Quantum-dot laser with periodic pumping

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We sketch the theory of a laser based on quantum dots which are pumped by a surface acoustic wave. Collective emission in the sense of superradiance is possible if the dots are identical and arranged along a straight line. In that case strong noise suppression is possible in the photocurrent spectrum, even for high output intensity. [S1050-2947(99)02512-3]

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I. INTRODUCTION

Semiconductor quantum dots have an interesting potential for quantum-optical applications. The growth of dots with transition frequencies in the optical range is very well controlled [1]. Such a zero-dimensional system leads to much higher gain than bulk or two-dimensional quantum-well structures, as shown theoretically as well as experimentally [1–4]. Dots as active media in semiconductor lasers have already been established, and even lasing of a single dot in a semiconductor microcavity can be achieved [5–7]. From a theoretical point of view, the discrete states allow one to treat dots much like atoms. This makes for a much simpler situation than, for example, the continua of states in quantum wells. Furthermore, the semiconductor samples are small compared to atomic beams or even clouds of trapped atoms.

If a dot is to be operated as a low-noise light source it had better be pumped in an as regular manner as possible. Imamoḡlus and Yamamoto's scheme of a single-photon turnstile device [8] driven by an alternating voltage source is one method for a regularized pump. A surface acoustic wave (SAW) could as well periodically deliver electrons and holes at a well localized array of dots or even a single dot, as already suggested in [9]. If (more or less) identical quantum dots could be arranged along a straight line a SAW with wave fronts parallel to that line would entail collective emission by the array; with suitable mirrors mounted one could thus realize a superradiant laser. The present paper sketches the theory of such a laser and demonstrates interesting potential for noise suppression. In contrast to the model of a superradiant laser with collectively pumped three-level atoms suggested in Refs. [10–13] the noise reduction in the present case is due to the periodic simultaneous pumping of the dots by the SAW and can be optimal even for strong output.

II. THE PUMP MECHANISM

To briefly explain our concept, let us consider a semiconductor quantum well surrounded by a piezoelectric material with an interdigital transducer (IDT) on top of the crystal (Fig. 1). A mechanical SAW is generated by applying a high-frequency signal to the IDT. The fundamental acoustic wavelength λ_0 and the frequency $f_0 = v/\lambda_0$ are established by the interdigital electrode spacing, where v is the sound velocity of the crystal. With $\lambda_0 \sim 1-3\ \mu\text{m}$ and $v \sim 3\ \text{km}\,\text{s}^{-1}$, frequencies in the GHz range are achievable. The acoustic wave is accompanied by a piezoelectric field which gives an additional potential for electrons and holes and so periodically modulates the band edges. For high enough SAW amplitudes, optically generated excitons in the quantum well will be dissociated by the piezoelectric field (inset of Fig. 1). A field strength of the order of 500 V/cm suffices and results in a wave amplitude of 50–150 meV, depending on the wavelength. Carriers are then trapped in the moving lateral potential superlattice of the sound wave and recombination becomes impossible: Electrons will stay in the minima of the wave, while holes move with the maxima [14–16]. A simple estimate of the spatial width Δd of the lateral ground state in the wave potential yields $\Delta d/\lambda < 0.02$. We thus obtain a series of equally spaced quantum wires moving in the plane of the quantum well. The

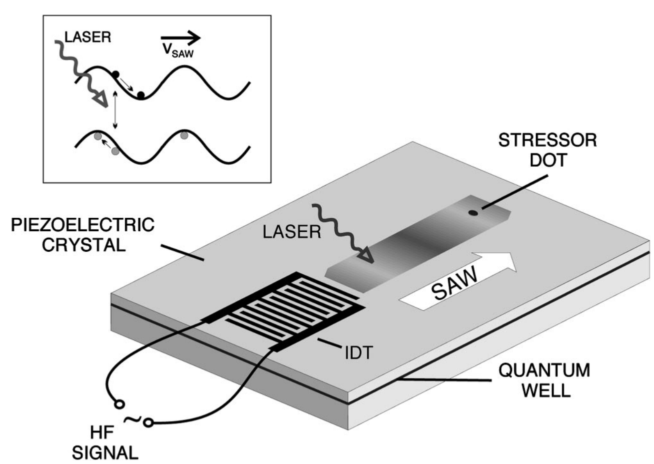

FIG. 1. Schematic sketch of a SAW sample. The material of the system may be, for example, GaAs for the piezoelectric crystal, InGaAs for the quantum well, and InP for the stressor. The inset depicts the storage of optically generated excitons in the potential of the surface acoustic wave.

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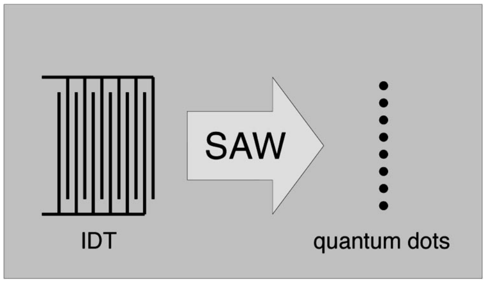

FIG. 2. Arrangement for parallel pumping of several identical quantum dots.

length of these wires is given by the width of the IDT's, typically 300 μm. The occupation of the wires with electrons and holes can be controlled by the pump strength of the laser and is of the order 10^3–10^4 carriers per wire.

A quantum dot for our purposes may be established by a stressor on top of the crystal which causes a local potential minimum in the quantum well underneath. The linear dimension of typical stressor dots with transition frequencies in the optical range is about 10–30 nm while their potential depths are about 100 meV for electrons and 50 meV for holes. For further investigations, we assume that the dot is so small that there is only one electron and one hole level. Occupation of these levels with two carriers of opposite spin is forbidden by Coulomb blockade. When both levels are occupied an exciton is formed, so there is just a single exciton state. We should speak of an excited, a semiexcited, and an unexcited dot for an exciton, only one carrier (electron or hole), and no carrier present. An excited/unexcited dot may then be treated as simple two-level system with pseudospin operators S_+, S_- creating and annihilating an exciton. This system may interact with a single-mode light field. In the semiexcited case no interaction with the light field is possible and the creation of an exciton is only possible by capturing the missing carrier. While being crossed by a moving quantum wire an empty dot may pluck one of the carriers offered: If the dot potential is deep enough a carrier will drop into it and stay there, while the wave is moving on.

The scheme just sketched may indeed produce the designed properties of the pump. First, the periodicity of arriving carriers is given by the SAW, as the moving wires are well separated. Second, with a density of about 3 carriers per 100 nm in a wire, there is a high probability for the dot to capture an electron or hole within the crossing time of a wire. Of course, a single dot makes but inefficient use of the moving wires, as only one of 10^4 carriers is used per cycle. If one had several dots lying in a row parallel to the wires, better pump yields could arise. This arrangement would be needed for a superradiant laser and is shown in Fig. 2. Another way to increase efficiency may be focusing the SAW onto one or few dots.

Our periodically pumped dot (PPD) will in practice suffer from degradation of complete regularity. One cause of pump fluctuations is the finite width of the lateral SAW ground state, as mentioned above. This leads to variations in the instant of pumping. As indicated above, this will be a few percent of the pump period T given by the SAW frequency. Pump noise also results when no carrier is plucked from a crossing wire; this may be minimized by a high-electron and hole generation rate in the SAW, so all wires are well occupied. For simplicity we neglect both types of noise here and consider the case of zero pump fluctuations. Such an idealized pump process allows for a simple analytic description.

For quantum-optical applications we have to couple the PPD to a single-mode light field inside a resonator. As the dot is pumped by a SAW, we cannot put a semiconductor Bragg mirror, which would block the mechanical wave, on top of the device. For the back side this would not be a problem. We, therefore, need at least one external mirror for the cavity. The carriers could be generated outside the cavity, since the electrons and holes can be transported over some millimeters by the SAW.

Theoretical description

For the theoretical description we work with the unexcited (ground) state $|0\rangle$, the excited state $|1\rangle$, and the semiexcited states $|e\rangle$ and $|h\rangle$, which have either an electron (e) or a hole (h) in the dot. As already mentioned the ground and excited states form a two-level system coupled by the pseudospin operators S_+, S_-. The operator S_+ generates an exciton from the ground state, whereas S_- annihilates the exciton in favor of the ground state. Both semiexcited states are annihilated by $S_\pm$,

$$S_\pm|e\rangle = S_\pm|h\rangle = 0; \tag{1}$$

these states do not couple to the light field. The density matrix of a single quantum dot may be written in the general form

$$\sigma(t) = \sigma_{11}(t)|1\rangle\langle 1| + \sigma_{00}(t)|0\rangle\langle 0| + \sigma_{01}(t)|0\rangle\langle 1| + \sigma_{10}(t)|1\rangle\langle 0| + \sigma_{ee}(t)|e\rangle\langle e| + \sigma_{hh}(t)|h\rangle\langle h|. \tag{2}$$

Note that no coherences except σ_{01} and σ_{10} are ever created. The pumping causes the following transition.

An excited quantum dot has no free state for an additional carrier so this state remains unchanged:

$$|1\rangle\langle 1| \rightarrow |1\rangle\langle 1|. \tag{3a}$$

For the following transitions we must distinguish if an electron or a hole arrives. For an incoming electron we find for the semiexcited states

$$|e\rangle\langle e| \rightarrow |e\rangle\langle e| \quad \text{and} \quad |h\rangle\langle h| \rightarrow |1\rangle\langle 1|, \tag{3b}$$

i.e., the single-electron state is occupied and remains unchanged while the single-hole state is brought to the excited state. For an incoming hole we correspondingly find

$$|e\rangle\langle e| \rightarrow |1\rangle\langle 1| \quad \text{and} \quad |h\rangle\langle h| \rightarrow |h\rangle\langle h|. \tag{3b$'$}$$

The ground state will be transferred to one of the semiexcited states,

$$|0\rangle\langle 0| \rightarrow |e\rangle\langle e| \quad \text{or} \quad |h\rangle\langle h|. \tag{3c}$$

Finally, all off-diagonal elements are destroyed,

$$|1\rangle\langle 0|, \quad |0\rangle\langle 1| \rightarrow 0. \tag{3d}$$

We may write the pump process described by Eqs. (3a)–(3d) in terms of the density matrix $\sigma_{ij}(t_0-0)$ for the quantum dot. Directly after the pump act with an electron we find at t_0+0

$$\begin{aligned}\sigma_{11}(t_0+0)&=\sigma_{11}(t_0-0)+\sigma_{hh}(t_0-0),\\ \sigma_{00}(t_0+0)&=0,\\ \sigma_{10}(t_0+0)&=\sigma_{01}(t_0+0)=0,\\ \sigma_{hh}(t_0+0)&=0,\\ \sigma_{ee}(t_0+0)&=\sigma_{ee}(t_0-0)+\sigma_{00}(t_0-0).\end{aligned} \tag{4}$$

For pumping with a hole we find the corresponding equations by interchanging index e and h in Eq. (4). If we couple the quantum dot to a single light mode the whole statistical operator has the form

$$\rho(t)=\sum_{j,k}\rho_{jk}|j\rangle\langle k|, \tag{5}$$

where the indices take on the values $j,k=0,1,e,h$, and the ρ_{jk} are operators acting in the Hilbert space of the field mode. According to Eq. (4) we may formally introduce the pump operators P_e for pumping with electrons and P_h for pumping with holes. We may write

$$\rho(t_i+0)=P_e\rho(t_i-0) \tag{6}$$

and

$$\rho(t_i+T/2+0)=P_h\rho(t_i+T/2-0). \tag{7}$$

If the time evolution of the statistical operator without pumping is given by the Liouville–von Neumann generator L as

$$\dot{\rho}=L\rho, \tag{8}$$

the formal solution over one period reads

$$\rho(t_0+T+0)=P_e e^{LT/2}P_h e^{LT/2}\rho(t_0+0), \tag{9}$$

and may be read as a stroboscopic quantum map.

III. SUPERRADIANT LASER

We now come to the description of our superradiant laser model for N quantum dots. As shown in Fig. 2 all quantum dots are assumed to lie in a row parallel to the pump wave. This ensures that all quantum dots are pumped at the same instant. Furthermore the coupling constant g should be the same for all quantum dots and the field mode is very weakly damped with the rate κ. To get a large number of photons in the cavity we require[1]

$$\frac{1}{T},\sqrt{N}g\gg\kappa. \tag{10}$$

If we neglect spontaneous emission and assume zero detuning between dots and field mode the interaction-picture master equation in between two pump acts reads

$$\begin{aligned}\dot{\rho}&=L\rho,\\ &=g\sum_{i=1}^{N}[aS_+^i-a^\dagger S_-^i,\rho]+\frac{\kappa}{2}\{[a,\rho a^\dagger]+[a\rho,a^\dagger]\}.\end{aligned} \tag{11}$$

The index i labels quantum dots. To solve the master equation we resort to certain approximations. The limit $\kappa\ll 1/T,\sqrt{N}g$, Eq. (10) allows an adiabatic elimination of the dot variables and to expand the reduced generator of the mode dynamics in powers of $\kappa T,\kappa/\sqrt{N}g$. In implementing that expansion we will follow here the strategy presented in [18] with appropriate modifications to account for the periodic pumping. By this expansion and going to the semiclassical limit of large photon numbers we will obtain a Fokker-Planck equation.

P representation

Since we are looking for a Fokker-Planck equation for the field mode consider a diagonal expansion with respect to coherent states $|\alpha\rangle$, i.e., we make use of the Glauber-Sudarshan P representation,

$$\rho(t)=\int P(\alpha,\alpha^*,t)|\alpha\rangle\langle\alpha|d^2\alpha. \tag{12}$$

Later on we will use polar coordinates for the complex field amplitude as

$$\alpha=\sqrt{n}e^{i\varphi}. \tag{13}$$

Now we can write the master equation (11) for the complete density operator P in the form

$$\dot{P}=LP=(L_0+L_1+\Lambda)P. \tag{14}$$

Here L_0+L_1 is the interaction operator resulting from the Jaynes-Cummings term in Eq. (11),

$$L_0P=\sum_{i=1}^{N}g[\alpha S_+^i-\alpha^* S_-^i,P]=\sum_{i=1}^{N}L_{0i}P, \tag{15}$$

$$L_1P=\sum_{i=1}^{N}g\left(\frac{\partial}{\partial\alpha}S_+^iP+\frac{\partial}{\partial\alpha^*}PS_-^i\right)=\sum_{i=1}^{N}L_{1i}P. \tag{16}$$

L_0 describes atomic motion under the parametric influence of the field mode, without back action on the latter; atomic back action on the field mode is accounted for by L_1; finally, Λ is the damping generator for the field mode,

$$\Lambda P=\frac{\kappa}{2}\left(\frac{\partial}{\partial\alpha}\alpha+\frac{\partial}{\partial\alpha^*}\alpha^*\right)P=\kappa\frac{\partial}{\partial n}nP. \tag{17}$$

The limit (10) allows us to treat $L_1+\Lambda$ perturbatively.

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[1]For the strong-coupling regime the condition on N is not restrictive; see Ref. [17] where $g/\kappa>1$ is advocated.

IV. SYSTEMATIC APPROXIMATION OF THE FIELD MODE GENERATOR

We now want to expand the propagator e^{Lt} in terms of $L-L_0$ which is possible if the stationary photon number is large. We write the formal solution of Eq. (14) after a pump act as

$$P(t_0+t)=e^{Lt}P(t_0+0). \tag{18}$$

If we take the pump process into account with the operators $P_e^N=\Pi_i P_e^i$ and $P_h^N=\Pi_i P_h^i$ the map (9) takes the form

$$P(t_0+T+0)=P_e^N e^{LT/2}P_h^N e^{LT/2}P(t_0+0). \tag{19}$$

Due to periodic pumping it makes sense to look for the stroboscopic stationary state which obeys

$$P(t_0+T+0)=P(t_0+0). \tag{20}$$

Since we are interested in the properties of the light field, we look for the solution of the reduced density operator of the field mode,

$$P_{\mathrm{f}}=\mathrm{Tr}_{\mathrm{dot}}^{N}P. \tag{21}$$

Here the partial trace $\mathrm{Tr}_{\mathrm{dot}}^{N}$ refers to all quantum dots, while we use $\mathrm{Tr}_{\mathrm{dot}\ i}$ for a single dot.

The expansion of the exponential term in Eq. (18) up to first order in $L-L_0$ reads

$$e^{Lt}\approx e^{L_0 t}+\int_0^t dt'\, e^{L_0(t-t')}(L-L_0)e^{L_0 t'}, \tag{22}$$

whereupon the master equation becomes

$$\dot{P}(t_0+t)=L_0 e^{L_0 t}P(t_0+0)+(L-L_0)e^{L_0 t}P(t_0+0). \tag{23}$$

We now proceed to the explicit form of this equation.

A. Zeroth-order treatment and stationary quantum-dot state

The zeroth-order terms in Eq. (23) yield

$$\dot{P}^{(0)}=L_0 P. \tag{24}$$

Since the field mode appears only parametrically in L_0 we can make a factorization ansatz

$$P^{(0)}(t)=P_{\mathrm{f}}\,(\alpha,t)\prod_{i=1}^{N}\sigma^i(\alpha,t), \tag{25}$$

with σ^i the density matrix of the ith quantum dot. By tracing over $N-1$ quantum dots we find the equation of motion for a single quantum dot

$$\dot{\sigma}^i=L_{0i}\sigma^i. \tag{26}$$

We see that all quantum dots obey the same equation of motion and all dots are decoupled. The single-dot dynamics in between the instants of pumping is that of a two-level atom driven by a constant electric field α with the Rabi frequency

$$\Omega=g|\alpha|. \tag{27}$$

Accounting for the jumps (4) at the instants of pumping and looking for the stationary solution of map (9) we get

$$\begin{aligned}
\sigma_{00}^i(t_0+0)&=0,\\
\sigma_{11}^i(t_0+0)&=\frac{1}{1+\sin^2(\Omega T/2)},\\
\sigma_{hh}^i(t_0+0)&=0,\\
\sigma_{ee}^i(t_0+0)&=\frac{\sin^2(\Omega T/2)}{1+\sin^2(\Omega T/2)},\\
\sigma_{10}^i(t_0+0)&=\sigma_{01}^i(t_0+0)=0
\end{aligned} \tag{28}$$

directly after the arrival of an electron, and

$$\begin{aligned}
\sigma_{00}^i(t_0+T/2+0)&=0,\\
\sigma_{11}^i(t_0+T/2+0)&=\frac{1}{1+\sin^2(\Omega T/2)},\\
\sigma_{ee}^i(t_0+T/2+0)&=0,\\
\sigma_{hh}^i(t_0+T/2+0)&=\frac{\sin^2(\Omega T/2)}{1+\sin^2(\Omega T/2)},\\
\sigma_{10}^i(t_0+T/2+0)&=\sigma_{01}^i(t_0+T/2+0)=0
\end{aligned} \tag{29}$$

after the arrival of a hole.

B. Higher-order terms

We split off the factorizing part from the complete density operator,

$$P(\alpha,t)=P_{\mathrm{f}}\,(\alpha,t)\prod_{i=1}^{N}\sigma^i(\alpha,t)+\pi(\alpha,t), \tag{30}$$

and write the nonfactorizing part π in the general form

$$\pi(t)=\sum_{\substack{l_1,\ldots,l_N\\ r_1,\ldots,r_N}}\pi_{l_1\ldots l_N,r_1\ldots r_N}(t)|l_1,\ldots,l_N\rangle\langle r_1,\ldots,r_N|. \tag{31}$$

The l_i and r_i describe the state of the ith dot, where the sum runs over 0, 1, e, and h. All $\pi_{l_1\ldots l_N,r_1\ldots r_N}(t)$ are operators. Since the trace of σ over all quantum dots is 1 the trace of π must vanish,

$$\mathrm{Tr}_{\mathrm{dot}}^{N}\,\pi(t)=0. \tag{32}$$

With the ansatz (30) we find from the master equation (23) evolution equations for $P_{\mathrm{f}}\,(\alpha,t)$ and $\pi(\alpha,t)$,

$$\dot{P}_{\rm f}=\mathrm{Tr}_{\rm dot}^{N}\left\{(L-L_0)\prod_{i=1}^{N}\sigma^i\right\}P_{\rm f}+\mathrm{Tr}_{\rm dot}^{N}\{(L-L_0)\pi\}, \tag{33}$$

$$\dot{\pi}=L_0\pi+(L-L_0)\prod_{i=1}^{N}\sigma^iP_{\rm f}-\mathrm{Tr}_{\rm dot}^{N}\left\{(L-L_0)\prod_{i=1}^{N}\sigma^i\right\}\prod_{i=1}^{N}\sigma^iP_{\rm f}. \tag{34}$$

To keep the expansion systematic we have taken only terms up to first order in $L-L_0$ in Eq. (34) for π, too. The zeroth-order approximation of π would be

$$\dot{\pi}^{(0)}=L_0\pi^{(0)}. \tag{35}$$

This is a homogeneous equation and we can choose arbitrary initial conditions. By setting $\pi(0)=0$ we see that the nonfactorizing terms vanish identically for all times in this approximation. So we need at least terms of first order to get quantum-mechanical corrections to the factorization ansatz. So the second term in Eq. (33) in all is of second order in $L-L_0$ and leads to field mode diffusion.

C. Drift and stationary photon number

We consider the first term in Eq. (33) since only derivatives of first order in the field variables are involved. By symmetry all σ^i are identical. This makes it easy to rewrite the drift term in the form

$$\mathrm{Tr}_{\rm dot}^{N}\left\{L_1\prod_{i=1}^{N}\sigma^i\right\}P_{\rm f}=\sum_{i=1}^{N}\mathrm{Tr}_{\mathrm{dot}\,i}\{L_{1i}\sigma^i\}P_{\rm f}=N\,\mathrm{Tr}_{\mathrm{dot}\,i}\{L_{1i}\sigma^i\}P_{\rm f}. \tag{36}$$

Next we again assume the relevant photon numbers to be large and, in addition, to have a sufficiently narrow distribution for the variation of the drift coefficient across its width to be negligible. This leads to

$$\dot{P}_{\rm f}^{(1)}=N\,\mathrm{Tr}_{\mathrm{dot}_i}(L_{1i}\sigma^i)P_{\rm f}+\Lambda P_{\rm f}=\frac{\partial}{\partial\alpha}\alpha\left(Ng\frac{\sigma_{10}^i}{\alpha}+\frac{\kappa}{2}\right)P_{\rm f}+\frac{\partial}{\partial\alpha^*}\alpha^*\left(Ng\frac{\sigma_{01}^i}{\alpha^*}+\frac{\kappa}{2}\right)P_{\rm f}. \tag{37}$$

The polarization σ_{10}^i in the interval $t_0<t<t_0+T/2$ is given by

$$\sigma_{10}^i(t)=-\frac{\alpha}{2|\alpha|}\sigma_{11}^i(t_0+0)\sin 2\Omega t. \tag{38}$$

Since we assumed κ to be small we may replace the polarization by its time average $\overline{\sigma_{10}^i}$.

$$\overline{\sigma_{10}^i}=-\frac{\alpha}{|\alpha|T}\sigma_{11}^i(t_0+0)\int_{t_0}^{t_0+T/2}\sin 2\Omega t\,dt=-\frac{\alpha}{|\alpha|}\frac{1}{\Omega T}\frac{\sin^2(\Omega T/2)}{1+\sin^2(\Omega T/2)}. \tag{39}$$

Inserting this into Eq. (37) leads to

$$\dot{P}_{\rm f}^{(1)}=\frac{\partial}{\partial n}n\left(\kappa-\frac{N}{nT/2}\frac{\sin^2(\Omega T/2)}{1+\sin^2(\Omega T/2)}\right)P_{\rm f}. \tag{40}$$

Note that $\dot{P}_{\rm f}^{(1)}$ vanishes in the stationary regime and we find for the stationary photon number in the cavity

$$n_s=\frac{2N}{\kappa T}\frac{\sin^2(\Omega_sT/2)}{1+\sin^2(\Omega_sT/2)}, \tag{41}$$

with the stationary Rabi frequency

$$\Omega_s=\sqrt{n_s}g\frac{T}{2}. \tag{42}$$

Similar to the atomic microlaser we may define two dimensionless parameters. The first one is the upper bound for the mean photon number

$$n_{\max}=\frac{N}{\kappa T}, \tag{43}$$

and the other one a dimensionless pump parameter,

$$\theta=g\sqrt{n_{\max}}\frac{T}{2}. \tag{44}$$

With this we find the equation for the normalized photon number $\tilde{n}$

$$\tilde{n}=\frac{n_s}{n_{\max}}=2\frac{\sin^2(\theta\sqrt{\tilde{n}}/2)}{1+\sin^2(\theta\sqrt{\tilde{n}}/2)}, \tag{45}$$

a transcendental equation to be solved numerically. The left-hand side represents the loss and the right-hand side the gain. The threshold is defined as the value of θ where the slope of gain and loss coincide at $n=0$. The pump parameter θ is normalized such that the threshold is at $\theta=1$. Stable solutions are the intersections of gain and loss curves where the slope of the gain curve is lower than the one of the gain. We display the graphical solution for a single value of θ in Fig. 3. For higher values of θ we find several stable solutions, but our semiclassical approach does not answer the question on which branch of the solution the laser operates or when transitions between the branches occur. Such deficiency is due to the neglect of noise and is already known from the atomic microlaser.

If we rewrite Eq. (43) with the help of Eq. (44) in the form

$$n_{\max}=\frac{N^2g^2}{4\theta^2\kappa^2}, \tag{46}$$

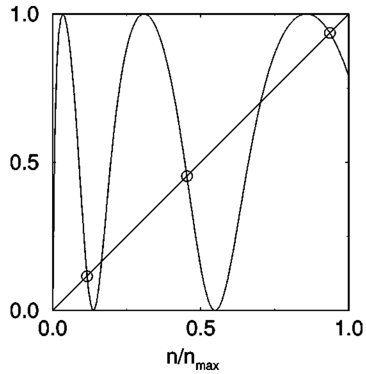

FIG. 3. Graphical solution for the stationary photon number according to Eq. (45).

we see that for a fixed value of θ the photon number scales as N^2, i.e., superradiantly. The reason for superradiance is the collectivity of the emission process. However, the collectivity is generated by a different mechanism than in the atomic superradiant laser of Refs. [10–13]. While in the latter case all transitions and even the pumping are coherent, we here get collectivity by pumping all quantum dots at the same instant.

Due to our assumptions of a large photon number and small fluctuations we may write

$$n=n_s+\nu, \quad n_s \gg \nu, \tag{47}$$

and get

$$\frac{\partial}{\partial n} n \approx n_s \frac{\partial}{\partial \nu}. \tag{48}$$

We now expand the drift coefficient in Eq. (40) around the stationary photon number n_s and find

$$\dot{P}_{\mathrm{f}}^{(1)}=\Gamma \frac{\partial}{\partial \nu} \nu P_{\mathrm{f}} \tag{49}$$

with

$$\Gamma=\kappa\left(1-\frac{\Omega_s T/2 \cot\left(\Omega_s T/2\right)}{1+\sin^2(\Omega_s T/2)}\right). \tag{50}$$

D. Semiclassical laser equation

At this point we interrupt our perturbative treatment of the back action ($\propto L-L_0$) and propose to interpret the foregoing results in terms of a familiar semiclassical laser equation.

With the help of Eq. (11) we may write the equation of motion for the mean photon number n as

$$\dot{n}=-2gN\alpha^* \sigma_{10}-\kappa n, \tag{51}$$

where we have neglected photon fluctuations. In this sense Eq. (51) is a semiclassical laser equation, since we consider a two-level system interacting with a parametric field. If we assume a high-Q cavity the influence of the periodic pumping is smoothed and we coarse grain by averaging σ_{10} over time as in Eq. (39). Then Eq. (51) becomes

$$\dot{n}=\left(\frac{A}{ngT/2}\frac{\sin^2(\Omega T/2)}{1+\sin^2(\Omega T/2)}-\kappa\right)n, \tag{52}$$

which indeed has the form of a semiclassical laser equation with gain and loss term. Here

$$A=\frac{g^2 NT}{2} \tag{53}$$

has the sense of a linear gain of the active medium. To reach the threshold of lasing the linear amplification inside the cavity must be higher than the field damping, $A>\kappa$. In the stationary regime $\dot{n}=0$ we recover the condition of vanishing drift (41),

$$n_s=\frac{2N}{\kappa T}\frac{\sin^2(\Omega_s T/2)}{1+\sin^2(\Omega_s T/2)}. \tag{54}$$

E. Diffusion terms

Similarly to the last section we rewrite the second term in Eq. (33),

$$\begin{aligned}\mathrm{Tr}_{\mathrm{dot}}^N \{(L-L_0)\pi\} &= \mathrm{Tr}_{\mathrm{dot}}^N\{D\pi\}=\sum_{i=1}^{N} \mathrm{Tr}_{\mathrm{dot}\ i}\{D_i \pi^i\}\\ &= N\, \mathrm{Tr}_{\mathrm{dot}\ i}\{D_i\pi^i\},\end{aligned} \tag{55}$$

where

$$\begin{aligned}\pi^i=\mathrm{Tr}_{\mathrm{dot}}^{N-1}\{\pi\} &= \pi_{11}^i(t)(|1\rangle\langle 1|)^i+\pi_{00}^i(t)(|0\rangle\langle 0|)^i\\ &+\pi_{01}^i(t)(|0\rangle\langle 1|)^i+\pi_{10}^i(t)(|1\rangle\langle 0|)^i\\ &+\pi_{ee}^i(t)(|e\rangle\langle e|)^i+\pi_{hh}^i(t)(|h\rangle\langle h|)^i\end{aligned} \tag{56}$$

is the trace of π over $N-1$ quantum dots. The last step in Eq. (55) again results from the equivalence of the dots. Now we find

$$\dot{P}_{\mathrm{f}}^{(2)}=Ng\left(\frac{\partial}{\partial\alpha}\pi^i{}_{10}+\frac{\partial}{\partial\alpha^*}\pi^i{}_{01}\right), \tag{57}$$

so what we need are the off-diagonal elements π_{10}^i and π_{01}^i. From Eq. (34) we get a set of equations for the matrix π^i by tracing over $N-1$ dots,

$$\dot{\pi}^i=L_{0i}\pi^i+D_i\sigma^i P_{\mathrm{f}}-\mathrm{Tr}_{\mathrm{dot}\ i}\{D_i\sigma^i\}\sigma^i P_{\mathrm{f}}. \tag{58}$$

In the Appendix we made some remarks on solving this set of equations. With the abbreviations

$$\delta_{\pm}=\frac{\partial}{\partial\alpha}\alpha \pm \frac{\partial}{\partial\alpha^*}\alpha^* \tag{59}$$

and

$$\sigma_{11}^i=\sigma_{11}^i(t_0+0), \tag{60}$$

we get the solution

$$\pi_{10}(t_0+t)=-\frac{\sigma_{11}}{4n}\frac{\alpha}{|\alpha|}\left(\Omega t+\frac{1}{2}\sin(2\Omega t)\right)\delta_- P_{\rm f}+\frac{\sigma_{11}}{4n}\frac{\alpha}{|\alpha|}\left[\sin(2\Omega t)\left\{\frac{1}{2}-\sigma_{11}\sin^2(\Omega t)\right\}+\Omega t\cos(2\Omega t)\right]\delta_+ P_{\rm f}$$
$$-\frac{\alpha}{4n^{3/2}}\sigma_{11}(1-\sigma_{11})\sin(2\Omega t)[\Omega T/2\cot(\Omega T/2)+\sigma_{11}]\delta_+ P_{\rm f}. \tag{61}$$

If we substitute π^i by its time average $\overline{\pi^i}$ and carry out all integrals, Eq. (55) becomes

$$Ng\left(\frac{\partial}{\partial\alpha}\overline{\pi}_{10}+\frac{\partial}{\partial\alpha^*}\overline{\pi}_{01}\right)=\kappa n_s\left[\frac{\Omega_s T/2\cot(\Omega_s T/2)}{1+\sin^2(\Omega_s T/2)}-\frac{1}{2}\sigma_{11}\sin^2(\Omega_s T/2)\frac{3+\sin^2(\Omega_s T/2)}{1+\sin^2(\Omega_s T/2)}\right]\frac{\partial^2}{\partial\nu^2}P_{\rm f}$$
$$+\frac{\kappa}{8n_s}\left(1+\frac{(\Omega_s^2T^2/4)}{\sin^2(\Omega_s T/2)}\right)\frac{\partial^2}{\partial\varphi^2}P_{\rm f}. \tag{62}$$

The first term is the diffusion of the photon number while the second describes phase diffusion.

F. Fokker-Planck equation

After the systematic expansion of the master equation we may write the result up to second order in the form

$$\frac{\partial}{\partial t}P_{\rm f}=\dot{P}_{\rm f}^{(1)}+\dot{P}_{\rm f}^{(2)}+\{\cdots\}$$
$$=\Gamma\frac{\partial}{\partial\nu}\nu P_{\rm f}+\Gamma\xi n_s\frac{\partial^2}{\partial\nu^2}P_{\rm f}+\frac{1}{2}D\frac{\partial^2}{\partial\varphi^2}P_{\rm f}+\{\cdots\}, \tag{63}$$

with the damping rate Γ for photon fluctuations inside the cavity.

$$\Gamma=\kappa\left(1-\frac{\Omega_s T/2\cot(\Omega_s T/2)}{1+\sin^2(\Omega_s T/2)}\right). \tag{64}$$

Here we have introduced Mandel's parameter

$$\xi=\frac{\Omega_s T/2\cot(\Omega_s T/2)-\frac{1}{2}\sin^2(\Omega_s T/2)\frac{3+\sin^2(\Omega_s T/2)}{1+\sin^2(\Omega_s T/2)}}{1-\Omega_s T/2\cot(\Omega_s T/2)+\sin^2(\Omega_s T/2)}, \tag{65}$$

which is a measure for the photon fluctuations $\overline{\Delta n^2}=n_s(1+\xi)$ with

$$\begin{aligned}\xi>0 &\quad \text{for super-Poissonian distributions,}\\ \xi=0 &\quad \text{for Poissonian distributions,}\\ -1\leqslant\xi<0 &\quad \text{for sub-Poissonian distributions.}\end{aligned} \tag{66}$$

The rate of phase diffusion is

$$D=\frac{\kappa}{4n_s}\left(1+\frac{\Omega_s^2T^2/4}{\sin^2(\Omega_s T/2)}\right) \tag{67}$$

and is the same as in a conventional laser [19].

The term $\{\cdots\}$ contains all higher-order derivatives with respect to ν and φ. In principal we are under obligation to keep all these derivatives since they might be important for nonclassical fields. But in this paper we are only concerned with the photoncurrent spectrum where higher-order terms do not contribute. Apparently Eq. (63) has the form of a Fokker-Planck equation. But notice that in the case of a narrow photon distribution Γ must be positive, while ξ is negative. Thus Eq. (63) has no solution for large times if the diffusion constant is negative. This reflects the fact that the P representation of a sub-Possonian field does not exist. Nevertheless the moments of the distribution may exist, and the system at hand is an example for such a situation. With such cautionary remarks in mind it still makes sense to work with Eq. (63). If an equation with an existing stationary solution is required one may switch to a different quasiprobability [18,20].

G. Photocurrent spectrum

The photocurrent spectrum may be written in the form

$$\overline{\langle i(0)i(\omega)\rangle}=i_{\rm SN}^{(2)}\left(1+\frac{2\kappa}{n_s}\mathrm{Re}\int_0^\infty\overline{\langle\nu(t)\nu(t+\tau)\rangle}e^{i\omega\tau}d\tau\right), \tag{68}$$

where $i_{\rm SN}^{(2)}$ is the shot-noise level. To calculate the photocurrent spectrum we have to find the correlation $\overline{\langle\nu(t)\nu(t+\tau)\rangle}$. Using a standard approach we can obtain

$$\frac{d}{d\tau}\overline{\langle\nu(t)\nu(t+\tau)\rangle}=\Gamma\overline{\langle\nu(t)\nu(t+\tau)\rangle}, \tag{69}$$

as well as

$$\dot{\overline{\nu^2}}=-2\Gamma\overline{\nu^2}+2\Gamma\xi n_s=0. \tag{70}$$

With this the desired correlation function reads

$$\overline{\langle\nu(t)\nu(t+\tau)\rangle}=\xi n_s e^{-\Gamma\tau} \tag{71}$$

and the photocurrent spectrum becomes

$$\overline{\langle i(0)i(\omega)\rangle}=i_{\rm SN}^{(2)}\left(1+2\xi\frac{\kappa\Gamma}{\Gamma^2+\omega^2}\right). \tag{72}$$

Since ξ is negative the second term reduces the spectrum below shot noise. The strength of shot-noise suppression for $\omega=0$ is given by

$$\Delta=2|\xi|\frac{\kappa}{\Gamma}=2\frac{y+1/2\sin^2(\Omega_sT/2)\dfrac{3+\sin^2(\Omega_sT/2)}{1+\sin^2(\Omega_sT/2)}}{[1+y+\sin^2(\Omega_sT/2)]^2/[1+\sin^2(\Omega_sT/2)]}, \tag{73}$$

where we have used the shorthand

$$y=-\Omega_sT/2\cot(\Omega_sT/2). \tag{74}$$

In the case where $\sin^2(\Omega_sT/2)\approx1$ this becomes

$$\Delta\approx4\frac{1+y}{(2+y)^2}. \tag{75}$$

For $y\ll1$ it is obvious that $\Delta\to1$, corresponding to complete shot-noise suppression. In contrast to that for $y\gg1$ we have no suppression and the light becomes Poissonian.

It might be helpful to compare the shot-noise suppression given by Eq. (73) with former results for different models. According to [21] the shot-noise suppression for a maser with regular atomic injection is

$$\Delta_{\rm M}=2\frac{y+1/2\sin^2(\Omega_sT/2)}{[1+y+\sin^2(\Omega_sT/2)]^2}. \tag{76}$$

The difference in the formulas (73) and (76) reflects the different physical conditions. In the maser case a regular distributed beam of fully excited atoms goes through a cavity. Since always only one atom is in the cavity at a time this corresponds to the single-quantum-dot laser. The difference here is that our quantum dot is not fully excited after every pump act, but is only with probability

$$p=\frac{1}{1+\sin^2(\Omega_sT/2)} \tag{77}$$

in the excited state. Nevertheless in both cases we find complete shot-noise suppression in the photocurrent spectrum. If we use the theory for a maser with regular distribution of atoms but only partial excitation with probability p before entering the cavity [21], the shot-noise suppression reads

$$\Delta_{{\rm M}p}=2\,\frac{y+1/2\sin^2(\Omega_sT/2)[1+\sin^2(\Omega_sT/2)]}{[1+y+\sin^2(\Omega_sT/2)]^2}. \tag{78}$$

Here we have phenomenologically chosen p as in the quatum-dot case (77). Now we find the maximum of $\Delta_{{\rm M}p}$ to be 1/2. Thus no complete suppression can be observed in such a system. It turns out that the phenomenological approach of Ref. [21] is not applicable for our quantum-dot laser, basically since our quantum dots are kept fixed in the cavity while Ref. (78) works with atoms traversing the cavity independently.

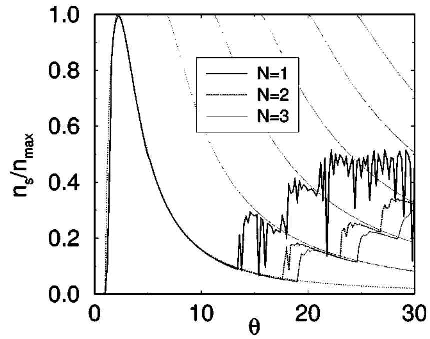

FIG. 4. Normalized stationary photon number vs dimensionless pump parameter in comparision between numerical simulation and semiclassical approximation (dotted).

V. NUMERICAL SOLUTION OF THE MASTER EQUATION

To check the analytical results we have made numerical simulations of Eq. (11). Starting from the vacuum state of the field mode we have iterated the quantum map (9) for fixed parameters κ, g, and T until the stationary state was reached. Since the theory predicts results independent of the number of quantum dots N we have limited our calculations up to $N=3$. Furthermore, we only looked at the stationary state inside the cavity and not at the photocurrent spectrum connected with the dynamics of the system.

A. Stationary photon number

The numerical results for the normalized stationary photon number are displayed in Fig. 4. The dotted lines are the stable solutions of Eq. (45). For $1<\theta<10$ we have nearly perfect agreement of theoretical and numerical results. Below threshold for $\theta<1$ Eq. (45) has no positive solution since the theory is not valid in this range. For higher values of θ we find transitions between different stable branches. For $N=1$ we addtionally see the well-known influence of the so-called trapping states also appearing in the atomic microlaser [22–24,21,25–27]. For $N>1$ these trapping states disappear and the transitions become sharp. Since Eq. (45) is only a parametric-field approximation we cannot explain these transitions since they are caused by fluctuations.

If we take a look at Eq. (46) we see that the intensity is not only increasing with N^2 but decreasing with θ^{-2}. This means that the range of small θ is the most interesting one for an experimental realization.

B. Variance and shot-noise suppression

In Fig. 5 (top) we have compared the numerical and theoretical results for the photon-number variance inside the cavity characterized by Mandel's parameter ξ. The analytical result is obtained by inserting the stable solution of Eq. (41) into Eq. (65). We have limited the comparison to the interesting θ range where no transitions between the solutions occur.

We find noise suppression inside the cavity up to over 90%, keeping in mind that this is independent of the number of quantum dots. In this range the theoretical approximation agrees very well with the numerical results. The deviation for lower noise suppression is mainly due our approximation

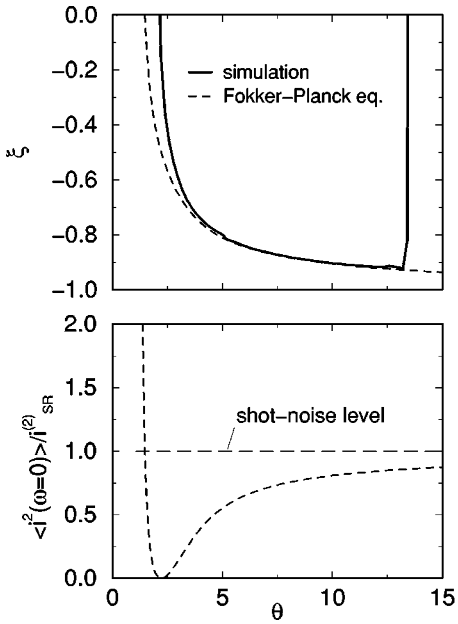

FIG. 5. Numerical solution of Mandel's parameter ξ in comparison to the semiclassical approximation (above). Below we show the shot-noise suppression of the photo current $i_{\rm SN}^{(2)}$ at $\omega=0$ as given by the semiclassical.

of small fluctuations (47), and Eq. (48).

In Fig. 5 (bottom) the prediction for shot-noise suppression in the photocurrent spectrum at $\omega=0$ is shown. It is interesting to notice that the ranges of large noise suppression inside and outside the cavity do not coincide as one might naively expect.

Finally, we would like to stress that in this model of a superradiant laser the range of large shot-noise suppression coincides with a large mean photon number. This is different from the atomic model given in Refs. [10–13].

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APPENDIX: THE EQUATIONS FOR THE MATRIX π

As for the quantum dot matrix σ we are after the stroboscopic stationary solution for π. Toward that goal we write the equations for the matrix elements in the time intervals between t_0-0 and t_0+T+0. The pump act at t_0 (electron) causes the transitions

$$\begin{aligned}
\pi_{11}(t_0+0)&=\pi_{11}(t_0-0)+\pi_{hh}(t_0-0),\\
\pi_{10}(t_0+0)&=\pi_{01}(t_0+0)=0,\\
\pi_{hh}(t_0+0)&=0,\\
\pi_{ee}(t_0+0)&=\pi_{00}(t_0-0),\\
\pi_{00}(t_0+0)&=0.
\end{aligned}\tag{A1}$$

Again the trace is conserved,

$$\pi_{11}+\pi_{ee}+\pi_{hh}+\pi_{00}=0. \tag{A2}$$

For the pump act at $t_0+T/2$ we find a set of equations equivalent to Eq. (A1). During the intervals $t_0+0<t<t_0+T/2-0$ and $t_0+T/2+0<t<t_1-0$ the interaction takes place. The equations of motion for the matrix elements up to first order in $L-L_0$ [Eq. (58)] read

$$\begin{aligned}
\dot{\pi}_{11}&=g\alpha\pi_{01}+g\alpha^*\pi_{10}+K_{11}P_{\rm f},\\
\dot{\pi}_{10}&=g\alpha(\pi_{00}-\pi_{11})+K_{10}P_{\rm f},\\
\dot{\pi}_{hh}&=K_{hh}P_{\rm f},\\
\dot{\pi}_{ee}&=K_{ee}P_{\rm f}.
\end{aligned}\tag{A3}$$

The operators K_{ik} in the inhomogeneous terms are defined by

$$K_{ik}=[(D\sigma)_{ik}-{\rm Tr}_Q(D\sigma)\sigma_{ik}], \tag{A4}$$

which in explicit form read

$$\begin{aligned}
K_{00}&={\rm Tr}_Q(D\sigma)(1-\sigma_{00}),\\
K_{11}&=-{\rm Tr}_Q(D\sigma)\sigma_{11},\\
K_{ee}&=-{\rm Tr}_Q(D\sigma)\sigma_{ee},\\
K_{hh}&=-{\rm Tr}_Q(D\sigma)\sigma_{hh},\\
K_{01}&=-{\rm Tr}_Q(D\sigma)\sigma_{01}+\frac{\partial}{\partial\alpha}\sigma_{11}.
\end{aligned}\tag{A5}$$

We now define

$$K_{\pm}=\alpha K_{01}\pm\alpha^* K_{10}. \tag{A6}$$

For a large photon number and small fluctuations the commutator of α and the differential operator $\partial/\partial\alpha$ may again be neglected, and with that approximation we get

$$K_+=\delta_+(\sigma_{11}-2|\sigma_{10}|^2) \tag{A7}$$

and

$$K_-=\delta_-\sigma_{11}. \tag{A8}$$

The operator ${\rm Tr}_Q(D\sigma)$ can be written in the form

$${\rm Tr}_Q(D\sigma)=\frac{\partial}{\partial\alpha}\sigma_{10}+\frac{\partial}{\partial\alpha^*}\sigma_{01}=\delta_{\pm}\frac{\sigma_{10}}{\alpha}=\delta_{\pm}\frac{\sigma_{01}}{\alpha^*}, \tag{A9}$$

with the shorthand

$$\delta_{\pm}=\frac{\partial}{\partial\alpha}\alpha\pm\frac{\partial}{\partial\alpha^*}\alpha^*. \tag{A10}$$

With these preparations the set of equations (A3) for π can be solved relatively easily. Imposing stroboscopic stationarity,

$$\pi(t_0+T+0)=\pi(t_0+0, \tag{A11}$$

we find the off-diagonal elements as

$$\begin{aligned}\pi_{10}(t_0+t)=&-\frac{\alpha}{2|\alpha|}\sin(2\Omega t)\pi_{11}(t_0+0)\\&+\left[\frac{g\alpha}{2|\alpha|}\{K_{00}-K_{11}\}\otimes\{\sin(2\Omega t)\}\right.\\&+\frac{g}{2\alpha^*}\{K_+\}\otimes\{\cos(2\Omega t)\}\\&\left.-\frac{g}{2\alpha^*}\{K_-\}\otimes\{1\}\right]P_{\mathrm{f}}.\end{aligned} \tag{A12}$$

Here the encircled product sign denotes temporal convolution,

$$\{f(t)\}\otimes\{g(t)\}=\int_0^t f(t')g(t-t')dt'. \tag{A13}$$

The stroboscopically stationary value of π_{11} reads

$$\begin{aligned}\pi_{11}(t_0+0)=&\frac{\sigma_{11}^2(t_0+0)}{2n}\sin^2(\Omega T/2)[\Omega T/2\cot(\Omega T/2)\\&+\sigma_{11}(t_0+0)]\delta_+P_{\mathrm{f}}.\end{aligned} \tag{A14}$$

On carrying out all convolution integrals and using $\sigma_{11}=\sigma_{11}(t_0+0)$ we find Eq. (61).

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