

Image and coherence transfer in the stimulated down-conversion process

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(Received 22 June 1999)

The intensity transverse profile of the light produced in the process of stimulated down-conversion is derived. A quantum-mechanical treatment is used. We show that the angular spectrum of the pump laser can be transferred to the stimulated down-converted beam, so that images can also be transferred from the pump to the down-converted beam. We also show that the transfer can occur from the stimulating beam to the down-converted one. Finally, we study the process of diffraction through an arbitrarily shaped screen. For the special case of a double-slit, the interference pattern is explicitly obtained. The visibility for the spontaneous emitted light is in accordance with the van Cittert–Zernike theorem for incoherent light, while the visibility for the stimulated emitted light is unity. The overall visibility is in accordance with previous experimental results. [S1050-2947(99)09412-3]

PACS number(s): 42.50.Ar, 42.25.Kb

I. INTRODUCTION

The parametric down-conversion process has become a very important tool for the study of the fundamentals of quantum mechanics. Until now, the production of pairs of correlated photons has been the most important feature of this process. In a cavity-free configuration, down-conversion can be accomplished in two ways: spontaneous and stimulated down-conversion. In the first case, pairs of photons are spontaneously emitted, while in the stimulated process, a second auxiliary laser is coupled to one of the down-conversion modes.

We will focus our attention on the stimulated down-conversion process. Even though the great interest in the spontaneous process is due to the quantum character of the correlations between the twin photons, as yet no quantum property [1] of the light produced in the stimulated process has been demonstrated. The coherence properties of this light field have been investigated by Mandel and co-workers [2] and also in Ref. [3]. It is possible to conclude from these works that essentially the light obtained in the stimulated down-conversion is a superposition of light due to the spontaneous-emission process, always present, and light due to the stimulating process. From the point of view of the first-order transverse coherence properties (field or phase coherence), the spontaneous emitted light is incoherent in the sense discussed in Ref. [4], and the stimulated emitted light is as coherent as the laser which induces the emission. For the case of the longitudinal coherence properties, it was shown that the degree of coherence of the stimulated light depends also on the pump beam temporal coherence properties [5]. For the transverse coherence properties, however, only a simple theory has been proposed [3], which does not take into account the pump field properties.

The aim of this paper is to show how transverse propagation properties (not only coherence properties) of the light beam produced in the stimulated down-conversion process

can be obtained from those of the pump, auxiliary laser, and the parametric interaction. The theory we present here is based on that developed in Ref. [6]. There, a theory was developed to take into account the transverse properties of the correlated twin photons. We do the same, but transverse propagation effects (image formation and coherence transfer) are obtained in the intensity profile instead of in coincidence. We obtain the intensity distribution of the stimulated down-converted beam as a function of the pump and auxiliary laser intensity distributions. Previous experiments are discussed and new ones are envisaged.

Finally, we would like to emphasize that the stimulated down-conversion process is a candidate for exhibiting interesting quantum features in the recent effort for generating multiple entangled particles.

II. STIMULATED INTENSITY PROFILE

A typical configuration for stimulated down-conversion is shown in Fig. 1. A nonlinear crystal is pumped by a laser and produces pairs of photons in the directions labeled by signal and idler. A second laser is aligned along the signal direction so that its modes are coupled to signal down-conversion modes. Now, the emission on the signal modes that are

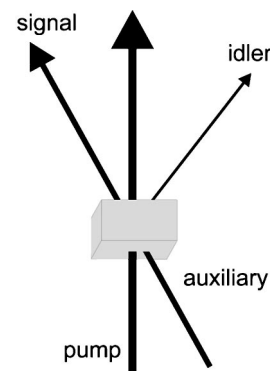


FIG. 1. Basic arrangement for the production of the stimulated down-conversion.

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coupled to the laser will be enhanced by stimulation. An immediate consequence of this stimulation process is the enhancement of the idler emission. This is a consequence of the fact that emissions are only allowed in pairs. We are going to proceed with the calculation of the idler beam intensity as a function of the transverse coordinates. This will provide us with the image formed on this beam.

By using a quantum down-conversion wave function, we will be able to use previous results. For a thin nonlinear crystal centered at the origin and pumped along the z direction, the state generated by SPDC (spontaneous parametric down-conversion) in the monochromatic and paraxial approximations can be represented by [6]

$$|\psi\rangle = |\text{vac}\rangle + C \int d\mathbf{q}_i \int d\mathbf{q}_s v_p(\mathbf{q}_i + \mathbf{q}_s) |1; \mathbf{q}_i\rangle |1; \mathbf{q}_s\rangle, \quad (1)$$

where $|1; \mathbf{q}\rangle$ represents a one-photon state with transverse wave-vector component \mathbf{q} , $v_p(\mathbf{q})$ is the angular spectrum of the pump field at $z=0$, and C is a constant.

For the stimulated parametric down-conversion, an analogous wave function is given by [2]

$$|\psi\rangle = |v_s(\mathbf{q})\rangle |\text{vac}\rangle + C \int d\mathbf{q}_i \int d\mathbf{q}_s v_p(\mathbf{q}_i + \mathbf{q}_s) |1; \mathbf{q}_i\rangle a^\dagger(\mathbf{q}_s) |v_s(\mathbf{q})\rangle, \quad (2)$$

where $|v_s(\mathbf{q})\rangle$ is a multimode coherent state in the continuous mode representation [7] with transverse wave-vector component \mathbf{q} , and $v_s(\mathbf{q})$ is the angular spectrum of the stimulating field at $z=0$.

The idler beam intensity is given by the second-order correlation function

$$I(\mathbf{r}_i) = \langle E_i^{(-)}(\boldsymbol{\rho}_i, z) E_i^{(+)}(\boldsymbol{\rho}_i, z) \rangle, \quad (3)$$

where $\boldsymbol{\rho}_i$ is the transverse component of \mathbf{r}_i and z is the longitudinal one. $E_i^{(+)}(\boldsymbol{\rho}_i, z)$ is the electric field operator given by [6]

$$E_i^{(+)}(\boldsymbol{\rho}_i, z) = \int d\mathbf{q}_i a(\mathbf{q}_i) \exp\left[i\left(\mathbf{q}_i \cdot \boldsymbol{\rho}_i - \frac{q_i^2}{2k_i} z\right)\right]. \quad (4)$$

The intensity distribution is then given by

$$\begin{aligned} I(\mathbf{r}_i) = & |C|^2 \int d\mathbf{q}_i \int d\mathbf{q}_s \int d\mathbf{q}'_i \int d\mathbf{q}'_s v_p(\mathbf{q}_i + \mathbf{q}_s) v_p^*(\mathbf{q}'_i + \mathbf{q}'_s) \\ & \exp\left[i\left(\mathbf{q}_i \cdot \boldsymbol{\rho}_i - \frac{q_i^2}{2k_i} z\right)\right] \exp\left[-i\left(\mathbf{q}'_i \cdot \boldsymbol{\rho}_i - \frac{q_i'^2}{2k_i} z\right)\right] \\ & \times \langle v_s(\mathbf{q}) | a(\mathbf{q}'_s) a^\dagger(\mathbf{q}_s) | v_s(\mathbf{q}) \rangle. \end{aligned} \quad (5)$$

By means of the commutation relation

$$[a(\mathbf{q}'_s), a^\dagger(\mathbf{q}_s)] = \delta(\mathbf{q}'_s - \mathbf{q}_s), \quad (6)$$

and the Fourier integral representation of the pump beam angular spectrum

$$v_p(\mathbf{q}_i + \mathbf{q}_s) = c \int d\boldsymbol{\rho} \mathcal{W}_p(\boldsymbol{\rho}) \exp[-i(\mathbf{q}_i + \mathbf{q}_s) \cdot \boldsymbol{\rho}], \quad (7)$$

Eq. (5) can be written as

$$\begin{aligned} I(\mathbf{r}_i) = & |C|^2 \int d\mathbf{q}_i \int d\mathbf{q}_s \int d\mathbf{q}'_i \int d\mathbf{q}'_s \int d\boldsymbol{\rho} \mathcal{W}_p(\boldsymbol{\rho}) \\ & \times \exp[-i(\mathbf{q}_i + \mathbf{q}_s) \cdot \boldsymbol{\rho}] \int d\boldsymbol{\rho}' \mathcal{W}_p^*(\boldsymbol{\rho}') \\ & \times \exp[i(\mathbf{q}'_i + \mathbf{q}'_s) \cdot \boldsymbol{\rho}'] \exp\left[i\left(\mathbf{q}_i \cdot \boldsymbol{\rho}_i - \frac{q_i^2}{2k_i} z\right)\right] \\ & \times \exp\left[-i\left(\mathbf{q}'_i \cdot \boldsymbol{\rho}_i - \frac{q_i'^2}{2k_i} z\right)\right] \\ & \times [\delta(\mathbf{q}'_s - \mathbf{q}_s) + v_s^*(\mathbf{q}_s) v_s(\mathbf{q}'_s)]. \end{aligned} \quad (8)$$

Now, let us perform the integrals in momentum variables:

$$\int d\mathbf{q}_i \exp\left[i\left(\mathbf{q}_i \cdot (\boldsymbol{\rho}_i - \boldsymbol{\rho}) - \frac{q_i^2}{2k_i} z\right)\right] = \exp\left[i|\boldsymbol{\rho}_i - \boldsymbol{\rho}|^2 \frac{k_i}{2z}\right], \quad (9a)$$

$$\int d\mathbf{q}_s \exp(-i\mathbf{q}_s \cdot \boldsymbol{\rho}) v_s^*(\mathbf{q}_s) = \mathcal{W}_s^*(\boldsymbol{\rho}). \quad (9b)$$

Using Eqs. (9a) and (9b) in Eq. (8), we obtain

$$\begin{aligned} I(\mathbf{r}_i) = & |C|^2 \left\{ \int d\boldsymbol{\rho} |\mathcal{W}_p(\boldsymbol{\rho})|^2 + \left| \int d\boldsymbol{\rho} \mathcal{W}_p(\boldsymbol{\rho}) \mathcal{W}_s^*(\boldsymbol{\rho}) \right. \right. \\ & \left. \left. \times \exp\left[i|\boldsymbol{\rho}_i - \boldsymbol{\rho}|^2 \frac{k_i}{2z}\right] \right|^2 \right\}. \end{aligned} \quad (10)$$

The first term in Eq. (10) is constant relative to the detection position $\boldsymbol{\rho}_i$. This is the spontaneous-emission term. Even without performing the integral in the second term, we can interpret it as being a consequence of the stimulation as it depends on the stimulating laser properties through $\mathcal{W}_s^*(\boldsymbol{\rho})$. This result stresses a known feature of the stimulated process. The light beam is a superposition of the light produced in the spontaneous- (first term) plus the light produced in the stimulated-emission process (second term).

The stimulated-emission term shows that the intensity profile is given by the product of the pump and the auxiliary lasers' transverse intensity distributions at the crystal ($z=0$), propagated by a Fresnel propagator to the idler detection plane. It works as if the stimulating field were diffracted by the transverse profile of the pump field and vice versa. This is equivalent to a photon-photon scattering process. This term can be much stronger than the first one, depending on the auxiliary laser intensity, and then dominating the idler intensity profile.

For example, if the auxiliary laser has a constant transverse intensity distribution, an image formed by the pump beam is transferred to the idler. Figure 2 illustrates this situation when we use a mask. This is analogous to the image transfer to the fourth-order correlation function demonstrated in Ref. [6], but now the image is transferred to the second-order correlation function (intensity).

If the pump beam profile is constant, one image formed by the auxiliary laser beam is transferred to the idler. However, in this case it is the image corresponding to the com-

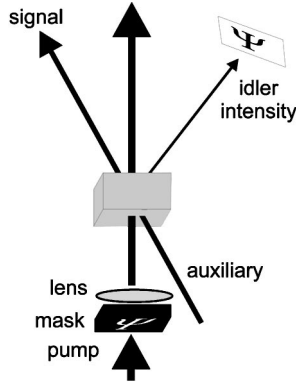


FIG. 2. Image transfer from the pump to the down-conversion beam.

plex conjugate of the auxiliary laser field. We have phase conjugation, which can be used in many ways for treating images and laser beams.

III. DOUBLE-SLIT EXPERIMENT WITH STIMULATED DOWN-CONVERSION

The degree of transverse coherence of the light produced in the stimulated down-conversion process has been studied experimentally in Ref. [3]. By performing double-slit experiments, it was shown that this light source behaves as a superposition of two kinds of light sources. One is incoherent, in the sense that the spatial correlation in the source is of the kind $\delta(\mathbf{r})$, if \mathbf{r} is the coordinate of one point in the source [4]. Another is coherent, so that the spatial correlation over the source surface is always unity. However, only a simple explanation was given.

Now, we are able to use the theory introduced in Ref. [6] to give a more general explanation for that experiment. The advantage of the present treatment is that the calculation can be carried out for an arbitrary shape of the screen and not only limited to the case of the double slits. Besides, it takes into account the spatial properties of the pump and stimulating beams in a more general way.

Let us think about the situation described in Fig. 3. The stimulating laser is aligned with the signal beam and the idler is passed through an aperture on the screen “S” whose shape is given by the function $\mathcal{A}(\rho)$ defined in the plane of the screen. One special case of the aperture is the double-slit shape. We want to calculate the intensity distribution of the

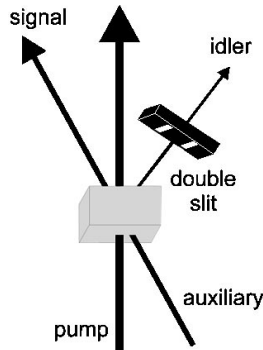


FIG. 3. Scattering of the stimulated down-conversion by a double slit.

idler beam after the screen. We will suppose that the distance crystal-screen and screen-detection plane are large enough to use the paraxial approximation. The intensity is given in the same way as before, by

$$I(\mathbf{r}_i) = \langle E_i^{(-)}(\boldsymbol{\rho}_i, z) E_i^{(+)}(\boldsymbol{\rho}_i, z) \rangle, \quad (11)$$

as well as the state of the down-converted light:

$$|\psi\rangle = |v_s(\mathbf{q})\rangle |vac\rangle + C \int d\mathbf{q}_i \int d\mathbf{q}_s v_p(\mathbf{q}_i + \mathbf{q}_s) |1; \mathbf{q}_i\rangle a^\dagger(\mathbf{q}_s) |v_s(\mathbf{q})\rangle. \quad (12)$$

The electric field operator, however, is changed in order to take into account the propagation through the aperture. This is done in the same way as in Ref. [8]:

$$E_i^{(+)}(\boldsymbol{\rho}_i, z) = \exp(ik_i z) \int d\mathbf{q}_i \int d\mathbf{q}' a(\mathbf{q}') T(\mathbf{q}_i - \mathbf{q}') \times \exp \left\{ i \left[\mathbf{q}_i \cdot \boldsymbol{\rho}_i - \frac{q_i^2}{2k_i} (z - z_A) - \frac{q'^2}{2k_i} z_A \right] \right\}, \quad (13)$$

where $T(\mathbf{q})$ is the Fourier transform of $\mathcal{A}(\boldsymbol{\rho})$, z_A is the longitudinal coordinate of the screen, and z is the longitudinal coordinate of the detection plane. The crystal is located at $z=0$.

The intensity after the screen is then given by

$$I(\mathbf{r}_i) = \left| C \int d\mathbf{q}_i \int d\mathbf{q}' \int d\mathbf{q}_s v_p(\mathbf{q}_i + \mathbf{q}_s) T(\mathbf{q}' - \mathbf{q}_i) \times \exp \left[i \left(\mathbf{q}' \cdot \boldsymbol{\rho}_i - \frac{q'^2}{2k_i} (z - z_A) - \frac{q_i^2}{2k_i} z_A \right) \right] \times a^\dagger(\mathbf{q}_s) |v_s(\mathbf{q})\rangle |0; \mathbf{q}_i\rangle \right|^2. \quad (14)$$

In order to perform the calculation, it is necessary to follow the same procedure as in the preceding section, using the commutation relation

$$[a(\mathbf{q}'_s), a^\dagger(\mathbf{q}_s)] = \delta(\mathbf{q}'_s - \mathbf{q}_s), \quad (15)$$

and performing integrals in momentum degrees of freedom,

$$\int d\mathbf{q}_s \exp(-i\mathbf{q}_s \cdot \boldsymbol{\xi}) v_s^*(\mathbf{q}_s) = \mathcal{W}_s^*(\boldsymbol{\xi}), \quad (16a)$$

$$\int d\mathbf{q}_i \exp[i\mathbf{q}_i \cdot (\boldsymbol{\eta} - \boldsymbol{\xi})] \exp \left[-i \frac{q_i^2}{2k_i} z_A \right] = \exp \left[i \left(|\boldsymbol{\eta} - \boldsymbol{\xi}|^2 \frac{k_i}{2z_A} \right) \right], \quad (16b)$$

$$\begin{aligned} & \int d\mathbf{q} \exp[i\mathbf{q} \cdot (\boldsymbol{\rho}_i - \boldsymbol{\eta})] \exp\left[-i \frac{q^2}{2k_i} (z - z_A)\right] \\ &= \exp\left[i|\boldsymbol{\rho}_i - \boldsymbol{\eta}|^2 \frac{k_i}{2(z - z_A)}\right]. \end{aligned} \quad (16c)$$

The intensity is now

$$\begin{aligned} I(\mathbf{r}_i) &= |C|^2 \left\{ \int d\xi |\mathcal{W}_p(\xi)|^2 \int d\boldsymbol{\eta} \mathcal{A}(\boldsymbol{\eta}) \right. \\ & \times \exp\left[i|\boldsymbol{\eta} - \xi|^2 \frac{k_i}{2z_A}\right] \exp\left[i|\boldsymbol{\rho}_i - \boldsymbol{\eta}|^2 \frac{k_i}{2(z - z_A)}\right] \Bigg|^2 \\ & + \left| \int d\xi \int d\boldsymbol{\eta} \mathcal{W}_p(\xi) \mathcal{W}_s^*(\xi) \mathcal{A}(\boldsymbol{\eta}) \right. \\ & \times \exp\left[i|\boldsymbol{\eta} - \xi|^2 \frac{k_i}{2z_A}\right] \exp\left[i|\boldsymbol{\rho}_i - \boldsymbol{\eta}|^2 \frac{k_i}{2(z - z_A)}\right] \Bigg|^2 \Bigg\}. \end{aligned} \quad (17)$$

We cannot proceed with the calculation without specifying the spectral and aperture functions. However, if we take the Fraunhofer limit, the phase functions become

$$\begin{aligned} \exp\left(-i|\boldsymbol{\eta} - \xi|^2 \frac{k_i}{2z_A}\right) &\rightarrow \exp\left(-i\boldsymbol{\eta} \cdot \boldsymbol{\xi} \frac{k_i}{z_A}\right), \\ \exp\left(-i|\boldsymbol{\rho}_i - \boldsymbol{\eta}|^2 \frac{k_i}{2(z - z_A)}\right) &\rightarrow \exp\left(-i\boldsymbol{\rho}_i \cdot \boldsymbol{\eta} \frac{k_i}{z - z_A}\right), \end{aligned} \quad (18)$$

and the intensity is changed to

$$\begin{aligned} I(\mathbf{r}_i) &= |C|^2 \left\{ \int d\xi |\mathcal{W}_p(\xi)|^2 |T(\beta_1 \xi + \beta_2 \boldsymbol{\rho}_i)|^2 \right. \\ & + \left. \left| \int d\xi \mathcal{W}_p(\xi) \mathcal{W}_s^*(\xi) T(\beta_1 \xi + \beta_2 \boldsymbol{\rho}_i) \right|^2 \right\}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \beta_1 &= \frac{k_i}{2z_A}, \\ \beta_2 &= \frac{k_i}{2(z - z_A)}. \end{aligned} \quad (20)$$

This result shows that the intensity pattern after a screen with arbitrary shape, in the Fraunhofer diffraction limit (which is quite acceptable for experimental situations), is given by the superposition of two terms. The first one is a convolution between the pump beam intensity distribution and the square modulus of the Fourier transform of the aperture function. This corresponds to an incoherent scattering through the aperture. The second term is the square modulus of a convolution between two field amplitudes and the Fourier transform of the aperture function. One of the fields is the pump and another is the conjugate of the auxiliary laser.

Thus, no matter what the shape of the aperture is, the scattered light will be composed of an incoherent plus a co-

herent part. Let us now carry out the calculation of the idler intensity distribution for a particular shape of the aperture.

IV. 1D UNIFORM SOURCE AND THIN DOUBLE SLITS

To illustrate the power of the above result, let us calculate explicitly the intensity profile for a one-dimensional case with the uniform pump and auxiliary lasers' intensity distributions. Also, let us consider thin double slits separated by a distance $2d$, so that we have the following configuration:

$$\begin{aligned} \mathcal{W}_p(\mathbf{x}) &= 0 & \text{if } -a > x > a, \\ \mathcal{W}_p(\mathbf{x}) &= w_p & \text{if } -a < x < a, \\ \mathcal{W}_s(\mathbf{x}) &= 0 & \text{if } -a > x > a, \\ \mathcal{W}_s(\mathbf{x}) &= w_s & \text{if } -a < x < a, \\ \mathcal{A}(\mathbf{x}) &= \delta(x + d) + \delta(x - d). \end{aligned} \quad (21)$$

Then the intensity after the slits is given by

$$\begin{aligned} I(x_i) &= |C|^2 w_p^2 4a \left\{ \left[1 + \frac{\sin(2\beta_1 da)}{2\beta_1 da} \cos(2\beta_2 dx_i) \right] \right. \\ & + \left. w_s^2 2a \left[\frac{\sin(\beta_1 da)}{\beta_1 da} \right]^2 [1 + \cos(2\beta_2 dx_i)] \right\}. \end{aligned} \quad (22)$$

By using the definitions

$$\begin{aligned} I_0 &= I_{\text{sp}} + I_{\text{st}}, \\ \mu &= \frac{I_{\text{sp}} \mu_{\text{sp}} + I_{\text{st}} \mu_{\text{st}}}{I_0}, \\ I_{\text{sp}} &= 4a |C|^2 w_p^2, \\ I_{\text{st}} &= 8a^2 \left[\frac{\sin(\beta_1 da)}{\beta_1 da} \right]^2 |C|^2 w_p^2 w_s^2, \\ \mu_{\text{sp}} &= \frac{\sin(2\beta_1 da)}{2\beta_1 da}, \\ \mu_{\text{st}} &= 1, \end{aligned} \quad (23)$$

the intensity distribution can be set in the form

$$I(x_i) = I_0 [1 + \mu \cos(2\beta_2 dx_i)]. \quad (24)$$

First of all, note that the overall visibility for the interference fringes μ is in complete agreement with that of Ref. [3], which has been experimentally tested. Note also that the visibility of the spontaneous-emission part μ_{sp} is exactly the visibility given by the van Cittert–Zernike theorem for spatially incoherent light, which was also checked for the down-conversion in Ref. [4].

The stimulated intensity I_{st} is a diffraction pattern of a coherent field through a slit with the source dimensions $2a$. This stresses the photon-photon character of the interaction between pump and auxiliary lasers.

V. CONCLUSION

As a conclusion, we have shown how the transverse properties (angular spectrum) of the light produced in the stimulated down-conversion process can be obtained from those of the pump, auxiliary laser beam, parametric interaction, and the shape of the apertures that can be eventually placed in any one of these beams. It is shown that images in the pump field, produced by a mask, for example, can be transferred to the down-converted field. Images in the stimulating field can also be transferred to the down-converted field, but through the conjugated field.

Some curious possibilities can be studied experimentally in further works. The first is to produce interference fringes in the down-converted field, placing slits in the pump beam. The second is to produce interference fringes in the down-converted field placing slits in the stimulating laser. Another one is to produce interference fringes in the idler beam, by placing one part of a double slit in the pump and another part in the auxiliary laser, as in Ref. [8]

Our results were also used to discuss a double-slit experiment with stimulated down-conversion light, showing generally the influence of the pump and auxiliary lasers' angular properties on the interference effects. We show that the the-

oretical description explains previous experimental results [3], where interference fringes were a consequence of two contributions, one due to the spontaneous-emission and another due to the stimulated-emission process.

We believe that this problem is already interesting from a fundamental point of view. However, some applications can be envisaged. For example, it is possible to produce a light signal containing one image superposed to a background. As the image information is contained in the coherent part of the field and the background is produced by the incoherent field, it is possible to filter image information from the background noise with spatial filters. Another interesting point is that the image transfer can occur with beams of different wavelengths. Many experimental situations can be thought of, by controlling pump and auxiliary laser angular properties and intensities. Finally, quantum properties of this kind of field can also be studied with the same formalism.

ACKNOWLEDGMENTS

The authors acknowledge financial support from the Brazilian agencies CNPq, FINEP, PRONEX, FAPERJ, and FAPEMIG.

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