Saturated transitions in exactly soluble models of two-state curve crossing with time-dependent potentials

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Two exactly soluble two-state curve crossing models, with potential matrices formed by a linear combination of time-independent terms and (I) inversely proportional ($\sim t^{-1}$) or (II) exponential time-dependent terms, are considered here. It is shown that the two models are related to each other by a simple transformation of the time variable. These models can be further transformed to a simpler one (the ''reduced'' model), in which the two crossing potentials are a constant (horizontal) one and an inversely proportional time-dependent one, and the interaction coupling them is time-independent. Analysis of the exact solution of the reduced model discloses that the nonadiabatic transition probability *saturates* at increasingly large coupling strengths without vanishing (in contradiction of the linear Landau-Zener theory and various adiabatic theories). Similar saturation can also occur in more general models, within certain ranges of the potential parameters involved. [S1050-2947(99)06011-4]

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I. INTRODUCTION

Curve-crossing problems have recently received renewed attention by their application to optical collisions of ultracold atoms in general, and to the problem of optical shielding (or suppression) in particular (see papers [1,2], reviews [3,4], and references therein). A dominant feature of the optical shielding of ultracold atoms is a saturation effect that keeps the probability of transmission through the crossing zone from vanishing as the radiative coupling is increased. This curve-crossing saturation effect is contrary to expectations based on the linear two-state Landau-Zener (LZ) model (see Refs. [5,6]). An adequate explanation of this effect (discussed in Refs. [1,2]) utilizes multistate curve crossings, including near-degenerate and ''counterintuitive'' transitions (see also Refs. [7,8]).

The present paper aims to show that, under certain conditions that may be relevant to a wide variety of curve-crossing problems in physics, saturation can take place even in two-state curve-crossing models, provided that they involve nonlinear potentials.

In atomic collision problems, curve crossings are correctly described by a system of coupled second-order differential equations in the radial (R-dependent) channel wave functions (see [9–11]). Exact solutions are known only for very few R-dependent two-state problems. These include the crossing of a horizontal and a slanted linear potential (solved in Ref. [12]; see also Ref. [7] for generalization to the multistate case including degeneracy), and an exponential problem constrained to a special combination of parameters (see Refs. [13,14]).

Assigning a common trajectory to all participating channels (see Refs. [9,10]), R-dependent problems can be reduced to strictly t-dependent problems. Time-dependent problems can appear, however, more naturally in other applications; e.g., when time-varying external fields are applied. Such problems, with which we are concerned here, can be described by a system of first-order differential equations involving time-dependent potentials. In addition to the linear t-dependent model solved exactly in Ref. [6], a few exactly soluble nonlinear models are known. Two models, involving an exponential potential gap [$\Delta V \sim \exp(-\kappa t)$] with (i) a t-independent interaction ($V_{12}=\text{const}$) and (ii) an exponential interaction [$V_{12}\sim\exp(-\kappa t)$], have been solved by Nikitin in Refs. [15,16] (see also Refs. [17,9,18]). Exact solutions are also known for the covalent-ionic model involving $\Delta V \sim t^{-1}$, up to an added constant, and $V_{12}\sim t^{-1}$ (see Refs. [19,20]), and for hypergeometric models (see Ref. [9]).

In the two-state linear curve-crossing problem the potentials diverge infinitely as the interatomic distance increases. This fact, combined with the use of a constant interaction, allows us to introduce *diabatic* boundary conditions at both asymptotes. In the nonlinear curve-crossing problems, in contrast, the ratio of the interaction strength to the potential gap does not vanish at one of the asymptotes, and thus *adiabatic* boundary conditions are required. The transition probabilities in these models cannot be expressed by the LZ formula, which is an exact solution for the linear t-dependent problem (as well as for the linear R-dependent problem with a horizontal potential).

The present paper is concerned, first of all, with a generalization of the two-state t-dependent models, with a potential matrix that is formed by a linear combination of constant (t-independent) terms and (I) inversely proportional or (II) exponential terms. The two exponential Nikitin models are special cases of the generalized ''exponential'' model, and the covalent-ionic model is a special case of the generalized ''inverse-t'' model. Moreover, we show here that the two kinds of models are related. A simple transformation, introduced in Sec. II, converts the exponential model to the inverse-t model. Still another transformation reduces the inverse-t model to a simpler model, that involves the crossing of an inversely proportional and a horizontal potential, coupled by a constant interaction — henceforth called the ''reduced'' model. This model is solved exactly in Sec. III. The interaction induced by a near-resonant laser field usually

varies with R only within a microscopic range. Therefore the reduced model is quite appropriate for the description of optical collisions. In this model, as well as in the more general models (within a certain range of the parameters), saturation of the transition probabilities can occur at large interaction strengths.

II. TRANSFORMATIONS LEADING TO THE REDUCED MODEL

A two-state t-dependent curve-crossing problem can be generally described by a system of two coupled equations of the form (see Refs. [9–11]), using atomic units,

$$\begin{aligned} i\frac{da_1(t)}{dt} &= \left(V_0(t) - \frac{1}{2}\Delta V(t) \right) a_1(t) + V_{12}(t) a_2(t), \\ i\frac{da_2(t)}{dt} &= V_{12}^*(t) a_1(t) + \left(V_0(t) + \frac{1}{2}\Delta V(t) \right) a_2(t). \end{aligned} \tag{1}$$

This problem can be easily reduced to the case $V_0(t)=0$ by a gauge transformation, and therefore problems that differ by $V_0(t)$ are simply related to each other.

Consider now the following two models.

(I) The generalized inverse-t model, in which

$$\Delta V(t) = \Delta_0 - \Delta_1/t, \quad V_{12}(t) = g_0 + g_1/t. \tag{2}$$

This model includes the covalent-ionic model [19,20] as a special case ($g_0=0$). The two states are defined so that $\Delta_0 \geqslant 0$, and the parameter g_0 can always be made real and positive by a simple phase change. This model is limited to $t \geqslant 0$.

(II) The generalized exponential model, in which

$$\begin{aligned} \Delta V(t) &= \Delta_0^{(e)} - \Delta_1^{(e)} \exp(-\kappa t), \\ V_{12}(t) &= g_0^{(e)} + g_1^{(e)} \exp(-\kappa t). \end{aligned} \tag{3}$$

This model includes the two Nikitin models (i) and (ii) (see Refs. [15,16]) as special cases ($g_1^{(e)}=0$ or $g_0^{(e)}=0$, respectively). The exponential model described by Eq. (3) can be transformed into the inverse-t model [Eq. (2)] by substituting a new time variable,

$$t' = \frac{1}{\kappa}\exp(-\kappa t). \tag{4}$$

The parameters of the inverse-t model are related to those of the exponential model, identified by the superscript (e), as follows:

$$\begin{aligned} \Delta_0 &= \Delta_1^{(e)}, \quad \Delta_1 = \Delta_0^{(e)}/\kappa, \\ g_0 &= -g_1^{(e)}, \quad g_1 = -g_0^{(e)}/\kappa. \end{aligned} \tag{5}$$

The inverse-t model [Eq. (2)] can be further reduced to a simpler model with a t-independent coupling. This is performed by applying the unitary matrix

$$\alpha_{jk} = (2q)^{-1/2} \begin{pmatrix} \sqrt{q+\Delta_1} & -\sqrt{q-\Delta_1}\, e^{i\theta} \\ \sqrt{q-\Delta_1}\, e^{-i\theta} & \sqrt{q+\Delta_1} \end{pmatrix} \tag{6}$$

in which $\theta = \arg g_1$ and

$$q = \sqrt{\Delta_1^2 + 4|g_1|^2}. \tag{7}$$

Definition of a new pair of variables $b_1(t)$, $b_2(t)$, by substituting

$$\begin{aligned} a_j(t) = \sum_{k=1}^{2} \alpha_{jk} b_k(t) t^{iq/2} \\ \times \exp\left(-i\int_0^t V_0(t')dt' + \frac{i}{2}Vt - i\delta_{k2}\arg g \right), \end{aligned} \tag{8}$$

where

$$\begin{aligned} g &= \frac{\Delta_0 g_1}{q} + g_0 \frac{q + \Delta_1 - (q-\Delta_1)e^{2i\theta}}{2q}, \\ V &= \frac{\Delta_0\Delta_1 - 4g_0 \mathrm{Re}\, g_1}{q}, \end{aligned} \tag{9}$$

leads to the equations

$$\begin{aligned} i\frac{db_1(t)}{dt} &= \frac{q}{t} b_1(t) + |g| b_2(t) \\ i\frac{db_2(t)}{dt} &= |g| b_1(t) + V b_2(t). \end{aligned} \tag{10}$$

These equations constitute the *reduced* model, solved exactly in Sec. III below.

III. SOLUTION OF THE REDUCED MODEL

The system of two first-order differential Eqs. (10) can be transformed to a single second-order differential equation for $b_2(t)$. Substitution of a new variable $c(t)$ using

$$b_2(t) = c(t)\exp\left(i\frac{\Omega - V}{2}t \right), \quad \Omega = \sqrt{V^2 + 4|g|^2}, \tag{11}$$

results in the *confluent hypergeometric* equation (see Ref. [21]),

$$t\frac{d^2c(t)}{dt^2} + (iq + i\Omega t)\frac{dc(t)}{dt} - q\frac{\Omega + V}{2}c(t) = 0. \tag{12}$$

The two independent solutions of this equation have the form (see Ref. [21])

$$\begin{aligned} c^{(1)}(t) &= -i\frac{|g|}{\Omega(1-iq)}(\Omega t)^{1-iq} M(1-id_-q, 2-iq, -i\Omega t), \\ c^{(2)}(t) &= M(id_+q, iq, -i\Omega t), \end{aligned} \tag{13}$$

where $M(x,y,z)$ is the confluent hypergeometric function, and

$$d_\pm = \frac{\Omega \pm V}{2\Omega}. \tag{14}$$

The pair of variables $b_j(t)(j=1,2)$ can thus have two independent solutions, $b_j^{(i)}(t)$ $(i=1,2)$, where $b_2^{(i)}(t)$ is given by Eq. (11), with $c^{(i)}(t)$ substituted for $c(t)$, and

$$b_1^{(i)}(t)=|g|^{-1}\left[i\frac{dc^{(i)}(t)}{dt}-\frac{\Omega+V}{2}c^{(i)}(t)\right]\exp\left(i\frac{\Omega-V}{2}t\right). \tag{15}$$

Using properties of the confluent hypergeometric function at a small value of its last (third) argument, and substituting Eqs. (13) into Eqs. (11) and (15), we obtain at $t\to 0$

$$b_1^{(i)}(t)\sim(\Omega t)^{-iq}\delta_{i1},\quad b_2^{(i)}(t)\to\delta_{i2}, \tag{16}$$

i.e., $|b_k^{(i)}(0)|=\delta_{ik}$.

Asymptotic expansion of the confluent hypergeometric function at large values of its last argument (see Ref. [21]) allows us to obtain the following asymptotes for $b_k^{(i)}(t)$ at $t\to\infty$,

$$\begin{pmatrix} b_1^{(i)}(t)\\ b_2^{(i)}(t)\end{pmatrix}=A_{+i}\begin{pmatrix}\sqrt{d_-}\\ \sqrt{d_+}\end{pmatrix}(\Omega t)^{-iqd_-}\exp(-iE_+t)$$
$$+A_{-i}\begin{pmatrix}-\sqrt{d_+}\\ \sqrt{d_-}\end{pmatrix}(\Omega t)^{-iqd_+}\exp(-iE_-t). \tag{17}$$

The two column vectors on the right-hand side of Eq. (17) form ''dressed'' states. These are eigenvectors of the energy matrix in Eq. (10) at $t\to\infty$, obeying

$$\begin{pmatrix}0 & |g|\\ |g| & V\end{pmatrix}\begin{pmatrix}\pm\sqrt{d_\mp}\\ \sqrt{d_\pm}\end{pmatrix}=E_\pm\begin{pmatrix}\pm\sqrt{d_\mp}\\ \sqrt{d_\pm}\end{pmatrix}, \tag{18}$$

in which the eigenvalues $E_\pm=(V\pm\Omega)/2$ are the corresponding dressed-state energies. These dressed states are the asymptotes of the adiabatic states that, at the other end ($t=0$), coincide with ''bare'' states corresponding to the coefficients $b_k(t)$. The ''Coulombic'' long-range nature of the inverse-t potential is disclosed by the appearance of the factors $(\Omega t)^{-iqd_\pm}$ in the asymptotic Eq. (17). The coefficients $A_{\pm k}$ in Eq. (17), given by

$$A_{+2}=-A_{-1}^*=\sqrt{P_a}\exp[i\arg\Gamma(iq)-i\arg\Gamma(iqd_+)], \tag{19}$$

$$A_{-2}=A_{+1}^*=\sqrt{1-P_a}\exp[i\arg\Gamma(iq)-i\arg\Gamma(iqd_-)], \tag{20}$$

are transition amplitudes relating the bare states at $t=0$ to the dressed states at $t\to\infty$, in which

$$P_a=\frac{\exp(-2\pi qd_-)-\exp(-2\pi q)}{1-\exp(-2\pi q)} \tag{21}$$

can be interpreted as the probability of transition between the adiabatic states. Finally, the parameters $d_\pm$ can be expressed in terms of the potential coefficients for the inverse-t model Eq. (2) as

$$d_\pm=\frac{1}{2}\left(1\pm\frac{\Delta_0\Delta_1-4g_0\mathrm{Re}\,g_1}{q\sqrt{\Delta_0^2+4g_0^2}}\right), \tag{22}$$

where q is defined by Eq. (7).

In real physical systems the interaction strength tends to zero at $t\to\infty$. In the case of an optical collision, for example, this may be caused by the exit from the illuminated region. However, the turning off of the interaction may be adiabatically slow. In systems that can be simulated by the reduced model, the interaction parameter g defined in Eq. (9) is actually turned off, and [as one can see from Eqs. (14) and (18)] the ''+'' and ''−'' dressed states correspond to the bare states 2 and 1, respectively, if $V>0$, or vice versa if $V<0$. In the generalized models that can be transformed to the reduced model by Eq. (6), only the parameter g_0 need be turned off, since the interaction corresponding to g_1 vanishes as $t\to\infty$. It follows from Eq. (6) that the ''+'' and ''−'' dressed states always correspond to the states 2 and 1, respectively, of the model described by Eq. (1). The apparent difference between the reduced and the more general models regarding the effect of the sign of V is due to a difference in notation. In the reduced model state 1 corresponds to a sloped potential and $\Delta_0=V$ may be negative, while in the generalized inverse-t model the states were defined so that $\Delta_0\geqslant 0$.

IV. RESULTS AND DISCUSSION

Equations (19)–(22) and (7) obtained in the previous sections allow us to evaluate transition amplitudes for the inverse-t model [described by Eqs. (1) and (2)], of which the covalent-ionic model considered in Refs. [19,20] is a special case, with $g_0=0$. Transition amplitudes for the exponential model [Eqs. (1) and (3)] can be evaluated by transformation to the corresponding inverse-t model with the use of Eqs. (5). The Nikitin exponential models, with (i) constant and (ii) exponential interactions (see Refs. [15–17]), are special cases of the exponential model, with $g_1=0$ or $g_0=0$, respectively. The results of the present paper are in agreement with those of the cases studies in Refs. [15–17,19,20] up to phase factors associated with differences in the choice of phases for the basis states.

The transition amplitudes obtained here relate to dressed states, rather than the bare states of the diabatic representation. In the reduced model Eq. (10) the dressed states coincide with the bare ones only at $t=0$.

The substitution of Eq. (4) transforms the Nikitin model (ii) with an exponential interaction to the reduced model while the time interval $(0,\infty)$ is transformed to $(\infty,-\infty)$. Therefore in the exponential model the dressed states coincide with the bare ones at $t\to\infty$. In the covalent-ionic model the transformation Eq. (8) converts the dressed states of the reduced model at $t\to\infty$ to the bare states that correspond to the coefficients a_j. However, at $t=0$ the same transformation converts the bare states of the reduced model to dressed states. The Nikitin exponential model (i) with constant interaction is connected to the covalent-ionic model by the substitution of Eq. (4), and thus it has a bare asymptotic state at $t\to-\infty$. In the more general case ($g_0\neq 0$ and $g_1\neq 0$), both

TABLE I. Asymptotic behavior of the adiabatic transition probability P_a for various coupling parameters.

		g	P_a
a		$g_0\to\infty$, $g_1=\text{const}$	$P_a\sim[\exp(-\pi q-2\pi\,\mathrm{Re}\,g_1)-\exp(-2\pi q)]/[1-\exp(-2\pi q)]$
b		$g_0=\text{const}$, $\lvert g_1\rvert\to\infty$	$P_a\to 0$
c		$g_0\to\infty$, $\lvert g_1\rvert\to\infty$, $\lim(\lvert g_1\rvert+\mathrm{Re}\,g_1)=\infty$	$P_a\to 0$
d		$g_0\to\infty$, $\lvert g_1\rvert\to\infty$, $\lim(\lvert g_1\rvert+\mathrm{Re}\,g_1)<\infty$	
	1	$\lvert g_1\rvert/g_0^2\to 0$	$P_a\sim\exp[-2\pi(\mathrm{Re}\,g_1+\lvert g_1\rvert)]$
	2	$\lvert g_1\rvert/g_0^2\sim\text{const}$	$P_a\sim\exp[-2\pi(\mathrm{Re}\,g_1+\lvert g_1\rvert)-\pi\Delta_0^2\lvert g_1\rvert/(4g_0^2)]$
	3	$\lvert g_1\rvert/g_0^2\to\infty$	$P_a\to 0$

boundary conditions should be defined in terms of dressed states.

The reduced model is approximately linear in the vicinity of the crossing point $t_x=q/V$ if the second term in the Taylor-series expansion of the potential q/t around t_x is small compared to the first term within the transition range $|g|t_x^2/q$. This approximation is valid if $g/V\ll 1$. An LZ exponent can be identified in terms of the model parameters as $\lambda=q|g|^2/V^2$. Thus, in order to avoid small values of λ, we have to choose $q\gg 1$. In this case Eq. (21) tends to the well-known LZ formula $P_a\approx\exp(-2\pi\lambda)$ for the transition probability between the adiabatic states. In the limit $g\ll V$, Eq. (6) yields $\alpha_{jk}\approx\delta_{jk}$ or $\alpha_{jk}\approx\delta_{j,3-k}$ for $\Delta_1>0$ or $\Delta_1<0$, respectively. Then the LZ formula is valid for the inverse-t model. Since the transition amplitudes for the exponential model are the same as for the corresponding inverse-t model, the LZ formula remains valid for the exponential model as well, as long as g is kept small enough and q is kept large enough.

At small values of $q\leqslant 1$, Eq. (21) departs significantly from the LZ formula. Moreover,

$$\lim_{q\to 0}P_a=d_+<1, \tag{23}$$

contrary to the unity-valued limit of the LZ formula at $\lambda\to 0$. This peculiar result, which actually means that the transition probability between diabatic states in nonlinear problems does not necessarily vanish in the limit of high relative velocity of the colliding particles, was already noted by Nikitin (see Ref. [9]) with regard to his exponential models.

In applications to optical collisions it is more interesting to study the dependence of the transition probability on the coupling strength, as g can be manipulated by changing the laser power. In the limit $g\to 0$ the diabatic transition probability vanishes, in agreement with the LZ formula. In the limit of strong coupling, however, the transformation to the reduced model Eq. (8) mixes the states appreciably. The asymptotic behavior of the transition probabilities then depends on both g_0 and g_1. The transition probabilities for various choices of the coupling parameters are presented in Table I. Case (a) includes the reduced model and the Nikitin model (ii) with an exponential interaction. Case (b) includes the covalent-ionic model and the Nikitin model (i) with a constant interaction. In cases (c) and (d), where both g_0 and g_1 tend to infinity, an asymptotic expansion for P_a, obtained from Eqs. (21), (22), and (7), is

$$P_a\sim\exp\left[-2\pi(\mathrm{Re}\,g_1+|g_1|)-\frac{\pi}{4}\left(\Delta_0^2\frac{|g_1|}{g_0^2}-2\frac{\Delta_0\Delta_1}{g_0}+\frac{\Delta_1^2}{|g_1|}\right)\right]. \tag{24}$$

This result depends on the behavior of the ratio $|g_1|/g_0^2$ in the asymptotic limit [subcases (d1), (d2), and (d3) of Table I]. As can be seen from Table I, a saturation of the transition probability takes place in cases (a), (d1), and (d2).

The dependence of the transition probability on the LZ exponent, which for the generalized models can be expressed as

$$\lambda=\Delta_1|g_0/\Delta_0+g_1/\Delta_1|^2, \tag{25}$$

is plotted in Fig. 1 for cases (a)–(c), and compared with the LZ formula. In the nonlinear system, the probability P_a of transition between the adiabatic states is initially lower than the expected from the LZ formula, but further on it declines more slowly. Saturation can then take place only in case (a), out of the three cases. However, in case (c), whenever $g_1\ll g_0$ the probability can remain quite high even at high values of the exponent λ (as can be seen from the long-dashed line).

Figure 2 displays the dependence of the transition probability P_a on the coupling strengths for the three (d) subcases. In these cases, whenever g_0 and g_1 have opposite

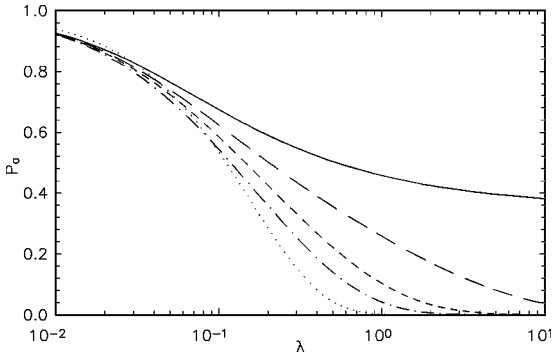

FIG. 1. Transition probabilities as functions of the LZ exponent for the coupling parameters $g_1=0$ (solid line), $g_1=g_0/9$ (long-dashed line), $g_1=9g_0$ (short-dashed line), and $g_1=0$ (dot-dashed line). Results are compared with the prediction of the LZ formula (dotted line). Other potential parameters used are $\Delta_0=\Delta_1=0.2$.

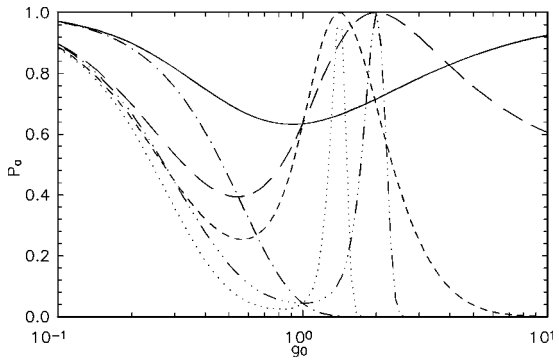

FIG. 2. Transition probabilities as functions of the coupling constant g_0, given $g_1=-0.5g_0$ (solid line), $g_1=-0.5g_0^2$ (long-dashed line), and $g_1=-0.5g_0^3$ (short-dashed line). Results are compared with the predictions of the LZ formula for the same three relations of g_1 to g_0 (dot-dashed, dot-dot-dashed, and dotted lines, respectively). Other potential parameters used are $\Delta_0=\Delta_1=2$.

signs, the dependence of λ on the coupling strengths is nonmonotonic, and this leads to a nonmonotonicity of the transition probability calculated even by using the LZ formula. Saturation takes place in cases (d1) and (d2) (solid and long-dashed lines, respectively).

In order to have a better appreciation of the mechanism that leads to the saturation effect, let us analyze the reduced model in terms of an adiabatic theory. According to Eq. (10), the adiabatic potentials $V_{\pm}$ cross at two points in the complex plane,

$$t_{\pm}=\frac{-V\pm 2ig}{V^2+4|g|^2}q. \tag{26}$$

The nonadiabatic transition probability may be estimated as (see Ref. [11])

$$P_a\sim\exp\left(-i\int_{t_-}^{t_+}[V_+(t)-V_-(t)]dt\right)=\exp(-2\pi q d_-). \tag{27}$$

This expression corresponds to the limit of Eq. (21) as $q\to\infty$, keeping fixed values of g and V. As $|g|\to\infty$, the crossing points $t_{\pm}$ tend to zero, while the exponent in Eq. (27) tends to a finite limit πq. The adiabatic theory is applicable only when this exponent is large, but a sufficient saturation takes place if this exponent is small; i.e., when the adiabatic theory is inapplicable. The difference in form between the limiting expression [case (a) in Table I] and Eq. (27) reflects the inapplicability of the adiabatic theory.

The *t*-dependent present results were obtained by using the common trajectory approach. However, analysis of known solutions of nonlinear *R*-dependent models may also reveal a saturation effect. The model considered in Ref. [13] involves two horizontal parallel potentials with exponential coupling. The associated time-dependent Demkov model (Ref. [22], see also Ref. [11]) can be transformed to the inverse-*t* model with $\Delta_0=g_1=0$. In this case the present approach gives a transition probability independent of the coupling strength, in agreement with the results of Ref. [13]. Another exactly soluble nonlinear *R*-dependent model was considered in Ref. [14]. It involves an exponential coupling of two potentials with exponential terms that are proportional to the coupling strength. Substitution of Eq. (4) transforms the associated *t*-dependent model into the reduced model with Δ_0 proportional to g_0. The transition amplitudes obtained in Ref. [14] are independent of the coupling strength, in accordance with Eqs. (19)–(22).

V. CONCLUSIONS

Exact solutions are presented here for two classes of two-state time-dependent curve-crossing problems with potential matrices having the form of a linear combination of time-independent terms and (I) inversely proportional or (II) exponential time-dependent terms. The generalized exponential model representing the second class [Eqs. (1) and (3)] is related to the generalized inverse-*t* model representing the first class by Eqs. (4) and (5). The inverse *t* model [Eqs. (1) and (2)] can be further transformed into a reduced one (10), involving a *t*-independent potential and an inversely proportional one, coupled by a *t*-independent interaction, by using the transformation Eq. (6).

It follows from the exact solution of the reduced model [see Eqs. (19)–(21)] that the transition probabilities do not tend to the limit predicted by LZ theory as the coupling strength increases, but saturate at intermediate values that lead to a nonvanishing transmission through the zone of potential crossing. A similar saturation effect can also occur in the more general models (see Table I), at certain ranges of the potential parameters. Saturation effects of the kind discussed here may quite likely occur in other models involving potentials with different types of nonlinear time (or coordinate) dependence.

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