Selection rule in the theory of laser-assisted charged-particle scattering

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It is shown that the exchange of an odd number of photons in laser-assisted charge-particle scattering is forbidden when the laser polarization is perpendicular to the scattering plane. It is proposed that this selection rule may be used to test the satisfaction of single-collision conditions under critical geometrical conditions where experimental count rates tend to become prohibitively small. [S1050-2947(99)08112-3]

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The Kroll-Watson (KW) approximation of laser-assisted electron-atom scattering has been the subject of several experimental [1–4] and theoretical [5–11] investigations in recent years (see, e.g., Ref. [12] for a recent review). Basically, the theory predicts that the laser-assisted amplitude for scattering from asymptotic initial momentum $\vec{q}_i$ to final momentum $\vec{q}_f$ in a short-range potential accompanied by the exchange of n photons from a harmonic field of frequency ω ($q_f^2/2 = q_i^2/2 - n\omega$) is given by [13] [atomic units (a.u.) are used throughout]

$$f_n(\vec{q}_f,\vec{q}_i) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt \exp\left[-i\left(n\omega t - \frac{A_0}{\omega}\vec{\epsilon}\cdot\vec{Q}\sin\omega t\right)\right] \times f(\vec{p}_f(t),\vec{p}_i(t)), \tag{1}$$

with $\vec{\epsilon}$ the linear polarization of the laser, A_0 the amplitude of the vector potential, $\vec{Q} = \vec{q}_f - \vec{q}_i$ the electron momentum transfer, and $f(\vec{p}_f(t),\vec{p}_i(t))$ the field-free off-shell amplitude evaluated at kinematical momenta $\vec{p}_j = \vec{q}_j + \vec{A}(t) (j = i,f)$.

In the classically allowed region where $|n|$ is less than the classical cutoff in the number of exchanged photons, $n_c = |(A_0/\omega)\vec{\epsilon}\cdot\vec{Q}|$ (see, e.g., Ref. [5] and references therein), Eq. (1) may be evaluated in the stationary-phase approximation to derive the well-known result of Kroll and Watson [13],

$$f_n(\vec{q}_f,\vec{q}_i) = J_n\left(\frac{A_0}{\omega}\vec{\epsilon}\cdot\vec{Q}\right) f(\vec{q}_f + \vec{\gamma}, \vec{q}_i + \vec{\gamma}). \tag{2}$$

Here $J_n(x)$ is a Bessel function of integer order and $\vec{\gamma} = n\omega\vec{\epsilon}/(\vec{\epsilon}\cdot\vec{Q})$ is exactly the momentum displacement needed to keep the field-free scattering amplitude on the energy shell, $|\vec{q}_f + \vec{\gamma}| = |\vec{q}_i + \vec{\gamma}|$. This expression is referred to as the KW approximation and we adhere to that convention in the following while the more general expression in Eq. (1), also given in the original paper of Kroll and Watson [13], will be referred to as the impulse approximation in accord with standard terminology in atomic collision theory.

Detailed experimental tests of Eq. (2) have been done in the regime where the momentum transfer in the direction of the laser field, $\vec{\epsilon}\cdot\vec{Q}$, is so small that exchange of photons is classically forbidden, $n_c < |n|$ [2,3]. When this is the case, the stationary phase argument no longer applies and Eq. (2) is not well founded. Instead, one has to consider the full expression of Eq. (1). This has been done in the case of a static model potential in Ref. [9] and in the case of a zero-range potential in Ref. [5]. In the latter case an exact analytical solution is possible. The zero-range potential may therefore serve as a unique testing ground for the validity of various theoretical approximation methods.

In a recent theoretical study [11], the possible role of double scattering has been discussed in order to explain the differences between the experimental results [2,3] and the KW approximation. The selection rule derived in this paper will provide a basis for an experimental investigation of the importance of multiple scattering events. As discussed in more detail below, deviations from the prediction of the selection rule will be a signature of the presence of double scattering or a low angular resolution in the experiment.

It is to be noted that the separation between classically allowed and forbidden regions is governed by the projection of the electron momentum transfer vector on the polarization vector of the field and that the orientation of the initial and final electron momenta only plays a minor role in the KW formula of Eq. (2). The situation is more complicated in the classically forbidden region. Laser-assisted scattering in this region depends in a delicate way on the individual field projections of initial and final momenta. As a clear manifestation, we shall demonstrate that the impulse approximation of Eq. (1) provides a selection rule in the classically forbidden region allowing only the exchange of an even number of photons if the polarization vector of the field is arranged to be orthogonal to the electron scattering plane. The KW approximation of Eq. (2) would give a vanishing cross section for the exchange of both even and odd numbers of photons due to the presence of the Bessel function.

For the derivation of the selection rule, we consider the special situation where the scattering plane is strictly orthogonal to the field polarization, $\vec{\epsilon}\cdot\vec{q}_i = \vec{\epsilon}\cdot\vec{q}_f = \vec{\epsilon}\cdot\vec{Q} = 0$. Inserting $\vec{\epsilon}\cdot\vec{Q} = 0$ into Eq. (1), we obtain

$$f_n(\vec{q}_f,\vec{q}_i) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt \exp[-in\omega t] f(\vec{p}_f(t),\vec{p}_i(t)). \tag{3}$$

When we expand $\vec{p}_j=\vec{q}_j+\vec{A}(t)(j=i,f)$ to zero order in $\vec{A}(t)$, we find

$$f_n(\vec{q}_f,\vec{q}_i)=f(\vec{q}_f,\vec{q}_i)\,\delta_{n,0}, \tag{4}$$

which represents, as expected, field-free scattering without exchange of photons. To study the inclusion of higher-order terms in $\vec{A}(t)$, a more explicit expression of the off-shell scattering amplitude $f(\vec{p}_f(t),\vec{p}_i(t))$ is needed. Generally, the off-shell scattering amplitude is defined in terms of the corresponding half-shell T matrix,

$$f(\vec{k}_f,\vec{k}_i)=-\frac{1}{2\pi}\langle\phi_{\vec{k}_f}|V|\Psi^{+}_{\vec{k}_i}\rangle, \tag{5}$$

where $\phi_{\vec{k}_f}$ is a plane wave and $\Psi^{+}_{\vec{k}_i}$ is an ordinary scattering state, both normalized to unit amplitude at large distances. Upon expansion in partial waves, assuming that the interaction potential between the incident electron and the target atom is represented by a spherical potential $V(r)$, the off-shell amplitude may be expressed as

$$f(\vec{k}_f,\vec{k}_i)=\sum_{l=0}^{\infty}(2l+1)a_l(k_f,k_i)P_l(\cos\theta_k), \tag{6}$$

where θ_k is the angle between $\vec{k}_i$ and $\vec{k}_f$ and where we have introduced the off-shell partial-wave amplitudes

$$a_l(k_f,k_i)=2e^{i\delta_l(k_i)}\int_0^{\infty}dr\,r^2 j_l(k_f r)V(r)R_l(k_i;r) \tag{7}$$

given in terms of ordinary scattering phase shifts, $\delta_l(k)$, spherical Bessel functions, $j_l(z)$, and radial waves with standard normalization at large distances, $R_l(k;r)\overset{r\to\infty}{\sim} j_l[kr+\delta_l(k)]$.

Note that the expression in Eq. (7) on the energy shell, i.e., for $k_f=k_i$, reduces to the ordinary partial-wave amplitude

$$a_l(k_i)=\frac{1}{k_i}e^{i\delta_l(k_i)}\sin\delta_l(k_i). \tag{8}$$

Let us now return to the expression in Eq. (3) and assume that the following property, trivially derived from Eq. (6) in the case of a spherical potential, is valid:

$$f(\vec{p}_f,\vec{p}_i)=f(p_f,p_i,\cos\theta_p), \tag{9}$$

with θ_p the angle between $\vec{p}_i$ and $\vec{p}_f$. Referring to the special geometry where the scattering plane is orthogonal to the polarization vector, we obtain

$$p_j(t)=\sqrt{q_j^2+A(t)^2}=q_j+O(A^2(t)) \tag{10}$$

and

$$\cos\theta_p=\cos\theta_q+O(A^2(t)), \tag{11}$$

where $\theta_q=\vec{q}_f\cdot\vec{q}_i/q_iq_f$ is the angle between $\vec{q}_i$ and $\vec{q}_f$. When these relations are applied in Eq. (9), we find

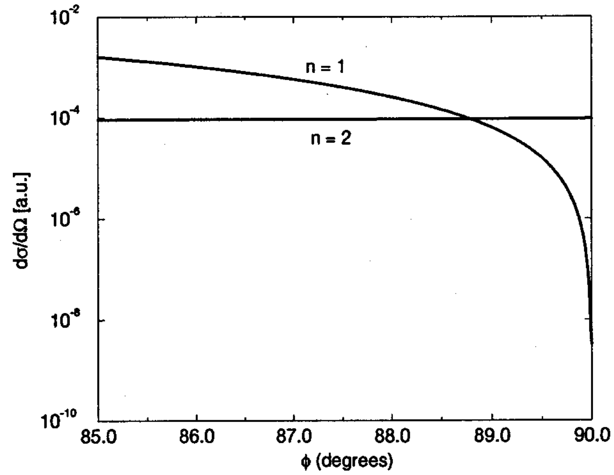

FIG. 1. Differential cross section in the one- and two-photon emission channel as a function of the (azimuthal) angle ϕ between the laser polarization and the scattering plane in a geometry where $\vec{\epsilon}\cdot\vec{Q}=0$. The scattering angle is 156°. The initial kinetic energy of the electron beam is 10 eV. The laser is a linearly polarized CO_2 laser ($\hbar\omega=0.117$ eV) operated at an intensity of $I_0=10^8$ W/cm^2. The potential is a δ-shell potential $V(r)=V_0\delta(r-a)$ with shell radius $a=5$ and potential strength $V_0=0.25$.

$$f(\vec{p}_f(t),\vec{p}_i(t))=f(\vec{q}_f,\vec{q}_i)+g(\vec{q}_f,\vec{q}_i)A^2(t)+O(A^4(t)), \tag{12}$$

where the explicit form of $g(\vec{q}_f,\vec{q}_i)$ is readily derived but is immaterial for the present discussion. Using the harmonicity of the field, we find

$$f_n(\vec{q}_f,\vec{q}_i)=f(\vec{q}_f,\vec{q}_i)\,\delta_{n,0}+\frac{1}{4}g(\vec{q}_f,\vec{q}_i)A_0^2(2\delta_{n,0}-\delta_{|n|,2})+O(A_0^4). \tag{13}$$

This clearly means that the laser-assisted differential cross section satisfies the following selection rule:

$$\frac{d\sigma}{d\Omega}(\vec{q}_f,\vec{q}_i;2n+1)=0\quad\text{for}\quad n=0,1,2,\ldots \tag{14}$$

in the considered geometry.

The selection rule in Eq. (14) was derived in this paper in the impulse approximation but applies more generally. This follows from purely geometrical considerations since, for a spherically symmetric potential, the system must remain invariant under simultaneous rotations of the laser field and incident and scattered electrons. This implies that the dependence on vector quantities such as electron momenta and vector potential only may enter in scalar combinations such as $\vec{A}\cdot\vec{q}$ and $\vec{A}^2$. Since the exchange of an odd number of photons clearly involves at least one interaction term of the type $\vec{A}\cdot\vec{p}$, where $\vec{p}$ is the electron momentum operator, the selection rule of Eq. (14) is readily established. The present derivation in the impulse approximation is, nevertheless, useful since it clearly reveals how the finite value of the cross section for the exchange of an even number of photons is intimately connected with the off-energy shell intermediatestate motion of the electron. Note that the KW approximation of Eq. (2) would predict a zero value for the cross section also for even values of n.

To illustrate the situation by a simple example, we consider laser-assisted scattering on a δ-shell potential. Figure 1 shows the laser-assisted differential cross section as a function of the angle ϕ between the scattering plane and the polarization vector of the laser. The scattering angle is 156° and the geometry is such that $\vec{\epsilon}\cdot\vec{Q}=0$. The parameters of the laser and the scattering potential as well as the kinetic energy of the electron are detailed in the caption. The one- and two-photon emission channels are considered. We note that the scattering for the chosen parameters takes place in the wings of a resonance centered around $\sim$9.5 eV and of width $\sim$1 eV. The cross sections for off-resonance scattering are somewhat smaller than the cross section in the figure, but the qualitative behavior is the same. First, the figure illustrates that the impulse approximation predicts a nonvanishing cross section in a geometry where the KW formula is ill-defined and where various lower-order theories would predict a vanishing cross section since the Bessel function, which determines the weight on the field-free cross section, fulfills $J_n((A_0/\omega)\vec{\epsilon}\cdot\vec{Q})=J_n(0)=\delta_{n,0}$ in this particular geometry (see, e.g., also the discussion in Ref. [5]). Second, the figure shows how the one-photon exchange signal will eventually decrease and drop to zero in the geometry where the selection rule applies. As predicted by the analysis above, the two-photon exchange signal is not influenced in any dramatic way in this particular geometry. Note that a resolution of about 1° would be sufficient to experimentally resolve the dramatic difference in the angular dependence of the one- and two-photon processes. Since the angular resolution in the earlier experiments by Wallbank and Holmes [2,3] is 2°–4°, the situation is considered to be quite adequate for an experimental realization of the effect.

In summary, we have derived a selection rule in the theory of laser-assisted charged-particle scattering. The rule was derived in the impulse approximation, but is was argued that it would apply generally for laser-assisted scattering in spherical symmetric potentials. We have illustrated the selection rule by a calculation showing how the two-photon exchange cross section will remain essentially unaffected while the one-photon exchange cross section vanishes as the special geometry with the laser polarization perpendicular to the scattering plane is approached. Accordingly, if a measurement in an experiment with an angular resolution of about 1° would give a result quantitatively different from the one presented in Fig. 1, it would be a signature that the experimental data are not properly analyzed with respect to the satisfaction of single-collision conditions [11].

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