta}{2E_{\mathbf{k}}}(1-\bar{n}_{\alpha\mathbf{k}}-\bar{n}_{\beta\mathbf{k}}). \quad (18)$$

The chemical potential is solved from $\rho_\nu=\rho_\nu(\bar{\mu})$. Here $\langle\psi_\downarrow(0)\psi_\uparrow(0)\rangle=-\Delta/(\hbar u_g)$ is ultraviolet-divergent, resulting from the assumption of the contact two-body interaction in Eq. (1). This contact interaction is momentum independent and it is not valid at high energies. To estimate the pairing we remove the high-energy divergence by introducing a high-momentum cutoff k_c. We may also replace $\hbar u_g$ by the two-body *T* matrix obtained from the Lippmann-Schwinger equation [4]. This is done by subtracting from Eq. (18)

$\Sigma \exp(i\mathbf{k}\cdot\mathbf{r})\Delta/(2\xi_{\mathbf{k}}V)$. Nevertheless, as argued in Ref. [4], the use of the T matrix may seriously underestimate the overlap in the case of large scattering length $|a_g| \gg r_u$.

We plot the line shift from Eq. (17) and the linewidth $\Gamma/\gamma = 1 - 6\pi\,\mathrm{Im}(\Sigma_1/\bar{\alpha})$ without the collisional shift (i.e., for $\bar{u}_g = \bar{u}_{ge}$) for $\lambda = 900$ nm, and $a_g = -2160a_0$. For the gap parameter at $T=0$ we use the weak-coupling approximation $\Delta \simeq 1.76 k_B T_c$ [11,4], where

$$k_B T_c \simeq \frac{8\epsilon_F}{\pi} e^{\gamma-2} \exp\left(-\frac{\pi}{2k_F|a|}\right), \tag{19}$$

with $\gamma \simeq 0.5772$ and $k_F = (6\pi^2\rho)^{1/3}$.

In Fig. 1(a) the solid line represents the linewidth in the absence of the superfluid state ($\Delta = 0$). The line narrows as a function of the density already at very low densities [10]. The presence of the superfluid state broadens the line. The linewidth is finite even without the regularization in the anomalous correlation (the dashed line). This is because the dipole radiation already involves a high-frequency cutoff [15] that regularizes small r behavior. We also plot the linewidth with the cutoff $k_c = 1/r_u$ and the realistic value $r_u = 100a_0$ of the triplet s-wave potential [4]. We found that the linewidth is almost independent of the cutoff from $r_u = 0$ to $500a_0$, indicating that the exact short-range behavior of the two-body potential is not very crucial for the linewidth. The line shift from the unregularized anomalous correlation in integral (15) diverges logarithmically for small r. Although the radiation kernel (5b) involves a cutoff, the Lamb shift is not treated rigorously. However, for the present purposes we may at least obtain an estimate for the shift by using the cutoff $k_c = 1/r_u$, as in the case of the linewidth. For the BCS state, even with $\bar{u}_g = \bar{u}_{ge}$, also the line shift is increased.

We studied the interaction of light with a two-species atomic superfluid gas. The analysis of the quasiparticles followed the standard BCS theory [11]. We assumed a translationally invariant system. Atoms in a harmonic trap may be considered as locally homogeneous [4], provided that the trap length scale $l = (\hbar/m\omega)^{1/2}$ is much larger than the correlation length, which for intraspecies is $\xi_{\nu\nu} \sim 1/k_F$ and for the interspecies $\xi_{\uparrow\downarrow} \sim \epsilon_F/(\Delta k_F)$ [11]. Other notable assumptions were zero temperature and low atom density. The present work could be extended to larger values of $|a_g|$ and $\bar{\rho}$ by going beyond the BCS weak-coupling limit and by including the cooperative optical linewidths and line shifts [15].

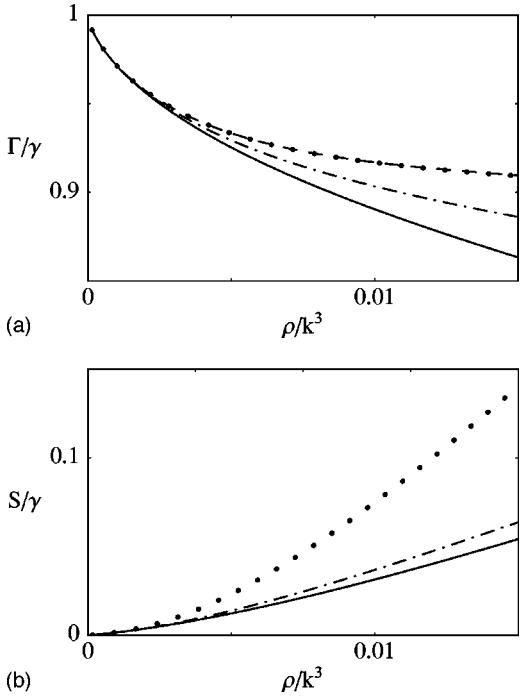

FIG. 1. The optical (a) linewidth and (b) line shift as a function of the atom density per cubic optical wave number of the driving light in the absence of the collisional shift. The dashed-dotted line corresponds to the regularization by the two-body T matrix, the dotted by the cutoff $k_c = 0.01a_0^{-1}$, and the dashed line is the unregularized case. The solid line has $\Delta = 0$.

We acknowledge financial support from the EC through the TMR Network, No. ERBFMRXCT96-0066.

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