Reply to "Comment on 'Limits of the measurability of the local quantum electromagnetic field amplitude'"

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We argue that the criticism by Hnizdo [preceding Comment, Phys. Rev. A **60**, 4212 (1999)] of the results obtained by Compagno and Persico [Phys. Rev. A **57**, 1595 (1998)] on the theory of measurement of the amplitude of the quantum electromagnetic field is unfounded. By a simple and direct approach we show that the quantities evaluated by Hnizdo in his Comment are incorrect and we present the correct results. [S1050-2947(99)05911-9]

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The origin of the misconceptions in Hnizdo's argument in the preceding Comment [1] can be best illustrated by an exact and simple evaluation of the relevant quantities appearing in his Comment, rather than by checking through the complicated mathematics developed in a previous paper [2].

We start from Hnizdo's definition, Eq. (4), for $f(t_1)$, which we write, using our notation and for a uniform charge density, as

$$f(t_1)=-\rho^2\int_\tau dt_2\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2\left(\frac{\partial^2}{\partial x_1\partial x_2}-\frac{1}{c^2}\frac{\partial^2}{\partial t_1\partial t_2}\right)G,$$

$$G=\frac{1}{r}\,\delta(t_2-t_1-r/c). \tag{1}$$

Using $\partial G/\partial x_1=-\partial G/\partial x_2$ and $\partial G/\partial t_1=-\partial G/\partial t_2$ as well as the spherical symmetry of the pointer, we have

$$f(t_1)=\rho^2\int_\tau dt_2\Bigg[\tfrac{1}{3}\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2\left(\Delta_2-\frac{1}{c^2}\frac{\partial^2}{\partial t_2^2}\right)G+\frac{2}{3c^2}\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2\frac{\partial^2}{\partial t_1\partial t_2}G\Bigg], \tag{2}$$

where Δ_2 is the Laplacian relative to $\mathbf{x}_2$. On the other hand, it is well known that

$$\left(\Delta_2-\frac{1}{c^2}\frac{\partial^2}{\partial t_2^2}\right)G=-4\pi\delta(\vec{x}_2-\vec{x}_1)\,\delta(t_2-t_1). \tag{3}$$

Consequently integration over t_2 yields

$$f(t_1)=-\frac{4\pi}{3}\rho^2V+\frac{2}{3c^2}\rho^2\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2\frac{1}{r}\frac{\partial}{\partial t_1}\times[\delta(t_1''-t_1-r/c)-\delta(t_1'-t_1-r/c)], \tag{4}$$

where we have assumed the weak definition of δ for convenience [3] and where t_1' and t_1'' are the extremes of τ. The usual retardation expansion leads to

$$f(t_1)=-\frac{4\pi}{3}\rho^2V-\tfrac{2}{3}\rho^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^{n+2}}\times[\delta^{(n+1)}(t_1''-t_1)-\delta^{(n+1)}(t_1'-t_1)]\langle r^{n-1}\rangle, \tag{5}$$

where

$$\delta^{(n)}\equiv\partial^n\delta/\partial t_1^n,\quad \langle r^{n-1}\rangle\equiv\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2 r^{n-1}. \tag{6}$$

Expression (5) shows that $f(t_1)$ is a highly singular function, in contrast with Hnizdo's expression (9). Thus the integration in Hnizdo's expression (3) for the time average of the self-force F_D cannot be performed as in Hnizdo's Comment. Consequently, Hnizdo's whole argument leading to his expression (17) must be considered as flawed.

Turning to the evaluation of $g(t_2)$, we use Hnizdo's definition given in the first equality of his expression (19) in the Comment. Assuming uniform charge density, we write this as

$$g(t_2)=-\rho^2\int_\tau dt_1\int_V d^3\vec{x}_1\int_V d^3\vec{x}_2\frac{\partial^2}{\partial x_1\partial x_2}G. \tag{7}$$

For the spherical case considered by Hnizdo, an argument parallel to that leading from Eqs. (1)–(5) above yields

$$g(t_2)=-\frac{4\pi}{3}\rho^2V-\tfrac{1}{3}\rho^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^{n+2}}[\delta^{(n+1)}(t_2-t_1'')-\delta^{(n+1)}(t_2-t_1')]\langle r^{n-1}\rangle, \tag{8}$$

where $\delta^{(n)}\equiv\partial^n\delta/\partial t_2^n$. Similarly to $f(t_1)$, also $g(t_2)$ is highly singular and different from the expression given by Hnizdo in his Eq. (19). Consequently, integrals involving $g(t_2)$ must be handled differently from the way described in Hnizdo's Comment.

Given that the expressions for f and g in Ref. [1] seem incorrect, also Hnizdo's expressions for the various time-averaged quantities he has chosen to evaluate must be incorrect, since f and g enter the definition of all these average quantities. In what follows, we shall illustrate this by an explicit evaluation of the same time averages, using the correct expressions (5) and (8) above for f and g.

The average self-force cannot be evaluated as (approximately) done by Hnizdo in expression (10) of the Comment in view of the singular character of f. However, use of our expression (5) and of term by term partial integration yields

$$\overline{F}_D \equiv \frac{1}{\tau}\int_\tau dt_1 Q(t_1) f(t_1)$$
$$= -\frac{4\pi}{3}\rho^2 V\overline{Q} - \tfrac{2}{3}\rho^2 \frac{1}{\tau}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^{n+2}}$$
$$\times[Q^{(n+1)}(t_1'') - Q^{(n+1)}(t_1')]\langle r^{n-1}\rangle. \tag{9}$$

The series on the right-hand side of this expression can be interpreted as the time average of the Abraham-Lorentz radiation-reaction force acting on the pointer in the absence of the neutralizing body [4], whereas the first term represents the nonretarded Coulomb pull from the neutralizing body, since we have defined

$$\overline{Q} \equiv \frac{1}{\tau}\int dt_1 Q(t_1). \tag{10}$$

Setting formally $Q(t_1)\equiv 1$ in Eq. (9), we find immediately

$$\frac{1}{(\rho V\tau)^2}\int_\tau dt_1 f(t_1) \equiv \bar{\bar{A}}_{xx}^{(II)} = -\frac{4\pi}{3}\frac{1}{V\tau} \tag{11}$$

which is in disagreement with Hnizdo's result (12) as well as with his conclusions in [2]. Using Eq. (11), the average self-force can be cast in the form

$$\overline{F}_D = \overline{F}_{\rm BR} - \tfrac{2}{3}\rho^2 \frac{1}{\tau}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^{n+2}}$$
$$\times[Q^{(n+1)}(t_1'') - Q^{(n+1)}(t_1')]\langle r^{n-1}\rangle, \tag{12}$$

where we have set

$$\overline{F}_{\rm BR} \equiv \rho^2 V^2 \tau \overline{Q}\bar{\bar{A}}_{xx}^{(II)}. \tag{13}$$

Expression (13) is the time average of the nonretarded Coulomb force exerted by the neutralizing body on the pointer. The fact that in the spherical case it has exactly the form proposed by Bohr and Rosenfeld (BR) for the total self-force should not obscure the facts (i) that it vanishes in the absence of the neutralizing body [hence the Compagno and Persico (CP) conclusion that no spring mechanism is necessary to evade quantum limitations on the measurability of the field amplitude [5]] and (ii) that the average self-force can be approximated by Eq. (13) only when the radiation reaction can be neglected (which is in agreement with CP [5] but in contrast with Hnizdo's contention that the time average of the radiation reaction force contains terms linear in Q).

Finally, we note that the time average of the retarded Coulomb attraction F_Q between the neutralizing body and the pointer cannot be evaluated as described by Hnizdo following expression (18) of his Comment, because of the singular character of $g(t_2)$. It is straightforward, however, to perform the integral in this expression (18) using Eq. (8) above. The correct result is

$$\overline{F}_Q \equiv \frac{1}{\tau}\int_\tau dt_2 Q(t_2) g(t_2)$$
$$= \overline{F}_{\rm BR} + \tfrac{1}{3}\rho^2 \frac{1}{\tau}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{1}{c^{n+2}}$$
$$\times[Q^{(n+1)}(t_1'') - Q^{(n+1)}(t_1')]\langle r^{n-1}\rangle, \tag{14}$$

which is in contrast with Hnizdo's expression (21).

In summary, our exact and direct approach for the spherical case indicates that Hnizdo's expressions for $f(t_1)$ and $g(t_2)$ are incorrect. Since the two quantities appear in the definition of all the average forces that Hnizdo has chosen to evaluate, also the expressions for these average forces are incorrect. Thus it is not surprising that the physical conclusions that Hnizdo draws from these expressions are unfounded. A particularly relevant example of this is Hnizdo's conclusion that the average value of $F_{\rm RR}$ should be proportional to Q in the absence of the neutralizing body. This can hardly be the case (even for the spherically symmetric pointer considered here) since it is in contrast with translational invariance of the pointer-field Lagrangian. It is also clear, in the light of the present analysis, that Hnizdo's contention that some of the findings of the CP paper [4] are incorrect must be regarded as unfounded.

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[1] V. Hnizdo, preceding Comment, Phys. Rev. A **60**, 4212 (1999).
[2] V. Hnizdo, J. Phys. A **32**, 2427 (1999).
[3] G. Barton, *Elements of Green's Functions and Propagation* (Oxford Science, New York, 1989).
[4] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).
[5] G. Compagno and F. Persico, Phys. Rev. A **57**, 1595 (1998).