Theoretical analysis of resonances in the polarization spectrum of a two-level atom driven by a polychromatic field

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Analytic solutions of the optical Bloch equations for a two-level atom interacting with a strong polychromatic field whose frequencies are symmetrically positioned with respect to the atomic frequency are used to obtain the polarization spectrum of the atom. The spectrum is found to consist of a series of discrete peaks or dips superimposed on the continuous part of the spectrum. Physical interpretation of resonances exhibited in the continuous part of the spectrum is given using a semiclassical dressed-atom approach.
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I. INTRODUCTION

The interaction of a two-level atom with an intense laser field is of fundamental importance in quantum optics and laser spectroscopy. The spectral properties of a two-level atom driven by a monochromatic laser field are now well understood, since the pioneering work of Mollow [1,2]. The fluorescence spectrum from the atom contains coherent and incoherent contributions which respectively consist of a discrete elastic peak at the line center and a three-peaked continuous spectrum with the peaks separated by the Rabi frequency. Attention has been paid mainly to the incoherent part of the spectrum, because the elastic peak in the fluorescence spectrum becomes negligibly small when the driving field is sufficiently strong to saturate the transition. The dressd-atom picture [3,4] has been particularly useful in this area of research, as it clearly explains the physical origin of the three-peaked structure exhibited in the fluorescence and absorption spectra.

Recent investigations [5-27] have shown that a significant variation of the three-peaked structure occurs when the driving field is polychromatic. Even under a bichromatic (fully amplitude modulated) field, fluorescence and absorption spectra are found to exhibit vastly different characteristics. The spectra again contain coherent and incoherent contributions [5,13–15,24]. The coherent intensity, however, no longer decreases steadily to zero as the intensity of the driving field is increased [5]. Furthermore, the incoherent part of the fluorescence and absorption spectra exhibits a series of resonances separated not by the Rabi frequency but by the modulation frequency [7,8]. Physical interpretation of these resonances have been obtained again with the help of the dressed-atom picture [7–10,14,19]. It is, however, no easy task to determine dressed-state energies of a two-level atom interacting with a bichromatic field, because it is necessary in principle to perform diagonalization of an infinite-dimensional matrix [7].

From the practical viewpoint, it is important to understand the interaction of an atom with a polychromatic field, because one frequently employs amplitude-modulated lasers and, in some cases, needs to use more than one laser. Theoretically, however, it is difficult to describe analytically the response of an atom when the field configuration is more complex than bichromatic. Dressed states and energies have been obtained for a two-level atom in a bichromatic field [7–10,14,19] for various configurations, e.g., when the bichromatic field is not resonant with the atomic transition (i.e., when the average frequency of the field is not equal to the atomic frequency) [8], and when the field consists of one strong and one weak frequency component [9]. To our knowledge, however, there exist only a few reports [5,16] in which calculation of dressed state energies for more complex field configurations than a bichromatic driving field is attempted. The problem gets even more difficult if one needs to calculate absorption spectra under the condition that the probe field is also allowed to be intense.

There exists one special situation in which the field has more than two frequency components and analytical expressions for emission and absorption spectra can still be obtained. It is when the field contains an arbitrary number of pairs of frequency components which are symmetrically positioned with respect to the atomic frequency with symmetrically arranged amplitudes. In addition, a single frequency component may be placed at the line center. In fact, it has been known [6,11] that optical Bloch equations describing a two-level atom interacting with a resonant, symmetrically positioned bichromatic field possess analytic solutions in terms of the Bessel functions. Only a straightforward extension of the method used in these treatments is required to treat the atom interacting with a field with more than one pair of symmetrically positioned frequency components. Vitushkin *et al.* [20,21] recently considered an atom interacting with a pentachromatic field, i.e., with a trichromatic pump field which consists of a single frequency component at the line center and a pair of symmetrically positioned frequency components and with an arbitrarily intense bichromatic

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probe field which consists of another pair of symmetrically positioned frequency components. They obtained an analytic expression for the polarization spectrum in terms of Bessel functions. The results of their calculations indicate that the spectrum contains a coherent part which consists of δ-function peaks which they called supernarrow resonances (SNR's) and an incoherent part which corresponds to a continuous spectrum. This appears to be the first investigation that reports a theoretical analysis of discrete absorption peaks. No detailed analysis of the continuous part of the spectrum is given, however; in particular, physical understanding of the resonances in reference to the dressed-atom picture seems desirable.

In this paper we present a theoretical analysis of the polarization spectrum of a two-level atom interacting with an intense trichromatic pump field and an arbitrarily intense bichromatic probe field. Starting with analytic solutions of the optical Bloch equations, an analytic expression for the polarization spectrum can be obtained, real and imaginary parts of which determine the dispersion and absorption spectra, respectively. While an earlier treatment [20,21] of this system focused on discrete peaks that occur in the coherent part of the polarization spectrum, our main consideration is given to resonances in the incoherent part of the spectrum. We introduce, in particular, a semiclassical method of determining dressed-state energies. Using this method we determine dressed-state energies of our atom interacting with a trichromatic pump field and a bichromatic probe field, which then provides physical interpretation of the resonances exhibited in the polarization spectrum. It is thus hoped that our analytical solutions for the polarization spectrum and physical interpretation of the spectrum on the basis of the dressed-atom picture provide important physical insight into the behavior of a two-level atom interacting with a polychromatic field.

In the next section we briefly review analytic solutions of the optical Bloch equations for a two-level atom interacting with a trichromatic pump field and a bichromatic probe field. An analytic expression for the polarization spectrum is presented in Sec. III, which consists of coherent and incoherent contributions. The location and strength of discrete peaks in the coherent part and resonances in the incoherent part are determined. In Sec. IV a semiclassical method of determining dressed-state energies is described and applied to our system. The dressed-atom picture described in this section is shown to provide physical interpretation of the resonances exhibited in the incoherent part of the polarization spectrum. Finally in Sec. V a brief summary is given.

II. SOLUTIONS OF THE OPTICAL BLOCH EQUATIONS

Let us consider a two-level atom (with a lower level $|1\rangle$ of energy ϵ_1 and an upper level $|2\rangle$ of energy ϵ_2) interacting with an arbitrarily intense polychromatic field. The field is assumed to consist of one central pump field component ω_0 resonant with the atomic frequency, $\omega_0=(\epsilon_2-\epsilon_1)/\hbar$, with an amplitude E_0, a pair of symmetrically positioned pump fields at frequencies $\omega_s^{\pm}=\omega_0\pm\delta_s$ with equal amplitudes E_s, and another pair of symmetrically positioned probe fields at frequencies $\omega_p^{\pm}=\omega_0\pm\delta_p$ with equal amplitudes E_p (see Fig. 1). The probe field as well as the pump field can be arbitrarily intense. We consider the situation in which the pair of frequency components $\omega_p^{\pm}$ are simultaneously varied in a symmetrical way, and absorption (or dispersion) is monitored at either of the frequencies, ω_p^{+} or ω_p^{-}. Our system with the field configuration depicted in Fig. 1 contains, as special cases, a two-level atom interacting with a resonant monochromatic pump field when we take $E_s=0$, a two-level atom interacting with a bichromatic pump field when we take $E_0=0$. The properties of the system monitored by a weak probe beam is obtained by taking the limit $E_p\to 0$.

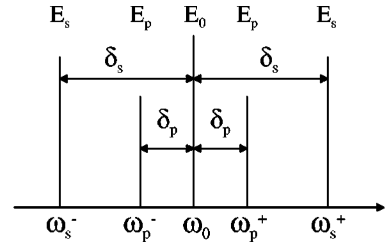

FIG. 1. The field configuration. The pump field has three frequency components ω_0, $\omega_s^{+}=\omega_0+\delta_s$, and $\omega_s^{-}=\omega_s-\delta_s$ (with amplitudes given by E_0, E_s, and E_s, respectively), and the probe field has two frequency components $\omega_p^{+}=\omega_0+\delta_p$ and $\omega_p^{-}=\omega_0-\delta_p$ (with an equal amplitude of E_p).

We assume that all the five frequency components of the field are associated with the same polarization and the same initial phase. The total electric field can then be written as

$$E(t)=\left(\frac{1}{2}E_0+E_s\cos\delta_s t+E_p\cos\delta_p t\right)e^{-i\omega_0 t}+\text{c.c.} \tag{2.1}$$

The interaction of our two-level atom with the field is described under the rotating wave approximation by the optical Bloch equations in the vector model

$$\frac{dU}{dt}=-\gamma U, \tag{2.2}$$

$$\frac{dV}{dt}=-\gamma V+2aW, \tag{2.3}$$

$$\frac{dW}{dt}=-\Gamma(W+1)-2aV, \tag{2.4}$$

where U and V are related to the off-diagonal elements of the density matrix as

$$\rho_{21}=\frac{1}{2}(U-iV)e^{-i\omega_0 t}, \quad \rho_{21}=\rho_{12}^{*}, \tag{2.5}$$

W represents population inversion

$$\rho_{22}-\rho_{11}=W, \tag{2.6}$$

$\gamma(=1/T_2)$ and $\Gamma(=1/T_1)$ are, respectively, the transverse (dipole) and longitudinal (population) relaxation rates, and the parameter a is given by

$$a=\frac{1}{2}\Omega_0+\Omega_s\cos\delta_s t+\Omega_p\cos\delta_p t. \tag{2.7}$$

In Eq. (2.7), Ω_0, Ω_s, and Ω_p are the Rabi frequencies defined by

$$\Omega_0 = \frac{\mu E_0}{\hbar}, \quad \Omega_s = \frac{\mu E_s}{\hbar}, \quad \Omega_p = \frac{\mu E_p}{\hbar}, \tag{2.8}$$

where μ is the component of the eletric dipole moment

$$\vec{\mu} = \langle 1 | e\vec{r} | 2 \rangle \tag{2.9}$$

along the direction of the electric field.

Equations (2.2)–(2.4) have Bessel function solutions when the relaxation rates γ and Γ are equal [6,11,20,21]. For our analytical investigation, we therefore take $\gamma = \Gamma$. Equations (2.3) and (2.4) can then be combined to yield a single equation

$$\frac{dy}{dt} = -\Gamma - (\Gamma + 2ia)y, \tag{2.10}$$

where y is defined as

$$y = W - iV. \tag{2.11}$$

We note from Eq. (2.2) that U decays exponentially with time and does not contribute to the steady-state solution. Using the Bessel identity

$$\exp(iz \sin xt) = \sum_{n=-\infty}^{+\infty} J_n(z) \exp(inxt), \tag{2.12}$$

one can obtain by straightforward algebra a steady-state solution to Eq. (2.10) as

$$y = -\Gamma \sum_k \sum_l \sum_m \sum_n J_k(-Z_s) J_l(-Z_p) J_m(Z_s) J_n(Z_p) \times \frac{\exp\{i[(k+m)\delta_s + (l+n)\delta_p]t\}}{\Gamma + i(\Omega_0 + m\delta_s + n\delta_p)}, \tag{2.13}$$

where $Z_s = 2\Omega_s/\delta_s$ and $Z_p = 2\Omega_p/\delta_p$, and the summations over k, l, m and n here and from now on run from $-\infty$ to ∞ unless otherwise stated. Equation (2.13) was obtained earlier by Vitushkin *et al.* [20,21]. The polarization spectrum is determined by the off-diagonal elements of the density matrix which in turn is determined by V, the imaginary part of y. We proceed in the next section to obtain the expression for the polarization spectrum and present discussion of the coherent and incoherent contributions to the spectrum.

III. POLARIZATION SPECTRUM

In this section we obtain the polarization spectrum, the real and imaginary parts of which give the dispersion and absorption spectra, respectively. The polarization is related to the off-diagonal elements of the density matrix by

$$P = N\mu(\rho_{21} + \rho_{12}) = \frac{N\mu}{2}[(U - iV)e^{-i\omega_0 t} + (U + iV)e^{i\omega_0 t}], \tag{3.1}$$

where N is the density of the atom. Since U decays exponentially with time, it does not contribute to P at steady state. The component of polarization oscillating at $\omega_p^{\pm} = \omega_0 \pm \delta_p$ is determined from

$$P(\omega_p^{\pm} = \omega_0 \pm \delta_p) = -\frac{iN\mu}{2} \langle V e^{\pm i\delta_p t} \rangle_t, \tag{3.2}$$

where $\langle \rangle_t$ indicates time averaging. Due to the symmetric field configuration we have

$$P(\omega_p^+) = -P^*(\omega_p^-) \tag{3.3}$$

and it is sufficient to consider $P(\omega_p^+)$ only.

It should be stressed that the polarization spectrum has its physical significance, because it is directly related to the refractive index and the absorption coefficient of the system which can be measured experimentally. The polarization component of Eq. (3.2) is related to the dimensionless first-order susceptibility $[\chi(\omega_p^+)/\chi_0]$ by

$$\chi(\omega_p^+)/\chi_0 = \frac{\Gamma/N\mu}{\Omega_p} P(\omega_p^+), \tag{3.4}$$

where $\chi_0 = N\mu^2/\hbar\Gamma$. The refractive index and absorption coefficient, respectively, are given by the real and imaginary parts of $[\chi(\omega_p^+)/\chi_0]$.

Using Eqs. (2.11), (2.13), and (3.2), it is a simple matter to obtain the following expression for $P(\omega_p^+)$:

$$\begin{aligned} P(\omega_p^+) = -\frac{N\mu\Gamma}{4} \sum_k \sum_l \sum_m \sum_n & \times \Bigg\{ \frac{J_k(-Z_s) J_l(-Z_p) J_m(Z_s) J_{n-1}(Z_p)}{\Gamma^2 + [\Omega_0 + m\delta_s + (n-1)\delta_p]^2} \\ & \times \langle e^{i[(k+m)\delta_s + (l+n)\delta_p]t} \\ & \times [\Gamma - i(\Omega_0 + m\delta_s + (n-1)\delta_p)] \rangle_t \\ & - \frac{J_k(-Z_s) J_l(-Z_p) J_m(Z_s) J_{n+1}(Z_p)}{\Gamma^2 + [\Omega_0 + m\delta_s + (n+1)\delta_p]^2} \\ & \times \langle e^{-i[(k+m)\delta_s + (l+n)\delta_p]t} \\ & \times [\Gamma + i(\Omega_0 + m\delta_s + (n+1)\delta_p)] \rangle_t \Bigg\}. \end{aligned} \tag{3.5}$$

It is clear from Eq. (3.5) that only the terms that satisfy the condition

$$(k+m)\delta_s + (l+n)\delta_p = 0 \tag{3.6}$$

survive the time averaging. Equation (3.6) is always satisfied regardless of δ_p and δ_s if

$$m = -k \quad \text{and} \quad n = -l. \tag{3.7}$$

The part of $P(\omega_p^+)$ which is obtained by imposing the condition, Eq. (3.7), on Eq. (3.5) corresponds to the continuous part of the polarization spectrum, which we refer to as the incoherent contribution. Equation (3.6) can also be satisfied at particular values of δ_p given by

$$\delta_p = -\frac{k+m}{l+n}\delta_s, \quad n \neq -l. \tag{3.8}$$

The condition given by Eq. (3.8) contributes δ-function type of peaks to the polarization spectrum, which we refer to as the coherent contribution. These peaks were called supernarrow resonances and investigated earlier by Vitushkin *et al.* [20–22]. We discuss below the coherent and incoherent contributions in more detail.

A. Coherent contribution

The coherent peaks (or dips) can be calculated by substituting Eq. (3.8) into Eq. (3.5). Since the indices k,l,m, and n run from $-\infty$ to ∞, there are in principle an infinite number of discrete peaks in the polarization spectrum. The strength of each peak of course is different for different peaks and is determined by the magnitude of the terms involving the Bessel functions in Eq. (3.5). Since these peaks were previously analyzed in detail [20,21], we give only a brief discussion.

It has been known for long that discrete peaks occur in the fluorescence spectrum of a two-level atom. Mollow [1] has shown that, in addition to the characteristic three-peaked continuous spectrum, there exists a discrete elastic peak at the line center in the fluorescence spectrum of a two-level atom driven by a resonant monochromatic field. The strength of the peak, however, is negligibly small when the intensity of the driving field is sufficiently strong to saturate the transition. A series of δ-function type of discrete peaks have been shown to appear in the fluorescence spectrum when a two-level atom interacts with a bichromatic or a trichromatic field [5,13–15,24]. It was found that these peaks can give significant contributions to absorption even when the driving field is strong, and that the peaks are separated by the modulation frequency, consistent with the condition of Eq. (3.8). With a few exceptions [20–22], however, no particular attention has been paid in the past to the coherent peaks in the absorption spectrum.

Calculation of the position and strength of the coherent peaks is simpler in the limit of a weak probe field $\Omega_p \to 0$. In the limit $\Omega_p \to 0$, only the terms with $l=0$ and $n=1$ in the first term in the curly bracket on the right hand side of Eq. (3.5) and those with $l=0$ and $n=-1$ in the second term contribute significantly to $P(\omega_p^+)$. It is then clear from Eq. (3.8) that only those peaks at $\delta_p = n\delta_s$ $(n=0,\pm1,\pm2,\ldots)$ are important. The strength of the peak at $\omega_p^+ = \omega_0 + n\delta_s$ $(n=0,1,2,\ldots)$ can be calculated from Eq. (3.5) and is given by

$$\mathrm{Re}\, P^C(\omega_p^+ = \omega_0 + n\delta_s) \cong -\frac{N\mu\Gamma^2}{4}\sum_m \frac{J_m(Z_s)}{\Gamma^2 + (\Omega_0 + m\delta_s)^2} \times [J_{-n-m}(-Z_s) - J_{n-m}(-Z_s)], \tag{3.9}$$

$$\mathrm{Im}\, P^C(\omega_p^+ = \omega_0 + n\delta_s) \cong \frac{N\mu\Gamma}{4}\sum_m \frac{J_m(Z_s)(\Omega_0 + m\delta_s)}{\Gamma^2 + (\Omega_0 + m\delta_s)^2} \times [J_{-n-m}(-Z_s) + J_{n-m}(-Z_s)], \tag{3.10}$$

where the superscript C stands for coherent contribution, and real and imaginary parts of the coherent contribution to the polarization spectrum are written separately. If we further take $\Omega_s = 0$ in Eq. (3.10), we find that all the peaks disappear [i.e., $\mathrm{Im}\, P^C(\omega_p^+ = \omega_0 + n\delta_s) = 0$] except the peak at the line center (i.e., at $\omega_p^+ = \omega_0$). The strength of this central peak is given by

$$\mathrm{Im}\, P^C(\omega_p^+ = \omega_0) \cong \frac{N\mu\Gamma}{2}\frac{\Omega_0}{\Gamma^2 + \Omega_0^2}. \tag{3.11}$$

Equation (3.11) indicates that there is a single peak at the line center in the absorption spectrum of a two-level atom driven by a resonant monochromatic field. On the other hand, when the atom is driven by an amplitude-modulated (bichromatic or trichromatic) field, a series of discrete peaks separated by the modulation frequency appear, as indicated by Eqs. (3.9) and (3.10).

B. Incoherent contribution

We now consider the continuous part of the polarization spectrum which can be obtained by substituting the condition, Eq. (3.7), into Eq. (3.5). After straightforward algebra we find

$$\mathrm{Re}\, P^I(\omega_p^+) = -\frac{N\mu\Gamma^2}{4}\sum_k \sum_l \times \frac{J_k(-Z_s)J_{-k}(Z_s)J_{-l}(Z_p)}{\Gamma^2 + (\Omega_0 - k\delta_s - l\delta_p)^2} \times [J_{l-1}(-Z_p) - J_{l+1}(-Z_p)], \tag{3.12}$$

$$\mathrm{Im}\, P^I(\omega_p^+) = \frac{N\mu\Gamma}{4}\sum_k \sum_l \times \frac{J_k(-Z_s)J_{-k}(Z_s)J_{-l}(Z_p)(\Omega_0 - k\delta_s - l\delta_p)}{\Gamma^2 + (\Omega_0 - k\delta_s - l\delta_p)^2} \times [J_{l-1}(-Z_p) + J_{l+1}(-Z_p)], \tag{3.13}$$

where the superscript I stands for incoherent contribution. Equations (3.12) and (3.13) show that resonances occur, in the dispersion and absorption spectra of the atom, at $\delta_p = \pm\Omega_0, \pm(\Omega_0 \pm \delta_s), \pm(\Omega_0 \pm 2\delta_s), \ldots$, and their subharmonics. The width of the resonances at $\delta_p = (\Omega_0 - k\delta_s)/l$, the lth subharmonic of $(\Omega_0 - k\delta_s)$, where $k=0,\pm1,\pm2,\ldots$, is given by Γ/l. In order to better understand these resonances, which may be called generalized Rabi resonances, we consider some simple cases below.

1. $\Omega_s = 0$

This is the case for a monochromatic pump field at ω_0 and a bichromatic probe field at ω_p^+ and ω_p^-. Equations (3.12) and (3.13) in this case become

$$\mathrm{Re}\, P^I(\omega_p^+) = -\frac{N\mu\Gamma^2}{4}\sum_l \frac{J_{-l}(Z_p)}{\Gamma^2+(\Omega_0-l\delta_p)^2}\times[J_{l-1}(-Z_p)-J_{l+1}(-Z_p)], \quad (3.14)$$

$$\mathrm{Im}\, P^I(\omega_p^+) = \frac{N\mu\Gamma}{4}\sum_l \frac{J_{-l}(Z_p)(\Omega_0-l\delta_p)}{\Gamma^2+(\Omega_0-l\delta_p)^2}\times[J_{l-1}(-Z_p)+J_{l+1}(-Z_p)]. \quad (3.15)$$

If we further assume that the probe field is weak, i.e., if we take the limit $\Omega_p \to 0$ in Eqs. (3.14) and (3.15), we obtain

$$\mathrm{Re}\, P^I(\omega_p^+) \cong -\frac{N\mu\Gamma^2 Z_p}{8}\left[\frac{2}{\Gamma^2+\Omega_0^2}-\frac{1}{\Gamma^2+(\Omega_0-\delta_p)^2}-\frac{1}{\Gamma^2+(\Omega_0+\delta_p)^2}\right], \quad (3.16)$$

$$\mathrm{Im}\, P^I(\omega_p^+) \cong -\frac{N\mu\Gamma Z_p}{8}\left[\frac{\Omega_0-\delta_p}{\Gamma^2+(\Omega_0-\delta_p)^2}-\frac{\Omega_0+\delta_p}{\Gamma^2+(\Omega_0+\delta_p)^2}\right]. \quad (3.17)$$

Equations (3.16) and (3.17) represent dispersion and absorption spectra for the case of a two-level atom driven by a strong resonant monochromatic pump field and monitored by a weak bichromatic probe field. One sees that the dispersion curve exhibits Lorentzian peaks at $\delta_p = \pm\Omega_0$, while the absorption curve exhibits a dispersion-like profile with resonances at $\delta_p = \pm\Omega_0$.

Comparison of Eqs. (3.14) and (3.15) with Eqs. (3.16) and (3.17) indicate that, as the probe intensity is increased, resonances occur not only at $\delta_p = \pm\Omega_0$ but also at their subharmonics $\delta_p = \pm\Omega_0/2, \pm\Omega_0/3, \ldots$. These subharmonic resonances for the case of a strong monochromatic pump field and a strong monochromatic probe field have been observed experimentally [6,23,28–30] and analyzed theoretically [6,31]. The subharmonic resonances can be understood as arising from multiphoton transitions between nonadjacent dressed-state pairs [29], as illustrated in Fig. 2.

2. $\Omega_0 = 0$

This corresponds to the case of a bichromatic pump field and a bichromatic probe field. Substituting $\Omega_0 = 0$ into Eqs. (3.12) and (3.13), we see immediately that resonances occur at frequencies $\delta_p = 0, \pm\delta_s, \pm 2\delta_s, \ldots$, and their subharmonics. As above, these subharmonic resonances arise from multiphoton transitions involving probe photons. If we further take the limit of a weak probe field, $\Omega_p \to 0$, we obtain

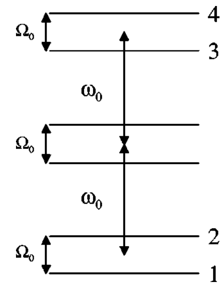

FIG. 2. Dressed states of a two-level atom driven by a resonant monochromatic field. A two photon transition between the two dressed states 1 and 4 gives rise to a resonance at $\omega_0 + \Omega_0/2$. Similarly, a two photon transition between the two dressed states 2 and 3 gives rise to a resonance at $\omega_0 - \Omega_0/2$.

$$\mathrm{Re}\, P^I(\omega_p^+) = -\frac{N\mu\Gamma^2 Z_p}{4}\sum_k \left[\frac{J_k(-Z_s)J_{-k}(Z_s)}{\Gamma^2+(k\delta_s)^2}-\frac{J_k(-Z_s)J_{-k}(Z_s)}{\Gamma^2+(k\delta_s-\delta_p)^2}\right], \quad (3.18)$$

$$\mathrm{Im}\, P^I(\omega_p^+) = -\frac{N\mu\Gamma Z_p}{4}\sum_k \frac{J_k(-Z_s)J_{-k}(Z_s)(k\delta_s-\delta_p)}{\Gamma^2+(k\delta_s-\delta_p)^2}. \quad (3.19)$$

Now, with a weak probe field, resonances occur at $\delta_p = 0$, $\pm\delta_s, \pm 2\delta_s, \ldots$, but not at their subharmonics, because multiphoton transitions involving probe photons are negligible.

The response of a two-level atom driven by a bichromatic pump field has been studied intensively in the past [5–27], because theoretically this problem represents a natural extension of the standard problem of a two-level atom driven by a monochromatic field and also because a bichromatic field corresponds to a practically important case of a fully amplitude-modulated field. It is already well known that the absorption spectrum in a bichromatic field differs significantly from that in a monochromatic field; instead of the triplet structure found for the case of a monochromatic driving field, the spectrum consists of a series of resonances separated not by the Rabi frequency but by the modulation frequency [7,8]. These resonances can be best interpreted with the help of the dressed-atom approach. An earlier theoretical investigation [7] has shown that dressed states for a two-level atom driven by a strong resonant bichromatic field comprise of manifolds of levels, with the neighboring manifolds separated in frequency by the atomic frequency $\omega_0 = (\epsilon_2 - \epsilon_1)/\hbar$. Each manifold consists of an infinite number of sublevels separated in frequency by the modulation frequency δ_s. The transition between two arbitrary dressed states belonging to neighboring manifolds should then produce resonances at frequency $\omega_0 + m\delta_s$ ($m = 0, \pm 1, \pm 2, \ldots$).

3. Trichromatic pump field

Returning to Eqs. (3.12) and (3.13), it is natural to expect that the resonances at $\delta_p = (\Omega_0 - k\delta_s)/l$ can be explained if dressed-state energies of the atom interacting with a trichro-

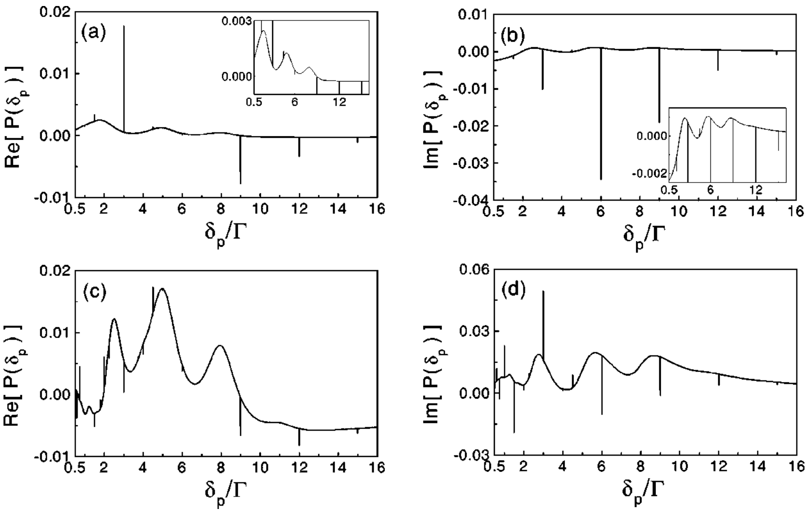

FIG. 3. Real [(a),(c)] and imaginary [(b),(d)] parts of the dimensionless polarization spectrum of a two-level atom interacting with a trichromatic pump field and a bichromatic probe field. The parameters are $\Omega_0/\Gamma=5$, $\Omega_s/\Gamma=2$, $\delta_s/\Gamma=3$, where the probe Rabi frequency is chosen to be $\Omega_p/\Gamma=0.1$ for (a) and (b), and $\Omega_p/\Gamma=2$ for (c) and (d).

matic pump field and a bichromatic probe field are determined. We note, however, that it is no easy matter to determine the dressed-state energies of an atom driven by a bichromatic field alone, as it required diagonalizing an infinite-dimensional matrix [7]. It seems very difficult to extend this method to the present case of a trichromatic pump field and an arbitrarily intense bichromatic probe field. We therefore introduce in the next section a semiclassical way of approximately determining dressed-state energies. This method is relatively simple and can be applied to obtain dressed-state energies for the present case of a two-level atom interacting with a pentachromatic field.

C. Example

In Figs. 3(a)–3(d) we present results of computation of the real and imaginary parts of the polarization spectrum for the case of a trichromatic pump field and a bichromatic probe field. The parameters were chosen to be $\Omega_0=5,\Omega_s=2$, and $\delta_s=3$, where all frequencies are measured with respect to Γ which is taken to be 1. Figures 3(a) and 3(b) show, respectively, the real and imaginary parts of the polarization spectrum when the probe field is weak ($\Omega_p=0.1$), whereas Figures 3(c) and 3(d) are drawn for the case when the probe field is as strong as the pump field ($\Omega_p=2$). Figures 3(a) and 3(b) indicate that, when the probe field is weak, coherent contributions are strong. The discrete peaks (or dips) at $\delta_p=3,\ 6,\ 9$, and 12 can clearly be identified. These locations are consistent with the formula $\delta_p=n\delta_s$ with $n=1,\ 2,\ 3$, and 4, as discussed in Sec. III A. The insets in Figs. 3(a) and 3(b) show more detailed views of the continuous part of the spectrum. The resonances at $\delta_p=2$,5, and 8 can clearly be identified. These locations of the resonances correspond to $\delta_p=\Omega_0-k\delta_s$ with $k=1,-1$, and -2 and are consistent with the locations of the resonances predicted by Eqs. (3.12) and (3.13), where $l=0$ should be taken for this case of a weak probe field. When the probe field is allowed to be strong, one sees from Figs. 3(c) and 3(d) that more discrete peaks and resonances appear. Discrete peaks occur not only at $\delta_p=n\delta_s$ but also at the subharmonics of these frequencies as indicated by Eq. (3.8), and resonances in the continuous part of the spectrum occur not only at $\delta_p=\Omega_0-k\delta_s$ but also at the subharmonics of these frequencies as indicated by Eqs. (3.12) and (3.13).

IV. DRESSED-ATOM APPROACH

In this section we use a semiclassical dressed-atom approach to explain the resonance structure exhibited in the continuous part of the polarization spectrum. The starting point of our semiclassical dressed-atom approach is the coupled equations for the probability amplitudes $c_1(t)$ and $c_2(t)$,

$$\frac{dc_1(t)}{dt}=i\left(\frac{\Omega_0}{2}+\Omega_s\cos\delta_s t+\Omega_p\cos\delta_p t\right)c_2(t), \tag{4.1}$$

$$\frac{dc_2(t)}{dt}=i\left(\frac{\Omega_0}{2}+\Omega_s\cos\delta_s t+\Omega_p\cos\delta_p t\right)c_1(t), \tag{4.2}$$

where the probability amplitudes are defined by

$$|\Psi(t)\rangle=c_1(t)e^{-i\omega_1 t}|1\rangle+c_2(t)e^{-i\omega_2 t}|2\rangle, \tag{4.3}$$

$\omega_1=\epsilon_1/\hbar,\omega_2=\epsilon_2/\hbar$, and, as in Sec. II, we treat the field classically and assume that the field has five frequency components shown in Fig. 1 so that the total electric field is given by Eq. (2.1), and the Rabi frequencies Ω_0,Ω_s, and Ω_p are defined in Eq. (2.8). Equations (4.1) and (4.2) are equivalent to the Schrödinger equation except that the rotating wave approximation is made to arrive at these equations. We assume that the atom is initially in the lower level $|1\rangle$ so that $c_1(0)=1,c_2(0)=0$.

In order to solve Eqs. (4.1) and (4.2) we make an ansatz

$$c_1(t)=\cos\xi(t),\quad c_2(t)=i\sin\xi(t) \tag{4.4}$$

and substitute it into Eqs. (4.1) and (4.2). One can easily find that Eqs (4.1) and (4.2) are solved if $\xi(t)$ is given by

$$\xi(t)=\frac{\Omega_0}{2}t+\frac{\Omega_s}{\delta_s}\sin\delta_s t+\frac{\Omega_p}{\delta_p}\sin\delta_p t. \tag{4.5}$$

The state vector $|\Psi(t)\rangle$ in Eq. (43) can then be written as

$$
\begin{aligned}
|\Psi(t)\rangle &= \cos\xi(t)e^{-i\omega_1 t}|1\rangle + i\sin\xi(t)e^{-i\omega_2 t}|2\rangle \\
&= \frac{1}{2}\left(e^{i(\Omega_0/2)t}e^{i(\Omega_s/\delta_s)\sin\delta_s t}e^{i(\Omega_p/\delta_p)\sin\delta_p t} + \text{c.c.}\right) \\
&\quad \times e^{-i\omega_1 t}|1\rangle + \frac{1}{2}\left(e^{i(\Omega_0/2)t}e^{i(\Omega_s/\delta_s)\sin\delta_s t}e^{i(\Omega_p/\delta_p)\sin\delta_p t}\right. \\
&\quad \left. - \text{c.c.}\right)e^{-i\omega_2 t}|2\rangle. \qquad (4.6)
\end{aligned}
$$

Using the identity, Eq. (2.12), we rewrite Eq. (4.6) as

$$
\begin{aligned}
|\Psi(t)\rangle = \frac{1}{2}\sum_n \sum_m \Bigg\{ &\Bigg[e^{-i(\omega_1-\Omega_0/2-n\delta_s-m\delta_p)t} J_n\left(\frac{\Omega_s}{\delta_s}\right) J_m\left(\frac{\Omega_p}{\delta_p}\right) \\
&+ e^{-i(\omega_1+\Omega_0/2-n\delta_s-m\delta_p)t} J_n\left(-\frac{\Omega_s}{\delta_s}\right) J_m\left(-\frac{\Omega_p}{\delta_p}\right)\Bigg]|1\rangle \\
&+ \Bigg[e^{-i(\omega_2-\Omega_0/2-n\delta_s-m\delta_p)t} J_n\left(\frac{\Omega_s}{\delta_s}\right) J_m\left(\frac{\Omega_p}{\delta_p}\right) \\
&- e^{-i(\omega_2+\Omega_0/2-n\delta_s-m\delta_p)t} J_n\left(-\frac{\Omega_s}{\delta_s}\right) J_m\left(-\frac{\Omega_p}{\delta_p}\right)\Bigg]|2\rangle \Bigg\}.
\end{aligned}
\qquad (4.7)
$$

Equation (4.7) is the main equation which provides the basic information for our semiclassical dressed-atom approach. We first consider simple cases.

A. $\Omega_s = 0$

Taking $\Omega_s = 0$ in Eq. (4.7) to consider the case of a monochromatic pump field, we obtain

$$
\begin{aligned}
|\Psi(t)\rangle = \frac{1}{2}\sum_m \Bigg\{ &\Bigg[e^{-i(\omega_1-\Omega_0/2-m\delta_p)t} J_m\left(\frac{\Omega_p}{\delta_p}\right) \\
&+ e^{-i(\omega_1+\Omega_0/2-m\delta_p)t} J_m\left(-\frac{\Omega_p}{\delta_p}\right)\Bigg]|1\rangle \\
&+ \Bigg[e^{-i(\omega_2-\Omega_0/2-m\delta_p)t} J_m\left(\frac{\Omega_p}{\delta_p}\right) \\
&- e^{-i(\omega_2+\Omega_0/2-m\delta_p)t} J_m\left(-\frac{\Omega_p}{\delta_p}\right)\Bigg]|2\rangle \Bigg\}. \qquad (4.8)
\end{aligned}
$$

If we further take the limit $\Omega_p \to 0$ in Eq. (4.8) to consider the case of a weak probe beam, Eq. (4.8) becomes

$$
\begin{aligned}
|\Psi(t)\rangle \cong &\frac{1}{2}\left[e^{-i(\omega_1-\Omega_0/2)t} + e^{-i(\omega_1+\Omega_0/2)t}\right]|1\rangle \\
&+ \frac{1}{2}\left[e^{-i(\omega_2-\Omega_0/2)t} - e^{-i(\omega_2+\Omega_0/2)t}\right]|2\rangle. \qquad (4.9)
\end{aligned}
$$

Inspection of the phase terms in Eq. (4.9) suggests that the interaction with the monochromatic pump field splits the bare atomic levels $|1\rangle$ and $|2\rangle$ each into two, as shown in Fig. 4(a). Dressed-state energies are obtained by adding photon energies to the energy of each of these levels. The dressed states so obtained can be grouped into manifolds having the same number of excitation units. For example, the upper pair of states in Fig. 4(a) combined with $(N-1)$ photons comprise manifold N as do the lower pair of states combined with N photons. The resulting structure of dressed states are shown in Fig. 4(b), which of course coincides with the well-known dressed-state structure [3,4] for a two-level atom driven by a resonant monochromatic field.

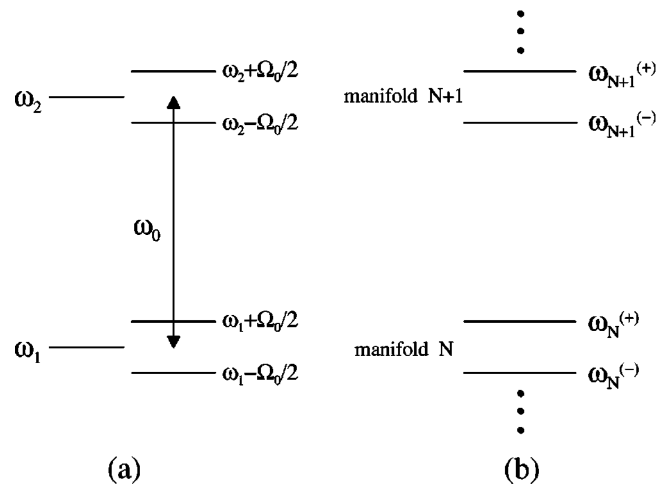

FIG. 4. (a) Splitting of the bare atomic levels caused by interaction with a monochromatic field. (b) Dressed states of a two-level atom driven by a monochromatic field. $\omega_N^{(+)} = \omega_1 + \Omega_0/2 + N\omega_0 = \omega_2 + \Omega_0/2 + (N-1)\omega_0$, $\omega_N^{(-)} = \omega_1 - \Omega_0/2 + N\omega_0 = \omega_2 - \Omega_0/2 + (N-1)\omega_0$.

Let us now return to Eq. (4.8) which applies to the case of a monochromatic pump field and an intense bichromatic probe field. Inspection of the phase terms in Eq. (4.8) indicates that the interaction with a strong probe beam splits the atomic levels of Fig. 4(a) further into sublevels separated by δ_p. As mentioned earlier, the physical mechanism responsible for this splitting is multiphoton transitions involving probe photons. These multiphoton transitions produce subharmonic resonances at $\delta_p = \pm\Omega_0/l$ $(l=2,3,4,\ldots)$ in the dispersion and absorption spectra, as illustrated in Fig. 2.

B. $\Omega_0 = 0$

We now consider the case of a bichromatic pump field $(\Omega_0 = 0)$ in the limit of a weak probe field $(\Omega_p \to 0)$. Substituting $\Omega_0 = 0$ and taking $\Omega_p \to 0$ in Eq. (4.7), we obtain

$$
\begin{aligned}
|\Psi(t)\rangle = \frac{1}{2}\sum_n \Bigg\{ &e^{-i(\omega_1-n\delta_s)t}\left[J_n\left(\frac{\Omega_s}{\delta_s}\right) + J_n\left(-\frac{\Omega_s}{\delta_s}\right)\right]|1\rangle \\
&+ e^{-i(\omega_2-n\delta_s)t}\left[J_n\left(\frac{\Omega_s}{\delta_s}\right) - J_n\left(-\frac{\Omega_s}{\delta_s}\right)\right]|2\rangle \Bigg\}.
\end{aligned}
\qquad (4.10)
$$

Since $J_n(-x) = (-1)^n J_n(x)$, the term in the first square bracket on the right hand side of Eq. (4.10) (i.e., the term multiplied by the state $|1\rangle$) vanishes for odd n and the term in the second square bracket (i.e., the term multiplied by the state $|2\rangle$) vanishes for even n. (It should be noted, however, that, if we assume that the atom is initially in the upper state $|2\rangle$, the term multiplied by the state $|1\rangle$ vanishes for even n and the term multiplied by the state $|2\rangle$ vanishes for odd n.) Equation (4.10) therefore indicates that the interaction of the

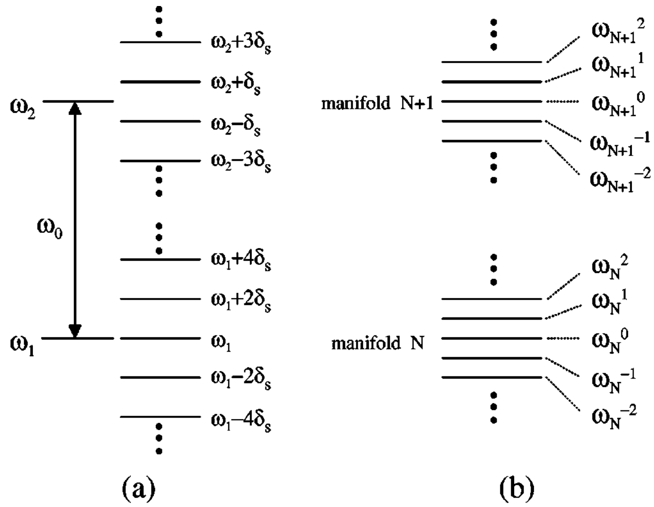

FIG. 5. (a) Splitting of the bare atomic levels caused by interaction with a bichromatic field. The atom is assumed to be in the lower level at the initial time. (b) Dressed states of a two-level atom driven by a bichromatic field. $\omega_N^{2m}=\omega_1+2m\delta_s+N\omega_0$, $\omega_N^{2m+1}=\omega_2+(2m+1)\delta_s+(N-1)\omega_0$, $m=0,\pm1,\pm2,\ldots$.

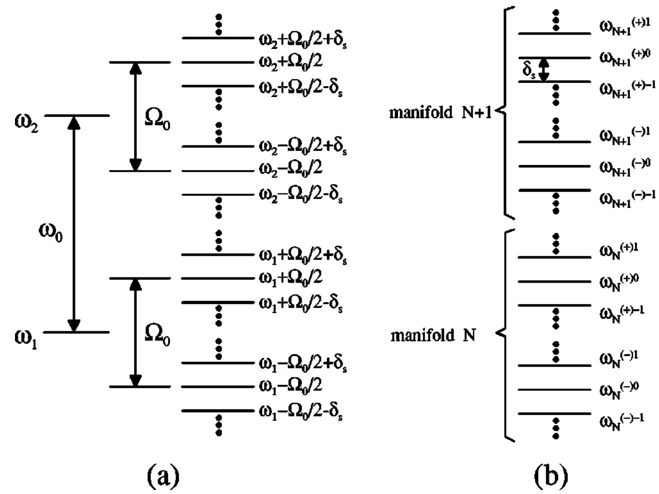

FIG. 6. (a) Splitting of the bare atomic levels caused by interaction with a trichromatic field. (b) Dressed states of a two-level atom driven by a trichromatic field. $\omega_N^{(+)m}=\omega_1+\Omega_0/2+m\delta_s+N\omega_0=\omega_2+\Omega_0/2+m\delta_s+(N-1)\omega_0$, $\omega_N^{(-)m}=\omega_1-\Omega_0/2+m\delta_s+N\omega_0=\omega_2-\Omega_0/2+m\delta_s+(N-1)\omega_0$, $m=0,\pm1,\pm2,\ldots$.

atom with the bichromatic pump field splits the atomic states $|1\rangle$ and $|2\rangle$ each into an infinite number of sublevels separated by $2\delta_s$, as illustrated in Fig. 5(a). As in Sec. IV A, the dressed states can be grouped into manifolds having the same number of excitation units, where the number of excitation units is given by $N=N_a+N_++N_-$, N_a is 1 if the atom is in the upper level and 0 if in the lower level, and N_+ and N_- are the numbers of photons of frequency $\omega_0+\delta_s$ and $\omega_0-\delta_s$, respectively. Noting that the bare atomic levels split differently according to whether the atom is initially in the lower or upper level, the dressed-level structure shown in Fig. 5(b) is obtained. One sees that each manifold consists of an infinite series of sublevels separated by the modulation frequency δ_s. The same dressed-level structure has already been obtained previously using a different method [7]. The series of resonances separated by the modulation frequency observed in the spectrum of a two-level atom driven by a bichromatic field can be easily explained with the help of the dressed-atom picture shown in Fig. 5(b).

If we take $\Omega_0=0$ but do not take the limit $\Omega_p\rightarrow0$ in Eq. (4.7), we obtain an expression for $\Psi(t)$ according to which the bare atomic levels $|1\rangle$ and $|2\rangle$ split into an infinite number of sublevels with energies given by $\hbar(\omega_1+n\delta_s+m\delta_p)$ and $\hbar(\omega_2+n\delta_s+m\delta_p)$, respectively, where n and m are integers. Clearly, a strong probe beam causes further splitting of dressed states from those shown in Fig. 5(b). Because multiphoton transitions involving probe photons become important as the intensity of the probe beam is increased, subharmonic resonances at $\delta_p=(-k\delta_s)/l$ $(l=\pm2,\pm3,\pm4,\ldots)$ appear in the polarization spectrum.

C. Trichromatic pump field

We now return to Eq. (4.7) and consider the case of a trichromatic pump field. In order to avoid complications arising from multiphoton transitions involving probe photons, we take the limit of a weak probe beam, $\Omega_p\rightarrow0$. Equation (4.7) then becomes

$$|\Psi(t)\rangle=\frac{1}{2}\sum_n\left\{\left[e^{-i(\omega_1-\Omega_0/2-n\delta_s)t}J_n\left(\frac{\Omega_s}{\delta_s}\right)+e^{-i(\omega_1+\Omega_0/2-n\delta_s)t}J_n\left(-\frac{\Omega_s}{\delta_s}\right)\right]|1\rangle+\left[e^{-i(\omega_2-\Omega_0/2-n\delta_s)t}J_n\left(\frac{\Omega_s}{\delta_s}\right)-e^{-i(\omega_2+\Omega_0/2-n\delta_s)t}J_n\left(-\frac{\Omega_s}{\delta_s}\right)\right]|2\rangle\right\}. \quad (4.11)$$

Equation (4.11) suggests the level structure shown in Fig. 6(a). The corresponding dressed states can be grouped into manifolds of levels, where manifold N is represented by a collection of states for which $N=N_a+N_0+N_++N_-$, N_a is 1 or 0 depending on whether the atom is in the upper or lower level, and N_0,N_+,N_- are the numbers of photons of frequency $\omega_0,\omega_0+\delta_s$, and $\omega_0-\delta_s$, respectively. The dressed-state structure for the present case, i.e., for a two-level atom interacting with a trichromatic field, is shown in Fig. 6(b). It is clear from Fig. 6(b) that resonances occur at $\pm(\Omega_0+n\delta_s)$, $n=0,\pm1,\pm2,\ldots$. If we relax the condition that the probe field is weak, then resonances should occur not only at $\pm(\Omega_0+n\delta_s)$ but also at the subharmonics of these frequencies. The resonances at $\delta_p=\pm(\Omega_0-k\delta_s)/l$ suggested by Eqs. (3.12) and (3.13) can thus be explained with the dressed-atom picture obtained with our simple method.

V. SUMMARY

Analytic solutions of the optical Bloch equations describing a two-level atom interacting with a trichromatic pump field of frequency components ω_0 and $\omega_0\pm\delta_s$ and with a bichromatic probe field of frequency components $\omega_0\pm\delta_p$ are presented and used to obtain an analytic expression for the polarization spectrum of the atom. It is found that the spectrum contains a coherent contribution which consists of a series of discrete δ-function peaks (or dips) and an incoherent contribution which gives rise to a continuous part of the spectrum. A simple semiclassical method of obtaining dressed-state energies is introduced and used to obtain dressed-state energies for the present system of a two-level atom interacting with a trichromatic pump field and a bichromatic probe field. Resonances that occur in the polarization spectrum can be successfully explained using our semiclassical dressed-atom approach.

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