Quantum teleportation with squeezed vacuum states

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We show how the partial entanglement inherent in a two-mode squeezed vacuum state admits two different teleportation protocols. These two protocols refer to the different kinds of joint measurements that may be made by the sender. One protocol is the recently implemented quadrature phase approach of Braunstein and Kimble [Phys. Rev. Lett. **80**, 869 (1998)]. The other is based on recognizing that a two-mode squeezed vacuum state is also entangled with respect to photon-number difference and phase sum. We show that this protocol can also realize teleportation; however, limitations can arise due to the fact that the photon-number spectrum is bounded from below by zero. Our examples show that *a given entanglement resource may admit more than a single teleportation protocol and the question then arises as to what is the optimum protocol in the general case*. [S1050-2947(99)08808-3]

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I. INTRODUCTION

One of the central results in the rapidly developing field of quantum information theory is the possibility of perfectly transferring an unknown quantum state from a target system at the sender's location A to another identical system at the receiver's location B. This is called teleportation and requires that the sender and receiver share a maximally entangled state, and further, that they communicate via a classical channel. The original proposal of Bennett *et al.* [1] was posed in terms of systems with a two-dimensional Hilbert space [quantum-bits (qubits) [2]]. However, recently Furasawa *et al.* [4], using a proposal of Braunstein and Kimble [3], have demonstrated that the method can also be applied to entangled systems with an infinite-dimensional Hilbert space, specifically for harmonic-oscillator states. In that work, a coherent state was teleported using an entanglement resource that consisted of a two-mode squeezed vacuum state. The joint measurements required for teleportation are the joint quadrature phase on the target system and that part of the entangled resource shared by the receiver. The essential feature exploited in the scheme of Furasawa *et al.* is the well-known fact that a two-mode squeezed vacuum state is an approximation to an EPR (Einstein-Podolsky-Rosen) state, which had previously been shown by Vaidman [5] to enable teleportation of continuous observables. However, a squeezed vacuum state is also (imperfectly) entangled in number and phase. Can this entanglement be used as a teleportation resource as well?

In this paper we show that by making joint number and phase measurements this entanglement can also be used for teleportation. However, because the number operator is bounded from below, there are limits on the ability to teleport a quantum state with this protocol.

Suppose that at some prior time a two-mode squeezed vacuum state is generated and that one mode is open to local operations and measurements at the sender's location A by observer Alice, while the other mode is open to local operations and measurements in the receiver's location B, by observer Bob. Alice and Bob can communicate via a classical communication channel. Thus Alice and Bob each have access to one of the two entangled subsystems described by

$$|\mathcal{E}\rangle_{AB}=\sqrt{(1-\lambda^2)}\sum_{n=0}^{\infty}\lambda^n|n\rangle_A\otimes|n\rangle_B\,. \tag{1}$$

This state is generated from the vacuum state by the unitary transformation

$$U(r)=e^{-r(a^\dagger b^\dagger-ab)}, \tag{2}$$

where $\lambda=\tanh r$ and where a,b refer to the mode accessible to Alice and the mode accessible to Bob, respectively.

The entanglement of this state can be viewed in two ways: First, as an entanglement between quadrature phases in the two modes (EPR entanglement); and second as an entanglement between number and phase in the two modes. We can easily show that this state approximates the entanglement of an EPR state in the limit $\lambda\to 1$ or $r\to\infty$. The quadrature phase entanglement is easily seen by calculating the effect of the squeezing transformation, Eq. (2), in the Heisenberg picture. We first define the quadrature phase operators for the two modes,

$$\hat{X}_A=a+a^\dagger, \tag{3}$$

$$\hat{Y}_A=-i(a-a^\dagger), \tag{4}$$

$$\hat{X}_B=b+b^\dagger, \tag{5}$$

$$\hat{Y}_B=-i(b-b^\dagger). \tag{6}$$

Then

$$\mathrm{Var}(\hat{X}_A+\hat{X}_B)=2e^{-2r}, \tag{7}$$

$$\mathrm{Var}(\hat{Y}_A-\hat{Y}_B)=2e^{-2r}, \tag{8}$$

where $\mathrm{Var}(A)=\langle A^2\rangle-\langle A\rangle^2$ is the variance. Thus in the limit of $r\to\infty$ the state $|\mathcal{E}\rangle$ approaches a simultaneous eigenstate

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of $\hat{X}_A+\hat{X}_B$ and $\hat{Y}_A-\hat{Y}_B$. This is the analogue of the EPR state with position replaced by the real quadratures $\hat{X}$ and the momentum replaced by the imaginary quadratures $\hat{Y}$.

This state is also entangled with respect to the correlation specified by the statement: *an equal number of photons in each mode*. However, it is not a perfectly entangled state, which would require the (unphysical) case of a uniform distribution over correlated states. It can approach a perfectly entangled state of photon number asymptotically in the limit $\lambda\to 1$. The reduced density operator of each mode is a thermal-like state with mean photon number

$$\bar{n}=\frac{\lambda^2}{1-\lambda^2}, \tag{9}$$

and thus the limit of a perfectly entangled state can only occur as the mean photon number goes to infinity, which is not physical. For finite excitation, the distribution of correlated states is very close to uniform for values $n<e^{2r}$. This suggests that in practice this state can be used as a perfectly entangled state of photon number, provided all other states available have significant support on the photon-number basis up to a maximum value of $n\ll e^{2r}$. We now show that this is indeed true if this state is used as a teleportation resource.

In the case of number and phase, it is obvious that the squeezed vacuum state is the zero eigenstate of the number difference operator

$$\hat{J}_z=\frac{1}{2}(a^\dagger a-b^\dagger b). \tag{10}$$

Not so obvious is the fact that, as $\lambda\to 1$, the two modes become anticorrelated in phase. To see this we compute the canonical joint phase distribution for the two modes using the projection operator valued measure,

$$|\phi_A,\phi_B\rangle=\sum_{n,m=0}^{\infty} e^{in\phi_A+im\phi_B}|n\rangle_A\otimes|m\rangle_B, \tag{11}$$

normalized on $[-\pi,\pi]$ with respect to the measure $d\phi_A d\phi_B/4\pi^2$. The joint distribution is

$$P(\phi_A,\phi_B)=\frac{1-\lambda^2}{|1-\lambda e^{i(\phi_A+\phi_B)}|^2}. \tag{12}$$

As $\lambda\to 1$, this distribution becomes very sharply peaked at $\phi_A=-\phi_B$. Thus the photon number in each mode is perfectly correlated while the phase in each mode is highly anticorrelated.

II. TELEPORTATION

A. Teleportation using a quadrature EPR state

We first show how teleportation of continuous variables is possible using a perfect quadrature phase QND (quantum nondemolition) measurement between two optical modes A and B to create the entanglement resource. The state that is produced is an optical analogue of the EPR state discussed by Vaidman [5]. Our presentation is completely equivalent to that given by Vaidman; however, we will use more conventional quantum optics notation.

Consider the following entangled state of two modes A and B:

$$|X_1,P_1\rangle_{AB}=e^{-i\hat{Y}_A\hat{X}_B}|X_1\rangle_A\otimes|Y_1\rangle_B, \tag{13}$$

where the quadrature phase operators $\hat{Y}_A,\hat{X}_B$ are defined in Eq. (6) and the states appearing in this equation are the quadrature phase eigenstates,

$$\hat{X}_A|X_1\rangle_A=X_1|X_1\rangle_A,$$

$$\hat{Y}_B|Y_1\rangle_B=Y_1|Y_1\rangle_B.$$

One then easily verifies that the state defined in Eq. (13) is a simultaneous eigenstate of $\hat{X}_A-\hat{X}_B$ and $\hat{Y}_A+\hat{Y}_B$ with respective eigenvalues, X_1,Y_1. The unitary transformation in Eq. (13) is generated by the perfect QND Hamiltonian $H=\hat{Y}_A\hat{X}_B$, which realizes a QND coupling between modes A and B. It is also the prototype measurement coupling Hamiltonian first defined by von Neumann.

It is important to realize that all perfect QND couplings are a source of entanglement and a potential resource for teleportation. For example, the original teleportation scheme [1] is based on qubit Bell states that can be generated by single qubit rotations together with a controlled not (C-NOT) gate. In this case the C-NOT gate provides the entanglement. The C-NOT gate is itself is based on an ideal QND interaction in which the target qubit is the apparatus for measuring the state of the control qubit. If the target qubit is prepared in the logical zero state it will only change if the control qubit state is in a logical one state, and in all cases the state of the control qubit is unchanged. The control not gate itself can be realized by a simple controlled phase shift gate between two qubits. Such an interaction does nothing unless the state of both qubits is a logical one in which case the state acquires a π phase shift. For example, if we code our logical states via bosonic Fock states, such that the logical zero is the zero Fock state and the logical one is the one Fock state, the mutual phase shift between two modes with annihilation operators a and b can be realized by the QND number measurement interaction $H_I=\hbar\kappa a^\dagger a b^\dagger b$ [6]. In optics this interaction has been realized at the level of very few photons in cavity QED with very small cavities [7].

Needless to say the EPR state is not a physical state, not because the QND interaction cannot be achieved, but because the quadrature phase eigenstates appearing in Eq. (13) are infinite energy states. However, we can use arbitrarily close approximations to these states, given a sufficient energy resource, as in the case of a squeezed vacuum state discussed below.

In the protocol for teleportation based on this state, we now consider another mode, the target mode T, in an unknown state $|\psi\rangle_T$. Joint quadrature phase measurements of $\hat{X}_T-\hat{X}_A$ and $\hat{Y}_T+\hat{Y}_A$ are made on modes T and A, yielding two real numbers, X_2 and Y_2, respectively. The total input state for the teleportation protocol is

$$|\Psi_{in}\rangle=|\psi\rangle_T\otimes|X_1,Y_1\rangle_{AB}. \tag{14}$$

The (unnormalized) conditional state of the total system after the measurement on A and T is given by the projection

$$|\tilde{\Psi}_{out}^{(X_2,P_2)}\rangle = {}_{AT}\langle X_2,Y_2|\psi\rangle_T |X_1,Y_1\rangle_{AB}\otimes|X_2,Y_2\rangle_{AT}. \tag{15}$$

Using Eq. (13) we may then write the conditional state of mode B as

$$|\phi^{(X_2,P_2)}\rangle_B = [P(X_2,Y_2)]^{-1/2}\hat{\Phi}(X_2,Y_2)|Y_1\rangle_B, \tag{16}$$

where

$$P(X_2,Y_2) = {}_B\langle Y_1|\hat{\Phi}^\dagger\hat{\Phi}|Y_1\rangle_B \tag{17}$$

is the probability for the results (X_2,Y_2). The state $|Y_1\rangle_B$ is an eigenstate of $\hat{Y}_B$ with eigenvalue Y_1, which is determined by the initial choice of entangled state for A and B. The operator $\hat{\Phi}$ acts only on mode B and is defined by

$$\hat{\Phi}(X_2,Y_2) = {}_{AT}\langle X_2,Y_2|e^{-i\hat{Y}_A\hat{X}_B}|\psi\rangle_T\otimes|X_1\rangle_A. \tag{18}$$

Using the definition of the state $|X_2,Y_2\rangle_{AT}$,

$$|X_2,Y_2\rangle_{AT} = e^{i\hat{X}_A\hat{Y}_T}|X_2\rangle_T\otimes|Y_2\rangle_A, \tag{19}$$

where $\hat{X}_T|X_2\rangle_T = X_2|X_2\rangle_T$ and $\hat{Y}_A|Y_2\rangle_A = Y_2|Y_2\rangle_A$, it is possible to show that

$$|\psi^{(X_2,P_2)}\rangle_B = e^{iX_2Y_2}e^{iX_2\hat{Y}_B}e^{-iP_2\hat{X}_B}|\psi\rangle_B. \tag{20}$$

Thus up to a phase factor and two simple unitary transformations, the conditional state of B is the same as the initial unknown state of the target T. If A now sends the results of the measurements (X_2,Y_2) to the receiver B, the phase factor and two unitary transformations can be removed by local operations that correspond to a displacement in phase space by X_2 in the real quadrature direction and Y_2 in the imaginary quadrature direction. The initial state of T has then been ''teleported'' to mode B at a distant location.

B. Squeezed-vacuum-state teleportation using quadrature measurements

In the Introduction we noted that the squeezed vacuum state

$$|\mathcal{E}\rangle = e^{-r(a^\dagger b^\dagger - ab)}|0\rangle_{AB} \tag{21}$$

is an approximation of the quadrature EPR state discussed in Sec. II A. In the limit of infinite squeezing, this state becomes equivalent to the EPR state. We now show that the two-mode squeezed vacuum state can be used for teleportation with a fidelity that approaches unity as the squeezing increases to infinity.

We again assume perfect projective measurements of the joint quadrature phase quantities $\hat{X}_T-\hat{X}_A$ and $\hat{Y}_A+\hat{Y}_B$ on the target state and the sender's part of the entangled mode, A, with the results X and Y, respectively. The (unnormalized) conditional state of the total system after the measurement is then seen to be given by

$$|\tilde{\Psi}^{(X,Y)}\rangle = {}_T\langle X|\otimes{}_A\langle Y|e^{i\hat{Y}_T\hat{X}_A}|\psi\rangle_T|\mathcal{E}\rangle_{AB}. \tag{22}$$

It is then easy to show that the state of mode B at the receiver is the pure state $|\phi_{XY}(r)\rangle_B$ with the wave function (in the $\hat{X}_B$ representation),

$$\phi_{XY}(x_1,x_2;r) = \int_{-\infty}^{\infty} dx_1dx_2e^{ix_1Y}\mathcal{G}(x_1,x_2;r)\psi(X-x_1), \tag{23}$$

where $\psi(x) = {}_T\langle x|\psi\rangle_T$ is the wave function for the target state we seek to teleport. The kernel is given by

$$\mathcal{G}(x_1,x_2;r) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{4}(x_1+x_2)^2e^{2r} - \frac{1}{4}(x_1-x_2)^2e^{-2r}\right]. \tag{24}$$

This state is clearly not the same as the state we sought to teleport. However, in the limit of infinite squeezing $r\to\infty$, we find that $\mathcal{G}(x_1,x_2;r)\to\delta(x_1+x_2)$ and the state of mode B approaches

$$|\phi_{XY}(r)\rangle_B \to e^{iXY}e^{-iY\hat{X}_B}e^{iX\hat{Y}_B}|\psi\rangle_B, \tag{25}$$

which, up to the expected unitary translations in phase-space, is the required teleported state.

C. Squeezed-vacuum-state teleportation using number and phase measurements

In this section we explore to what extent teleportation is possible using the number phase entanglement implicit in the squeezed vacuum state. In this case we expand the target state in the photon number basis as

$$|\psi\rangle_T = \sum_{m=0}^{\infty} c_m|m\rangle_T. \tag{26}$$

Thus the input state to the receiver and sender is

$$|\Psi_{in}\rangle = (1-\lambda^2)^{1/2}\sum_{n,m=0}^{\infty}\lambda^n c_m|m\rangle_T\otimes|n\rangle_A\otimes|n\rangle_B. \tag{27}$$

To facilitate the description of the joint measurements that need to be made on T and A modes at the receiver, we define the eigenstates of the operator

$$\hat{J}_z = \frac{1}{2}(\hat{N}_T-\hat{N}_A) \tag{28}$$

where $\hat{N}_T,\hat{N}_A$ are the number operators for modes T and A, respectively. These eigenstates can be written as pseudoangular momentum states as

$$\hat{J}_z|j,k\rangle_{AT} = k|j,k\rangle_{AT}, \tag{29}$$

where the eigenvlaue j of $\hat{J}^2$ is determined by the result $\hat{J}^2 = \hat{N}/2(\hat{N}/2+1)$, where $\hat{N}=\hat{N}_A+\hat{N}_T$ is the total photon number operator for modes T and A with eigenvlaue N

$=0,1,2,\ldots$. In that case $j=N/2$. The relationship between the state Eq. (29) and the original product number basis is

$$|j,k\rangle_{AT}=|j+k\rangle_T\otimes|j-k\rangle_A. \tag{30}$$

The combined state of the entire system may now be written

$$|\Psi_{in}\rangle=(1-\lambda^2)^{1/2}\sum_{j=0}^{\infty}\sum_{k=-j}^{j}\lambda^{j-k}c_{j+k}|j,k\rangle_{AT}\otimes|j-k\rangle_B. \tag{31}$$

Note that in this equation the sum over j,k is over half integers as well as integers.

The teleportation protocol for number and phase requires that Alice make two measurements of a joint quantity on A and T. In this case the first measurement will seek to determine one half the photon number difference as represented by $\hat{J}_z$, while the second measurement will seek to determine the phase sum of the two modes. For the first measurements the possible results are $k=\{0,\pm\frac{1}{2},\pm 1,\pm\frac{3}{2},\ldots\}$. Consider first the case of $k>0$. The conditional (unnormalized) state of the entire system is

$$|\Psi^{(k)}\rangle=(1-\lambda^2)^{1/2}\sum_{n=0}^{\infty}\lambda^n c_{n+2k}|n+2k\rangle_T\otimes|n\rangle_A\otimes|n\rangle_B, \tag{32}$$

where we have returned to the product number basis in preparation for the next measurement of the phase sum. If the measurement result was negative $k<0$, the conditional unnormalized state is

$$|\Psi^{(-k)}\rangle=(1-\lambda^2)^{1/2}\sum_{n=0}^{\infty}\lambda^{n+2k}c_n|n\rangle_T\otimes|n+2k\rangle_A \otimes|n+2k\rangle_B. \tag{33}$$

The *second* measurement is a measurement of the joint total phase operator for modes T and A, defined by the projection operator valued measure

$$|\phi_+\rangle\langle\phi_+|=\sum_{n,m=0}^{\infty}\sum_{k=-min(n,m)}^{k=min(n,m)}|n,k\rangle_{AT}\langle k,m|e^{-i(n-m)\phi_+}. \tag{34}$$

Now, it must be said at once that such measurements are unphysical; however, they do represent the limit of perfectly valid (though rather impractical) discrete phase measurements [6]. As a result of this measurement, Alice has a value ϕ_+ for the phase. The corresponding conditional state of mode B, given a positive number difference measurement, is

$$|\psi^{(k,\phi_+)}\rangle_B=\frac{(1-\lambda^2)^{1/2}}{\sqrt{P_+(k)}}\sum_{n=0}^{\infty}\lambda^n c_{n+2k}e^{-i\phi_+(n+k)}|n\rangle_B, \tag{35}$$

while if a negative number difference result were obtained the state of mode B would be

$$|\psi^{(-k,\phi_+)}\rangle_B=\frac{(1-\lambda^2)^{1/2}}{\sqrt{P_-(-k)}}\sum_{n=0}^{\infty}\lambda^{n+2k}c_n e^{-i(n-k)\phi_+}|n+2k\rangle_B, \tag{36}$$

where $P(k)$ is in fact the probability for Alice to obtain the result k. This is given by

$$P_+(k)=(1-\lambda^2)\sum_{n=0}^{\infty}\lambda^{2n}|c_{n+2k}|^2, \tag{37}$$

$$P_-(-k)=(1-\lambda^2)\sum_{n=2k}^{\infty}\lambda^{2n}|c_{n-2k}|^2, \tag{38}$$

with k taken as positive in both equations.

Now it only remains for Alice to communicate to Bob what value she got for the two measurements, that is, the values k and ϕ_+, and for Bob to find the appropriate conditional unitary transformations to reconstruct the state. The phase displacement part is quite straightforward. The receiver B applies the local unitary transformation

$$U(\pm k,\phi)=e^{i\phi(\hat{N}_B\pm k)}, \tag{39}$$

where N_B is the number operator for the mode B. After this transformation the states become

$$|\psi^{(k)}\rangle=\frac{(1-\lambda^2)^{1/2}}{\sqrt{P_+(k)}}\sum_{n=2k}^{\infty}\lambda^{n-2k}c_n|n-2k\rangle_B, \tag{40}$$

$$|\psi^{(-k)}\rangle=\frac{(1-\lambda^2)^{1/2}}{\sqrt{P_-(-k)}}\sum_{n=0}^{\infty}\lambda^{n+2k}c_n|n+2k\rangle_B, \tag{41}$$

with $k>0$ in both cases. Naively one might think that we can now apply a number displacement operator, either up or down by $2k$, to reconstruct the state in a fashion analogous to the case of quadrature teleportation. While formally we can construct such an operator (see below), there is going to be a problem with the case $k>0$, as all the coefficients for photon numbers less than $2k$ will be *missing*. This result is directly attributable to the fact that the spectrum of the number operator is bounded below by zero. We must accept this as a limit to teleportation when number phase measurements are used and keep this in mind when trying to find more general teleportation schemes in the future.

What is the number displacement operator? The generator of displacements for number must be the canonical phase. Formally this is defined by

$$\mathcal{D}(k)=\int_{-\pi}^{\pi}d\phi e^{ik\phi}|\phi\rangle\langle\phi|, \tag{42}$$

where

$$|\phi\rangle=\sum_{n=0}^{\infty}e^{in\phi}|n\rangle. \tag{43}$$

The fact that these basis states are not normalizable indicates that it is impossible in practice to realize a true number displacement operator. However, there are schemes that can reproduce arbitrarily well a number displacement [8,9].

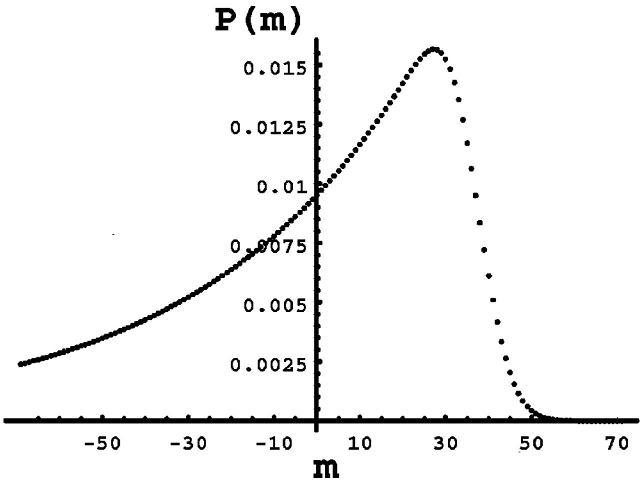

FIG. 1. The probability distribution for obtaining a result m for the number difference operator $N_T - N_A$ for a coherent state in the target with $\alpha = 6.0, \lambda = 0.99$.

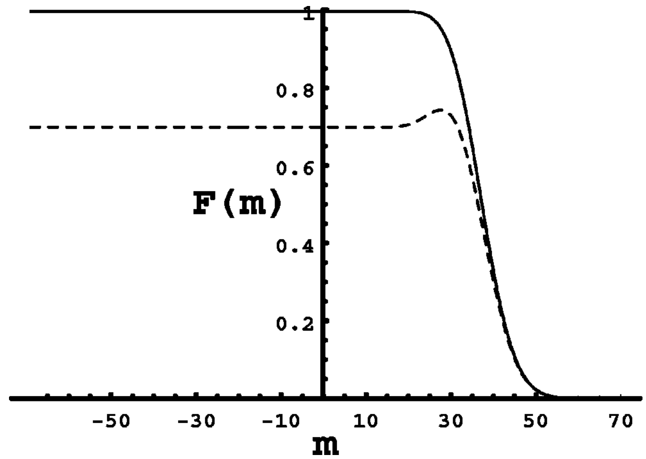

FIG. 2. The fidelity versus the result m for the number difference operator $N_T - N_A$ for a coherent state in the target with $\alpha = 6, \lambda = 0.9$ (dashed) and $\alpha = 6.0, \lambda = 0.99$ (solid).

We first consider the example of the target state prepared in the number state $|N\rangle$. In this case the probability of obtaining a result m for the measurement of the photon-number difference operator $2\hat{J}_z$ on A and T is

$$P(m) = \begin{cases} (1-\lambda^2)\lambda^{2(N-m)}, & m \leqslant N \\ 0, & m > N \end{cases} \tag{44}$$

where $m = 0,1,2,\ldots$. The most probable result is $m = N$, in which case the teleported state is the vacuum state $|0\rangle_B$, which, given the data $m = N$ may be displaced back to $|N\rangle_B$, *independent of the value of* λ. Indeed, it is easy to see that we can teleport a number state perfectly, regardless of the value of λ, provided that we can make number displacements. This is in contrast to the quadrature case where fidelity does depend on λ. This is a consequence of the perfect correlation between photon number for each mode in the squeezed vacuum state. However, the probabilities for different values of the photon number difference in A and T do depend on the value of λ.

Next consider the case of a coherent state $|\alpha\rangle$. This state has a Poisson photon-number distribution with a mean of $\bar{n} = |\alpha|^2$. The probability of observing a photon-number difference m between the target and the sender mode A is

$$P(m) = \begin{cases} \lambda^{-2|m|}(1-\lambda^2)e^{-|\alpha|^2(1-\lambda^2)}, & m < 0 \\ (1-\lambda^2)e^{-|\alpha|^2}\sum_{n=0}^{\infty}\lambda^{2n}\dfrac{|\alpha|^{2(n+m)}}{(n+m)!}, & m \geqslant 0 \end{cases} \tag{45}$$

where $m = 2k$ is an integer. This distribution is shown in Fig. 1, with $\alpha = 6, \lambda = 0.99$. Note that the distribution is relatively flat around $m = 0$, that is, around equal photon numbers in both A and T. It is easy to see that, when $\lambda \to 1$, the rapid fall-off occurs for values $m > \bar{n}$. This is not too surprising, as the most likely photon number in mode T is just $\bar{n}$, and thus this is the largest possible value for the photon-number difference between modes A and T. However, the minimum value for m (which is negative) is determined by the largest photon number in mode A, which, as $\lambda \to 1$, can be a large negative number. For this reason the distribution is highly asymmetric and falls off quite slowly for $m < 0$.

One performance measure for teleportation is the fidelity between the target state for mode B and the actual state teleported. We will calculate the fidelity for the transported state after the appropriate number displacement operator has acted. This is defined by

$$F(m) = |{}_B\langle\psi|\tilde{\psi}^{(m)}\rangle_B|^2, \tag{46}$$

where $|\tilde{\psi}^{(m)}\rangle_B$ is the teleported and displaced state at the receiver B, given a photon number difference measurement result, m, at the sender, A and T. The fidelity is given by

$$F(m) = \begin{cases} \dfrac{(1-\lambda^2)}{P_+(m)}e^{-2|\alpha|^2}\left|\sum_{n=0}^{\infty}\lambda^n\dfrac{|\alpha|^{2(n+m)}}{(n+m)!}\right|^2, & m \geqslant 0 \\ \exp[-|\alpha|^2(1-\lambda)^2], & m < 0. \end{cases} \tag{47}$$

The fidelity is plotted in Fig. 2 for $\alpha = 6$ and two values of λ. We see that for $\lambda \to 1$ the fidelity is very close to unity until there is a chance of obtaining a positive photon-number difference that exceeds the average photon number in the target state we wish to teleport. However, we see from Fig. 1 that this is likely to happen with rapidly decreasing probability.

Given the current difficulty of realizing a photon-number displacement operator, it is of interest to determine the fidelity when no attempt is made to displace the final state. If we assume that the target state is a coherent state with amplitude α, the fidelity when the results of the photon-number difference measurement is zero, $m = 0$, is

$$F(0) = e^{-|\alpha|^2(1-\lambda)^2}. \tag{48}$$

If we note that the mean photon number in the entanglement resource shared by A and B is just that for a squeezed vacuum state, $\bar{n}_{SV} = \lambda^2/(1-\lambda^2)$, we may write the fidelity as

$$F(0) = \exp\left\{-\frac{\bar{n}}{\bar{n}_{SV}}\lambda^2\right\}. \tag{49}$$

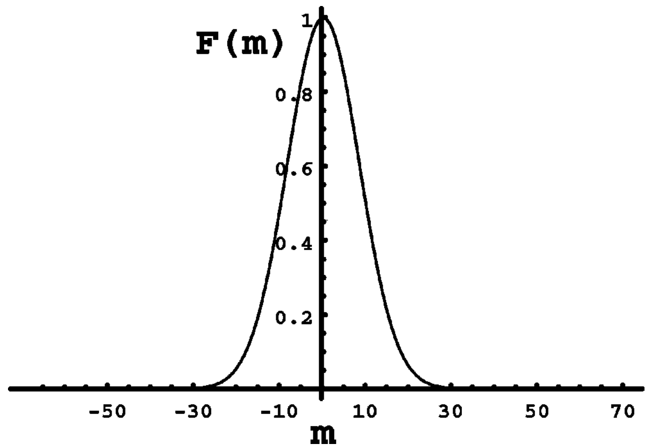

FIG. 3. The fidelity versus the result m for the number difference operator $N_T - N_A$ when no attempt is made to displace the teleported state conditioned in this result, for a coherent state in the target with $\alpha=6, \lambda=0.9$ (dashed).

This indicates that when the mean photon number in the entanglement resource is significantly greater than that in the target state, the teleportation has high fidelity. Indeed, in the limit $\lambda \rightarrow 1$, the teleportation for a result $m=0$ is perfect. Of course the fidelity falls off if $m \neq 0$, unless we act with the number displacement operator to shift the received state. If we do not (or possibly cannot) do that, the fidelity falls off in a Gaussian-like fashion, which for $|\alpha| \gg 1$ has a width that scales like half the mean photon number in the target state, $\bar{n}/2$. This is shown in Fig. 3.

III. CONCLUSION

We have shown how the imperfect entanglement of a two-mode squeezed vacuum state can be used for teleportation of an unknown quantum state for two different measurement protocols at the sender. One protocol is based on quadrature phase measurements and is suggested by the fact that a squeezed vacuum state is an approximation of an EPR correlated state for quadrature phase amplitude variables. However, a squeezed vacuum state is also entangled with respect to the photon-number difference and phase sum in the two modes. This suggests a protocol based on number and phase measurements at the sender. While such measurements are just beyond the reach of current experiments in quantum options, our examples suggest that a given entanglement resource admits more than one teleportation protocol. In the case of a squeezed vacuum state the quadrature phase protocol is simpler, based on current technology. However, this may not be true for other entanglement resources, or other realizations of the entanglement. In fact, any perfect QND interaction between two systems is a potential entanglement resource, and determining the best teleportation protocol may be a nontrivial exercise.

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