

## Measurements of quantum noise in optical phase conjugation via four-wave mixing in an atomic vapor

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We present the results of a comprehensive investigation of the quantum-noise properties of a continuous-wave phase-conjugate mirror (PCM) formed using backward-four-wave mixing in potassium vapor. We characterize the quantum-noise properties of the PCM as functions of the vapor density, pump detuning from resonance, and relative frequency detuning of the signal beam from the probe beam. We compare the noise measurements of the PCM with the predicted noise of an ideal quantum-noise-limited PCM and determine the value of the minimum signal that can be used to perform phase conjugation with unity signal-to-noise ratio. For the range of vapor densities studied, we find that the PCM operates nearest the quantum-noise limit and that the value of the minimum signal is lowest under conditions in which the reflectivity is maximized. These results demonstrate that it is possible to perform phase conjugation with signals as weak as 14 fW with near-unity reflectivity. Our measurements are in qualitative agreement with the predictions of a quantum theory of phase conjugation via nearly degenerate four-wave mixing in a two-level system. [S1050-2947(99)02212-X]

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### I. INTRODUCTION

An understanding of phase-conjugating optical amplifiers is important for fundamental investigations in quantum and nonlinear optics and has implications for the development of technologically relevant applications such as optical communications. For example, optical phase conjugation can be used to compensate in real time for the effects of dispersion and nonlinearities encountered in the propagation of pulses along optical fibers [1–3] and for the removal of aberrations from a signal wave front [4,5]. In the latter case, the phase-conjugate amplifier is called a phase-conjugate mirror (PCM). Phase-conjugate mirrors based on stimulated Brillouin scattering (SBS) have been used to perform phase conjugation of weak signals [6–9] as well as aberration correction of pulses through a turbulent atmosphere over a 6-km path [5]. Four-wave-mixing phase-conjugate mirrors (FWM PCMs) based on the resonant nonlinearity of atomic vapors have been studied extensively [10–16], and an atomic-vapor-based FWM PCM has been used to perform aberration correction of continuous-wave signals over a 2-km path [17].

The noise properties of phase-conjugating amplifiers have been the subject of numerous studies [6–8,18–25]. An understanding of these quantum-noise properties provides insight into possible choices of a PCM for a particular application. For example, Agarwal *et al.* [26] have shown that the beam generated at the output of a phase-conjugate resonator

is predicted to be quadrature squeezed under conditions in which the PCM is operated near the ideal quantum-noise limit. An understanding of the noise properties of phase-conjugating amplifiers is also important in the development of applications that require the ability to phase-conjugate very weak signals, since the total noise determines the value of minimum signal that can be amplified for a specified value of the signal-to-noise ratio (SNR).

Recent quantum-mechanical treatments of the noise properties of PCMs have shown that excess quantum noise is inherent to the phase-conjugation process [18,19,21,22] and additional contributions to the noise can arise from the physical mechanism that gives rise to phase conjugation and from fluctuations (e.g., collisions) in the nonlinear medium, as has been predicted for the nonlinear process of phase-preserving amplification by two-beam coupling in atomic vapors [27–29]. For Brillouin-enhanced four-wave-mixing (BEFWM) PCMs, the additional noise arises from the presence of thermal phonons. For atomic-vapor-based FWM PCMs, the additional noise is a result of resonance fluorescence. Recently, we used a high-reflectivity (up to 670%), wide-bandwidth (up to 230 MHz) potassium-vapor-based FWM PCM [30] to achieve phase conjugation and aberration correction of weak (250 fW) continuous-wave optical signals with a SNR of 18:1 [31]. Previously, the quantum-noise properties of nonlinear optical processes in atomic vapors such as intracavity four-wave mixing [32], forward four-wave mixing [33,34], and phase-preserving amplification [27–29] have been studied.

In this paper, we present detailed results of a comprehensive experimental and theoretical investigation of the quantum-noise properties of a continuous-wave PCM implemented using nearly degenerate four-wave mixing in potassium vapor [30]. Our work is motivated by the desire to determine the experimental parameters with which a PCM

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can perform phase conjugation on the smallest possible cw signals. In our investigation, we measure the phase-conjugate reflectivity and the total noise generated during the phase conjugation process as functions of the potassium vapor density, the pump detuning from resonance, and the relative frequency detuning between the signal beam and the pump beams. From the noise and reflectivity measurements, we determine the number of excess noise photons generated by the phase conjugation process, the photon noise factor, and the minimum signal level that is required to perform phase conjugation with a signal-to-noise ratio of unity. Our results show that over a wide range of parameters, the PCM operates nearest the quantum-noise limit under conditions that maximize the reflectivity. We use a fully quantum-mechanical theory [35,36] of phase conjugation via nearly degenerate four-wave mixing in a two-level system to model our results. In this theory, the signal and conjugate waves are quantized, while the strong pump waves are treated classically. The theory includes the effects of the collisions between the atoms but not the effect of grating-washout due to atomic motion. We find that our measurements are in good qualitative agreement with the theoretical predictions.

## II. PHENOMENOLOGICAL ANALYSIS OF THE ORIGIN OF QUANTUM NOISE IN PHASE CONJUGATION

We illustrate the origin of quantum noise in the phase-conjugation process with the following phenomenological analysis. For a PCM, the annihilation operator  $\hat{a}_c$  of each conjugate field mode is related to the creation operator  $\hat{a}_s^\dagger$  of a corresponding signal field mode via [22]

$$\hat{a}_c = \sqrt{R_{pc}} \hat{a}_s^\dagger + \hat{L}, \quad (1)$$

where  $R_{pc}$  is the phase-conjugate reflectivity and  $\hat{L}$  is the Langevin noise operator which obeys the commutation relation  $[\hat{L}, \hat{L}^\dagger] = R_{pc} + 1$  and satisfies the condition  $\langle \hat{L} \rangle = 0$ . For the case in which phase conjugation is achieved via backward FWM in a lossless Kerr medium,  $\hat{L}$  is identified with the amplified vacuum field mode incident on the rear port of the PCM [20]. The expectation value  $n_c$  of the photon number in the conjugate field is given by the expression

$$n_c = \langle \hat{a}_c^\dagger \hat{a}_c \rangle = R_{pc} n_s + R_{pc} + N_n, \quad (2)$$

where  $n_s = \langle \hat{a}_s^\dagger \hat{a}_s \rangle$  is the expectation value of the photon number in the signal field and  $N_n = \langle \hat{L}^\dagger \hat{L} \rangle$  is the number of excess noise photons produced by the PCM in the conjugate field mode. We define the quantum-noise limit (QNL) of an ideal PCM to be to the case in which  $N_n/R_{pc} = 0$ . The number of excess noise photons,  $N_n$ , depends on the physical mechanism that gives rise to phase conjugation. For example, in BEFWM and in SBS, the number of excess noise photons results from spontaneous Brillouin scattering. In FWM in an atomic vapor, the number of excess noise photons results from the resonance fluorescence of the atoms that are being strongly driven by the pump waves. Equation (2) shows that the total number of noise photons in each mode of the conjugate field is  $N_n + R_{pc}$  and that this number reaches a minimum value of  $R_{pc}$  for the case in which the

PCM is quantum-noise limited. Therefore, the total number of noise photons  $N_n + R_{pc}$  determines the level of the weakest signal that can be conjugated for a specified value of the SNR of the conjugate field.

The experimental technique we use to measure the noise produced by the PCM is optical heterodyne detection, in which the conjugate field and a strong local-oscillator (LO) field are combined at a glass beam splitter. The positive-frequency part of the combined field  $\hat{E}^{(+)}(t)$  incident at the photodetector is given by the expression

$$\hat{E}^{(+)}(t) = \sqrt{T} \hat{E}_c^{(+)}(t) + E_{lo}^{(+)}(t) + i\sqrt{1-T} \hat{E}_v^{(+)}(t), \quad (3)$$

where  $\hat{E}_c^{(+)}(t)$ ,  $E_{lo}^{(+)}(t)$ , and  $\hat{E}_v^{(+)}(t)$  represent positive-frequency parts of the field produced by the PCM, the LO field, and the vacuum field reflected from the beam splitter, respectively, and where  $T \leq 1$  is the transmission coefficient of the beam splitter. The LO field is measured after the beam splitter. The fields  $\hat{E}_c^{(+)}(t)$  and  $\hat{E}_v^{(+)}(t)$  are quantized, and we decompose the fields into their individual frequency components such that

$$\hat{E}_c^{(+)}(t) = \sum_j \hat{a}_{c_j}(t) = \sum_j \hat{a}_{c_j} \exp[-i2\pi(\nu_o + \delta\nu_j)t], \quad (4)$$

$$\hat{E}_v^{(+)}(t) = \sum_j \hat{a}_{v_j}(t) = \sum_j \hat{a}_{v_j} \exp[-i2\pi(\nu_o + \delta\nu_j)t], \quad (5)$$

where the sum is taken over the frequencies  $\delta\nu_j$ , where  $\nu_o$  is the frequency of the pump beams. The value of  $\delta\nu_j$  represents the frequency shift of each component of the phase-conjugate beam relative to the frequency  $\nu_o$ . The photon annihilation operator for the conjugate field mode is given by the expression

$$\hat{a}_{c_j} = \sqrt{R_{pc}} \hat{a}_{s_j}^\dagger + \hat{L}_j, \quad (6)$$

where  $\hat{a}_{s_j}$  and  $\hat{L}_j$  are the annihilation operator and the Langevin noise operator, respectively, at frequency  $\nu_o + \delta\nu_j$  for each corresponding signal field mode at frequency  $\nu_o - \delta\nu_j$ . The monochromatic LO field  $E_{lo}^{(+)}(t)$  is treated classically and is given by the expression,

$$E_{lo}^{(+)}(t) = E_{lo} \exp[-i2\pi\nu_o t], \quad (7)$$

where the frequency  $\nu_o$  of the local oscillator field is equal to the frequency of the pump fields.

The total field  $\hat{E}(t)$  given in Eq. (3) can be written as the sum of the positive-frequency part  $\hat{E}^{(+)}(t)$  and the negative-frequency part  $\hat{E}^{(-)}(t)$  according to the expression

$$\hat{E}(t) = \hat{E}^{(+)}(t) + \hat{E}^{(-)}(t). \quad (8)$$

The intensity of the combined field  $\hat{E}(t)$  is then detected by a high-quantum-efficiency, fast photodiode, and the photocurrent is amplified and measured by an electronic spectrum analyzer for its frequency content. Following the treatment of Kauranen *et al.* [37] to derive the noise power

spectrum of the phase-conjugate beam, we introduce the photocurrent operator  $\hat{I}(t)$  that describes the photocurrent resulting from the photodetection process,

$$\hat{I}(t) = \frac{ec}{L} \hat{E}^{(-)}(t) \hat{E}^{(+)}(t), \quad (9)$$

where  $e$  is the electric charge,  $c$  is the speed of light, and  $L$  is the linear size of the quantization volume. The correlation function  $C(\tau)$  of the photocurrent is given by the expression

$$C(\tau) = \frac{1}{2} \langle \hat{I}(t) \hat{I}(t+\tau) + \hat{I}(t+\tau) \hat{I}(t) \rangle. \quad (10)$$

The power spectrum  $S(f)$  of the amplified photocurrent is given by the Wiener-Khintchine theorem as the Fourier transform of the correlation function according to the following expression:

$$S(f) = R_L \frac{G^2}{4} \int_{-\infty}^{\infty} C(\tau) \exp[i2\pi f\tau] d\tau, \quad (11)$$

where  $R_L$  is the load resistor,  $G$  is the gain of the amplifier, and the frequency  $f$  is in the range  $-\infty \leq f \leq \infty$ .

We substitute Eqs. (3)–(10) into Eq. (11) and obtain an expression for the power spectrum  $S(f)$  displayed by the spectrum analyzer for positive values of the frequency  $f$  for the case in which the signal field incident at the PCM is in a single mode and the frequency of the signal field is  $\nu_s$ . In this case, we find that the power spectrum is given by the expression

$$S(f) = S_o \left[ 1 + T \left\{ P_s \frac{\eta}{h\nu_s} R_{pc}(\nu_s) \delta(f - \nu_s + \nu_o) + R_{pc}(\nu_o + f) + R_{pc}(\nu_o - f) + N_n(\nu_o + f) + N_n(\nu_o - f) \right\} \right], \quad (12)$$

where  $\nu_o$  is the frequency of the pump beams,  $P_s$  is the signal power,  $\eta$  is the quantum efficiency of the photodiode,  $h$  is Planck's constant,  $f \geq 0$  is the spectrum analyzer frequency, and  $S_o$  is the shot noise of the local oscillator and is given by the expression  $S_o = R_L(G^2/2)(\eta e^2/h\nu_{lo})P_{lo}$ , where  $P_{lo}$  is the power of the local-oscillator field. The values of the phase-conjugate reflectivity and the number of excess noise photons at the frequencies  $\nu_o \pm f$  are  $R_{pc}(\nu_o \pm f)$  and  $N_n(\nu_o \pm f)$ , respectively.

The second term in curly brackets on the right-hand side (RHS) of Eq. (12) is the contribution of the conjugated signal field and is equal to the number of photons detected per unit time per unit frequency. The last four terms on the RHS result from the noise photons produced by the PCM. We define the noise power of the photocurrent relative to the power spectral density of the shot noise  $S_o$  produced by the LO field to be  $S_{rel}(f) = S(f)/S_o$ .

An expression for the minimum signal power  $P_s^{min}$  (for SNR = 1:1) at the signal frequency  $\nu_s$  is obtained by integrating Eq. (12) over a bandwidth  $\Delta f$  to obtain the expres-

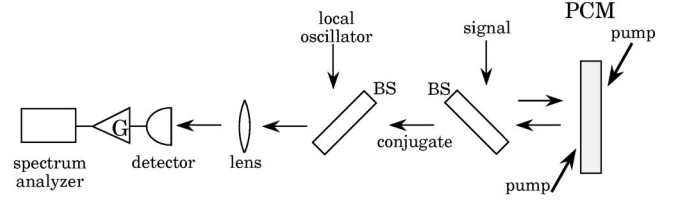


FIG. 1. Schematic of the experimental setup. BS, beam splitter; PCM, phase-conjugate mirror; G, amplifier gain.

$$P_s^{min} = \frac{h\nu_s \Delta f}{\eta R_{pc}(\nu_s)} \left[ \frac{1}{T} + R_{pc}(\nu_s) + R_{pc}(2\nu_o - \nu_s) + N_n(\nu_s) + N_n(2\nu_o - \nu_s) \right], \quad (13)$$

where  $\Delta f$  is the resolution bandwidth of the detection system.

We now assume that  $R_{pc}(\nu_o + f)$  equals  $R_{pc}(\nu_o - f)$  and that  $N_n(\nu_o + f)$  equals  $N_n(\nu_o - f)$ . We believe that these assumptions are valid since our theoretical analysis shows that the values of the phase-conjugate reflectivity  $R_{pc}$  and the number of excess noise photons  $N_n$  at each sideband are nearly equal when the frequency  $f$  is much smaller than the pump detuning, as is the case in our experiments that are discussed in the next section. Also, previous measurements in atomic vapors have shown that  $R_{pc}$  is symmetric about the frequency of the pump waves under these conditions [13,14]. Under these assumptions, we find that the relative power spectral density  $S_{rel}(f)$  of the photocurrent and the minimum signal  $P_s^{min}$  are given by the expressions

$$S_{rel}(f) = 1 + T \left\{ P_s \frac{\eta}{h\nu_s} R_{pc}(\nu_s) \delta(f - \nu_s + \nu_o) + 2R_{pc}(\nu_o + f) + 2N_n(\nu_o + f) \right\}, \quad (14)$$

$$P_s^{min} = \frac{h\nu_s \Delta f}{\eta R_{pc}(\nu_s)} \left[ \frac{1}{T} + 2R_{pc}(\nu_s) + 2N_n(\nu_s) \right]. \quad (15)$$

We further quantify the quantum-noise properties of our PCM by introducing the photon noise factor

$$N_{pc}(f) = 1 + \frac{N_n(\nu_o + f) + N_n(\nu_o - f)}{R_{pc}(\nu_o + f) + R_{pc}(\nu_o - f)} \quad (16)$$

of the PCM which is equal to the ratio of the total number of noise photons produced by the PCM to the number of noise photons produced by an ideal PCM. For an ideal quantum-noise-limited PCM,  $N_{pc}(f) = 1$ .

### III. MEASUREMENTS OF QUANTUM NOISE IN PHASE CONJUGATION IN AN ATOMIC VAPOR

To perform the reflectivity and noise measurements described here, we use a high-reflectivity, wide-bandwidth PCM that operates via nearly degenerate backward FWM in a 2-mm potassium vapor cell [30]. A schematic of the experimental setup is shown in Fig. 1. A frequency-stabilized continuous-wave titanium-sapphire laser with 2 W of output

power is tuned near the resonance frequency  $\nu_{D_2}$  of the potassium  $D_2$  line and is used to produce the two counterpropagating pump beams and the signal beam which are all linearly polarized in the same direction. Each pump beam can have up to 600 mW of power and is focused such that the confocal parameter is significantly larger than the entire 2-mm interaction length, which results in an intensity of up to  $10^3$  W/cm<sup>2</sup> at the cell. Two acousto-optic modulators (operating at 30–50 MHz and 60–120 MHz, respectively) are used to shift the frequency of the signal beam to  $\nu_s$  which is shifted relative to the pump frequency  $\nu_o$  by an amount called the signal-pump detuning  $\delta\nu$ . The signal beam can have up to 350  $\mu$ W of power and is more tightly focused, with an intensity as high as 3 W/cm<sup>2</sup> at the cell. The cell is heated to 300 °C, and the potassium reservoir is heated up to 230 °C.

We use optical heterodyne detection to measure the noise properties of the continuous-wave FWM PCM. A schematic of the experimental setup is shown in Fig. 1. The phase-conjugate beam and the local oscillator fields are combined at an uncoated glass beam splitter and are detected by a high-quantum-efficiency ( $\eta \sim 0.76$ ), fast (350 MHz) photodiode. The power of the LO field after the beam splitter is  $P_0 \sim 4$  mW, and its frequency is the same as the frequency of the pump fields. The photocurrent is amplified, and its frequency spectrum is measured with an electronic spectrum analyzer. The power spectral density of the photocurrent  $S(f)$  is given by Eq. (11). In our experiments, the amplifier gain  $G$  is 26 dB, and we measure the noise for frequencies  $f \leq 120$  MHz. All of our data have been taken with the laser tuned below the atomic resonance (i.e.,  $\nu_o - \nu_{D_2} \sim -1$  to  $-8$  GHz), since in this case the self-defocusing of the pump waves is much less deleterious to the four-wave mixing interaction than is the self-focusing that occurs when the pump waves are tuned above resonance.

We have measured the dependence of noise and the phase-conjugate reflectivity of the PCM on three experimental parameters: the potassium vapor density  $N$ , the signal-pump frequency detuning  $\delta\nu$ , and the pump detuning below atomic resonance. We have used these noise measurements and Eqs. (14), (15), and (16) to determine the dependence of the number of excess noise photons  $N_n$ , the minimum signal  $P_s^{min}$ , and the photon noise factor  $N_{pc}$  on these parameters. Figure 2 shows the reflectivity  $R_{pc}$ , the photon noise factor  $N_{pc}$ , and the minimum signal level  $P_s^{min}$  for three values of the potassium vapor density. These values have been chosen to illustrate the characteristics of the PCM at the lowest density studied [ $N = 2 \times 10^{13}$  cm<sup>-3</sup>, as shown in Figs. 2(a), 2(b), and 2(c)] and at the density which permits phase conjugation of the smallest signals [ $N = 2 \times 10^{14}$  cm<sup>-3</sup>, as shown in Figs. 2(a'), 2(b'), and 2(c)'].

Figures 2(a) and 2(a') summarize the behavior of the phase-conjugate reflectivity  $R_{pc}$  as a function of the laser detuning below resonance for three values of the signal-pump detuning ( $\delta\nu = \nu_s - \nu_o = 30, 60, 120$  MHz). A peak in the phase-conjugate reflectivity is observed for all three values of the signal-pump detuning at a pump detuning of approximately  $-1.6$  GHz. This value of the pump detuning yields the best compromise between maximizing the FWM nonlinearity and minimizing the absorption of the pump, sig-

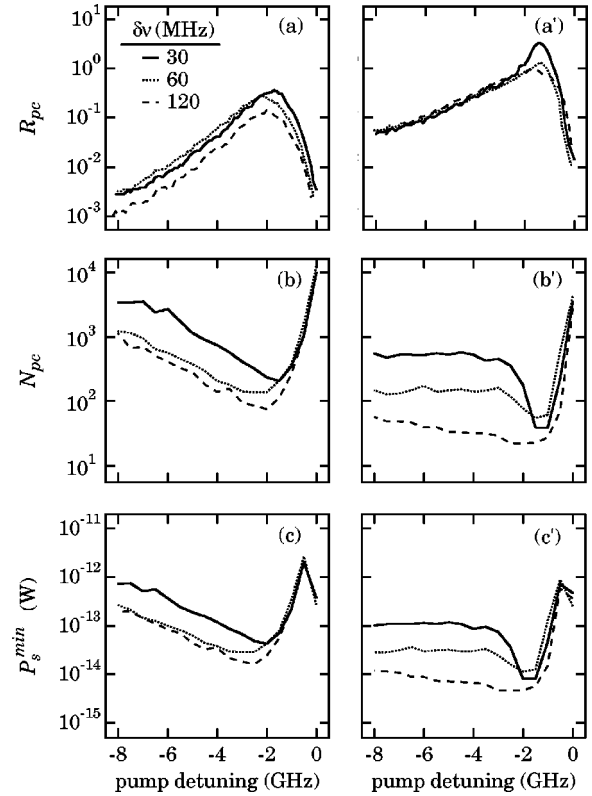


FIG. 2. Measured values of the phase-conjugate reflectivity  $R_{pc}$  (a), (a'), photon noise factor  $N_{pc}$  (b), (b'), and minimum signal power  $P_s^{min}$  (c), (c') as a function of the laser detuning below resonance for three values of the signal-pump detuning and for two values of the potassium vapor density: (a)–(c)  $N = 3 \times 10^{13}$  cm<sup>-3</sup>, (a')–(c')  $N = 2 \times 10^{14}$  cm<sup>-3</sup>.

nal, and conjugate waves at each value of the potassium vapor density [30].

Figures 2(b), 2(b') and 2(c), 2(c') summarize the dependence of the photon noise factor  $N_{pc}$  and the minimum phase-conjugate signal  $P_s^{min}$ , respectively, on the pump detuning from resonance for three values of the relative probe-pump detuning. The values of the minimum signal power  $P_s^{min}$  are calculated using a value of the resolution bandwidth  $\Delta f$  equal to 300 Hz. There are several general trends which are evident from these figures. First, our measurements show that as the potassium vapor density is increased, the resonance fluorescence and thus the number of excess noise photons  $N_n$  increases. Nevertheless, the values of the photon noise factor  $N_{pc}$  and the minimum signal  $P_s^{min}$  tend to decrease since the reflectivity increases faster than the number of excess noise photons does. Second, the number of excess noise photons and the photon noise factor tend to decrease as the value of the relative signal-pump detuning is increased, for each value of the potassium vapor density, which is consistent with squeezing experiments in atomic vapors [32].

Inspection of Figs. 2(a), 2(a') and 2(b), 2(b') shows that the value of  $N_n$  is at least several times larger than the number of noise photons expected for an ideal PCM since  $N_n \gg R_{pc}$  at each value of the potassium vapor density. A comparison of Figs. 2(a), 2(a') and 2(c), 2(c') shows that for each value of the potassium vapor density, the value of the minimum signal  $P_s^{min}$  is smallest under conditions in which the



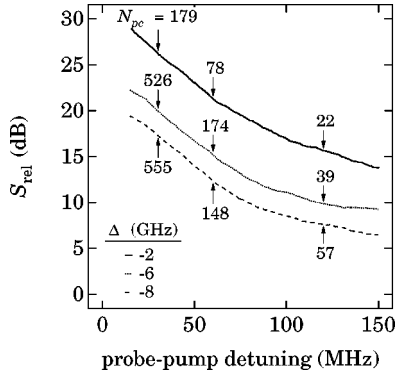


FIG. 3. Noise power spectral density  $S_{rel}$  in the phase conjugate beam relative to the shot noise of the local oscillator as a function of the probe-pump detuning for three values of the laser detuning from resonance, where the potassium vapor density is  $N=2 \times 10^{14} \text{ cm}^{-3}$ . The corresponding values of the photon noise figure  $N_{pc}$  at three values ( $\delta\nu=30, 60, 120$  MHz) of the probe-pump detuning are also shown.

phase-conjugate reflectivity is near its maximum value. We have determined that the minimum value of  $P_s^{min}$  is  $\sim 3$  to 4 fW (with  $\text{SNR} = 1:1$ ) and that this value can be achieved under conditions in which the signal-pump detuning is 120 MHz, and the pump frequency is tuned approximately 2 GHz below resonance. Under these conditions, the PCM is operating 22 times above the quantum-noise limit.

Figure 3 summarizes the behavior of the noise power spectral density  $S_{rel}$  in the phase conjugate beam relative to the shot noise of the LO as a function of the probe-pump detuning for three values of the laser detuning from resonance, where the potassium vapor density is  $2 \times 10^{14} \text{ cm}^{-3}$ . The corresponding values of the photon noise factor  $N_{pc}$  at three values of the probe-pump detuning ( $\delta\nu=30, 60, 120$  MHz) are also shown in the figure. For our focusing geometry, we find that the number of excess noise photons,  $N_n$ , per second can be as few as one in a 1-Hz bandwidth under conditions in which the atomic vapor density is  $3 \times 10^{13} \text{ cm}^{-3}$ , the laser is tuned 8 GHz below resonance, and the relative probe-pump detuning is 120 MHz.

#### IV. COMPARISON OF THE EXPERIMENTAL RESULTS WITH A QUANTUM THEORY OF PHASE CONJUGATION IN A TWO-LEVEL SYSTEM

We use our recently developed theory [35,36] of phase conjugation by nearly degenerate FWM in a two-level system to model our results. In this model, the pump waves are treated classically, and the signal and conjugate waves are quantized. The effects of collisions between the atoms are included, while Doppler broadening and grating-washout effects due to atomic motion are not included. Proper inclusion of Doppler broadening and grating washout effects into the quantum theory is beyond the scope of this paper; however, we find that our theoretical results give qualitative agreement with our experimental results. The coupled operator equations for the signal and conjugate are given by

$$\frac{d\hat{a}_s}{dz} = \gamma_s \hat{a}_s + i\kappa_s^* \hat{a}_c^\dagger + \hat{L}_1, \quad (17)$$

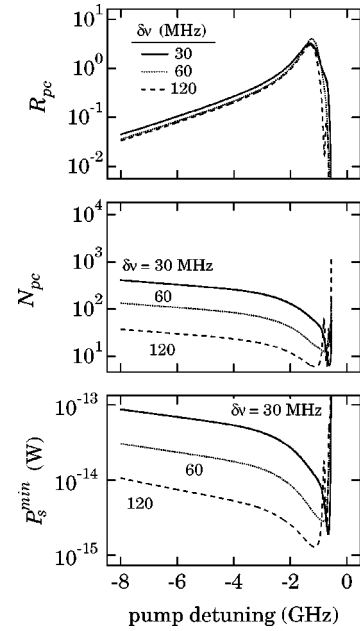


FIG. 4. Theoretical prediction of the phase-conjugate reflectivity  $R_{pc}$ , the photon noise factor  $N_{pc}$ , and the minimum signal power  $P_s^{min}$  as a function of the pump detuning from resonance for three values ( $\delta\nu=30, 60, 120$  MHz) of the probe-pump detuning, and where the effects of pump absorption are included.

$$\frac{d\hat{a}_c^\dagger}{dz} = -\gamma_c \hat{a}_c^\dagger + i\kappa_c \hat{a}_s + \hat{L}_2, \quad (18)$$

where the coefficients  $\gamma_j$  and  $\kappa_j$  ( $j=s,c$ ) are those given by the semiclassical model of a two-level atom [38], and where the moments of the noise operators  $\hat{L}_1$  and  $\hat{L}_2$  are determined from a general quantum formalism that takes into account the fluctuations of the decay of the population inversion and the dephasing of the atomic dipole moment [36].

We expect that the theoretical predictions for smaller pump detunings ( $-1$  to  $0$  GHz) are not valid since the effects of strong pump absorption play a dominant role in this regime [27]. For larger values of the pump detuning from resonance (i.e.,  $\nu_o - \nu_{D_2} \sim -2$  to  $-8$  GHz), this theory predicts values for the phase-conjugate reflectivity  $R_{pc}$ , the photon noise factor  $N_{pc}$ , and the minimum signal  $P_s^{min}$  that are in qualitative agreement with our experimental results reported above. Figure 4 is a plot of the (a) phase-conjugate reflectivity  $R_{pc}$ , (b) the photon noise factor  $N_{pc}$ , and (c) the minimum signal power  $P_s^{min}$  as functions of the pump detuning from resonance for three values of the signal-pump detuning. To obtain the results shown in this figure, we use our measurements of the pump transmission through the cell to approximate in the theory the effects of weak pump absorption. All (except one) of the parameters are chosen to correspond to the experimental conditions in which the potassium vapor is at  $210^\circ\text{C}$  and the resolution bandwidth  $\Delta f=300$  Hz. In Fig. 4, the ratios of the spontaneous emission rate and the Rabi frequency (associated with each pump-field amplitude) to the dipole-dephasing rate ( $1/2\pi T_2=10.6$  MHz) are 0.6 and 153, respectively. The value of the remaining parameter, the absorption coefficient  $\alpha_0$ , is then chosen to be  $\alpha_0 = 1.15 \times 10^4 \text{ cm}^{-1}$  to give a good fit to  $R_{pc}$  ( $\delta\nu=30\text{MHz}$ )

as a function of pump detuning. This value is within the uncertainty in our estimate of the experimental value. As observed in the experiments, the theoretical results show that  $P_s^{min}$  reaches a minimum at the pump detuning where  $R_{pc}$  is near its maximum. The occurrence of the minimum signal near the peak value of the reflectivity is found to persist over a wide range of parameters. Schirmer *et al.* [35] have shown theoretically that the region in which the minimum signal can be phase conjugated is near the quantum-noise limit only in the radiatively broadened regime.

## V. SUMMARY AND CONCLUSIONS

In conclusion, we demonstrate that the noise produced during the phase conjugation process using four-wave mixing an atomic vapor is typically much greater than is predicted for an ideal quantum-noise-limited phase-conjugate mirror. Nevertheless, the power level of the minimum signal

that is required to perform phase conjugation with a SNR =1:1 can be as low as a few femtowatts with near-unity reflectivity. Under these conditions, the PCM operates nearest to the quantum-noise limit. These characteristics are essential for developing low-noise optical devices for applications in optical communications and optical signal processing.

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