

---



---

**COMMENTS**


---



---

*Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

---

**Comment on “Limits of the measurability of the local quantum electromagnetic-field amplitude”**

V. Hnizdo\*

*Department of Physics, Schonland Research Centre for Nuclear Sciences, and Centre for Nonlinear Studies,  
University of the Witwatersrand, Johannesburg 2050, South Africa*

(Received 30 November 1998)

It is argued that the findings of a recent reanalysis by Compagno and Persico [Phys. Rev. A **57**, 1595 (1998)] of the Bohr-Rosenfeld procedure for the measurement of a single space-time-averaged component of the electromagnetic field are incorrect when the field measurement time is shorter than that required for light to traverse the measurement's test body. To this end, the time-averaged “self-force” on the test body, assumed for simplicity to be of a spherical shape, is evaluated in terms of a one-dimensional quadrature for the general trajectory allowed for the test body by Compagno and Persico, and in closed form for the limiting steplike trajectory used by Bohr and Rosenfeld. [S1050-2947(99)05611-5]

PACS number(s): 12.20.Ds

In a recent paper, Compagno and Persico (CP) [1] revisited the famous analysis of the measurability of the electromagnetic field by Bohr and Rosenfeld (BR) [2]. CP analyze the BR procedure for the measurement of a single space-time-averaged component of the electromagnetic field by treating the interaction of an extended test body with the local quantized electromagnetic field quantum mechanically in the electric dipole approximation, which is valid for field measurement times  $\tau > a/c$  ( $a$  characterizes the linear dimensions of the test body). They obtain a minimum uncertainty in the measured field component that they claim is different from that obtained by BR and which the latter authors eliminated by connecting the test body to the reference frame by a compensating spring. CP eliminate their minimum uncertainty simply by removing from the measurement the neutralizing body employed in the BR procedure to minimize the test body's field effects.

To investigate why their findings differ from the widely accepted BR results, CP recalculate the force on the test body due to the field created by the neutralizing body and the test body itself using classical electrodynamics as the BR analysis, and thus not restricting the field measurement times  $\tau$  to only  $\tau > a/c$ . However, CP here relax the BR assumption that the test body's unpredictable displacement resulting from the initial momentum measurement stays constant throughout the time period of the field measurement. Using this calculation, CP confirm the results of their quantum-mechanical treatment of the problem and identify the reason for what they believe is the difference between their results

and those of BR in the approximation, according to CP incorrect, of a constant displacement of the test body, which allows us to take the test body's trajectory outside a time integration and recovers the BR results. CP thus draw a far-reaching conclusion that a single space-time-averaged component of the electromagnetic field can be measured with arbitrary accuracy without any use of compensating forces even when the field measurement time  $\tau < a/c$ ; in their opinion, the necessity for compensating forces of a nonelectromagnetic nature would indicate that quantum electrodynamics is not self-consistent as a physical theory.

Using the Fourier-transform methods of a recent work [3], where the geometric factors of the field commutators and spring constants employed in the BR analysis are calculated, we evaluate here, as explicitly as is possible in general terms, the average force on a spherical test body that is due to the fields, calculated assuming classical electrodynamics, of both the test and neutralization bodies; following CP, we call this force the test body's average self-force. Using this evaluation, we show that the limiting average self-force obtained by BR with a steplike trajectory of the test body's constant displacement approximates well the time-average self-force obtained with a trajectory that, while conforming to the condition that the test body's maximum speed  $v_{\max} \ll c$ , approaches sufficiently closely the BR steplike trajectory. This provides a rigorous justification of the fact that the use of a steplike trajectory is fully consistent with the physical assumptions of the BR analysis, and refutes the implication of CP that such an approximation is incorrect.

We show also that the BR average self-force for a given field measurement time  $\tau < a/c$  has a component that is the steplike-trajectory limit of the average of what CP call the radiation-reaction component of the self-force and which is not affected by the removal of the neutralizing body. Con-

---

\*Present address: National Institute for Occupational Safety and Health, M/S L-3030, 1095 Willowdale Road, Morgantown, WV 26505.

rary to the conclusion of CP, this implies the need for a BR compensating spring even if the removal of the neutralizing body would leave the “radiation-reaction” component as the net self-force. This is because the time-average of the “radiation-reaction” component for field measurement times  $\tau < a/c$  cannot be reduced arbitrarily when the test body’s trajectory is of a sufficiently steplike character—and such a kind of trajectory is necessitated by the requirements on the type of momentum measurements that have to be performed on the test body.

Our starting point is expression (37) of CP, which they obtained for the self-force on a test body that describes a trajectory  $Q(t_1)$  along a given direction, say the  $x$  direction, during the field measurement period  $0 \leq t_1 \leq \tau$ . This self-force is due to the fields of the test body itself and of a neutralizing body charged oppositely and occupying permanently the space region of the test body’s initial location, and CP assumed in its derivation that the test body’s displacement  $Q$  and velocity  $\dot{Q}$  are such that  $|Q| \ll a$  and  $|\dot{Q}| \ll c$ . We assume that the test body has a constant charge density  $\rho_c$  and is spherical with radius  $R$ , and we describe its spatial region using a uniform distribution normalized to unit volume,  $\rho(r) = (1/V)\Theta(R-r)$ ,  $V = (4/3)\pi R^3$ . The test body’s self-force  $F(t_2)$  for  $0 \leq t_2 \leq \tau$  is thus given as

$$F(t_2) = \rho_c^2 V^2 \int \rho(r_1) d\mathbf{r}_1 \times \int \rho(r_2) d\mathbf{r}_2 \int_0^\tau dt_1 Q(t_1) A_{xx}^{(1,2)}(t, \mathbf{r}), \quad (1)$$

where the quantity  $A_{xx}^{(1,2)}$  is the distribution

$$A_{xx}^{(1,2)}(t, \mathbf{r}) = - \left( \frac{\partial^2}{\partial x_1 \partial x_2} - \frac{\partial^2}{\partial t_1 \partial t_2} \right) \frac{\delta(t-r)}{r}, \quad (2)$$

with  $t = t_2 - t_1$ ,  $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ , and  $r = |\mathbf{r}|$ . Units in which the speed of light  $c = 1$  are used henceforth.

The quantity of our interest is the time-averaged self-force  $\bar{F} = (1/\tau) \int_0^\tau dt_2 F(t_2)$ , which can be written using Eq. (1) as

$$\bar{F} = \frac{\rho_c^2 V^2}{\tau} \int_0^\tau dt_1 Q(t_1) f(t_1), \quad (3)$$

where

$$f(t_1) = \int \rho(r_1) d\mathbf{r}_1 \int \rho(r_2) d\mathbf{r}_2 \int_0^\tau dt_2 A_{xx}^{(1,2)}(t, \mathbf{r}). \quad (4)$$

With the spherically symmetric distribution  $\rho(r)$ , only the monopole component  $A_{xx0}^{(1,2)}(t, r)$  in a multipole expansion of the distribution  $A_{xx}^{(1,2)}(t, \mathbf{r})$  contributes to the double space integral in Eq. (4),

$$A_{xx0}^{(1,2)}(t, r) = - \frac{2}{3} \frac{\delta''(t-r)}{r} - \frac{2}{3} \lim_{\epsilon \rightarrow 0} \frac{\epsilon}{(r+\epsilon)^3} \frac{\delta(t-r)}{r}. \quad (5)$$

Here, the second term arises from a regularization of the space derivative part in Eq. (2) by

$$\frac{\partial^2}{\partial x_1 \partial x_2} \frac{\delta(t-r)}{r} = \lim_{\epsilon \rightarrow 0} \frac{\partial^2}{\partial x_1 \partial x_2} \frac{\delta(t-r)}{r+\epsilon}, \quad (6)$$

where the limit  $\epsilon \rightarrow 0$  is understood to be taken only after a two-dimensional integration; it is the only term with which the regularization can contribute to the multidimensional integral (4) that defines the function  $f(t_1)$ . To evaluate this integral, we perform first the integration of the monopole component (5) with respect to time  $t_2$ :

$$\begin{aligned} \bar{A}_{xx0}^{(1,2)}(t_1, r) &= \int_0^\tau dt_2 A_{xx0}^{(1,2)}(t, r) = - \frac{2}{3} \frac{\delta'(\tau - t_1 - r)}{r} \\ &\quad - \frac{2}{3} \lim_{\epsilon \rightarrow 0} \frac{\epsilon}{(r+\epsilon)^3} \frac{\Theta(\tau - t_1 - r)}{r}. \end{aligned} \quad (7)$$

Here, use was made of the fact that  $\delta'(-t_1 - r) = 0$  and  $\Theta(t_1 + r) = 1$  for  $t_1 > 0$ . The function  $f(t_1)$  of Eq. (4) is now given by

$$f(t_1) = \int \rho(r_1) d\mathbf{r}_1 \int \rho(r_2) d\mathbf{r}_2 \bar{A}_{xx0}^{(1,2)}(t_1, r), \quad (8)$$

where the double space integration can be done in closed form using the Fourier-transform method for evaluation of folding integrals [3]:

$$\begin{aligned} f(t_1) &= - \frac{6}{\pi R^2} \int_0^\infty [j_1(qR)]^2 \{2 \cos[q(\tau - t_1)] + 1\} dq \\ &= \frac{1}{2R^3} (\chi - 2)(2 - 2\chi - \chi^2) \Theta(2 - \chi) - \frac{1}{R^3}, \end{aligned} \quad (9)$$

where  $\chi = (\tau - t_1)/R$ . The above momentum-space integral involves the Fourier transform  $-(4\pi/3)\{2 \cos[q(\tau - t_1)] + 1\}$  of the “folding” function  $\bar{A}_{xx0}^{(1,2)}(t_1, r)$  and the Fourier transform  $3j_1(qR)/qR$  of the uniform distribution  $\rho(r)$ ; its evaluation was done with the help of the computing system MATHEMATICA [4]. Equations (3) and (9) give the average self-force  $\bar{F}$  in terms of a one-dimensional quadrature involving the test-body’s so-far unspecified trajectory  $Q(t_1)$ . We now assume that the trajectory  $Q(t_1)$  is of a steplike character, i.e., in an initial time interval  $(0, \Delta t)$  with  $\Delta t \ll \tau$ , the displacement  $Q(t_1)$  goes smoothly from 0 to a value  $Q$ , then  $Q(t_1) = Q = \text{const}$  for  $\Delta t \leq t_1 \leq \tau - \Delta t$ , and in a final interval  $(\tau - \Delta t, \tau)$  the displacement  $Q(t_1)$  returns smoothly from  $Q$  back to 0. As the test body’s maximum speed  $v_{\max} \equiv \max|\dot{Q}(t_1)|$ ,  $0 \leq t_1 \leq \tau$  must satisfy the condition  $v_{\max} \ll c$ , the constant  $Q$  is such that  $|Q| < v_{\max} \Delta t \ll c \Delta t$ , and if one defines the mean speed  $\bar{v}$  in the initial and final intervals by  $\bar{v} \Delta t = |Q|$ , one also has that  $\bar{v} \ll c$ . When the duration  $\Delta t$  of the initial and final time intervals is decreased, which needs to be done in order to approach the BR steplike trajectory, the constant  $Q$ , while staying finite, must decrease accordingly for a given test body’s maximum speed  $v_{\max}$ . The average force (3) can now be written as

$$\bar{F} = \frac{\rho_c^2 V^2}{\tau} \left[ Q \int_{\Delta t}^{\tau - \Delta t} dt_1 f(t_1) + \int_0^{\Delta t} dt_1 Q(t_1) f(t_1) + \int_{\tau - \Delta t}^{\tau} dt_1 Q(t_1) f(t_1) \right]. \quad (10)$$

To simplify their calculations, BR obtained the average self-force on the test body assuming for it a strictly steplike trajectory  $Q_{\text{BR}}(t_1) = Q \Theta(t_1) \Theta(\tau - t_1)$ , which is a limit  $\Delta t \rightarrow 0$  of our trajectory  $Q(t_1)$ . A formal substitution of the BR trajectory  $Q_{\text{BR}}(t_1)$  in Eq. (3) gives the time-averaged self-force  $\bar{F}_{\text{BR}}$ , obtained by BR in a different way in their analysis:

$$\bar{F}_{\text{BR}} = \rho_c^2 V^2 \tau Q \bar{A}_{xx}^{(1,1)}, \quad (11)$$

where

$$\begin{aligned} \bar{A}_{xx}^{(1,1)} &= \frac{1}{\tau^2} \int_0^{\tau} dt_1 f(t_1) \\ &= -\frac{1}{8R^4 \kappa} (4 + \kappa)(2 - \kappa)^2 \Theta(2 - \kappa) - \frac{1}{R^4 \kappa}, \end{aligned} \quad (12)$$

with  $\kappa = \tau/R$ , is a BR geometric factor for coinciding spherical space-time regions, which we evaluated here in closed form. According to Eqs. (11) and (12), the average BR self-force for a field measurement time  $\tau \geq 2R$  reduces to a force  $-\rho_c^2 V^2 Q/R^3$ , which is the electrostatic force of attraction between the test and neutralization bodies when their centers are displaced by a distance  $|Q| \ll R$ . Without the use of a compensating spring, Eq. (12) [together with Eq. (48) of BR] leads to a minimum uncertainty  $\Delta \bar{E}_x \sim (\hbar |\bar{A}_{xx}^{(1,1)}|)^{1/2} \sim (\hbar/\tau V)^{1/2}$  in the measured field component  $\bar{E}_x$  for both  $\tau \geq 2R$  and  $\tau < 2R$ —which in fact agrees with the uncertainty (28) of CP, obtained by them for  $\tau > 2R$  [5].

CP contend that the BR use of the steplike trajectory, which leads to the BR result (11), is incorrect, presumably as it implies that the velocity of the test body diverges in the vicinities of the beginning  $t_1 = 0$  and end  $t_1 = \tau$  of the measurement period. However, with our evaluations (10) and (12) of the average self-force  $\bar{F}$  and BR geometric factor  $\bar{A}_{xx}^{(1,1)}$ , it is easy to show that the BR self-force approximates correctly the self-force obtained with a “physical” trajectory of a sufficiently steplike character. Dividing the average self-force (10) by the BR average self-force (11), we get

$$\frac{\bar{F}}{\bar{F}_{\text{BR}}} = \frac{1}{\bar{A}_{xx}^{(1,1)} \tau^2} \int_{\Delta t}^{\tau - \Delta t} dt_1 f(t_1) + \frac{\Delta \bar{F}_i}{\bar{F}_{\text{BR}}} + \frac{\Delta \bar{F}_f}{\bar{F}_{\text{BR}}}, \quad (13)$$

where the quantities  $\Delta \bar{F}_i$  and  $\Delta \bar{F}_f$  arise from the time intervals of duration  $\Delta t$  at the beginning  $t_1 = 0$  and end  $t_1 = \tau$  of the trajectory, respectively. We find easily an upper bound on the absolute value of the quantity  $\Delta \bar{F}_i$  using the fact that the maximum value of the function  $|f(t_1)|$  is  $3/R^3$  for  $0 \leq t_1 \leq \tau$  [see Eq. (9)] and that  $|Q(t_1)| < v_{\text{max}} \Delta t$  in the initial time interval:

$$\begin{aligned} |\Delta \bar{F}_i| &= \frac{\rho_c^2 V^2}{\tau} \left| \int_0^{\Delta t} dt_1 Q(t_1) f(t_1) \right| \\ &\leq \frac{\rho_c^2 V^2}{\tau} \int_0^{\Delta t} dt_1 |Q(t_1)| |f(t_1)| < \rho_c^2 V^2 v_{\text{max}} \frac{3}{R^3} \frac{\Delta t^2}{\tau}. \end{aligned} \quad (14)$$

We find in the same way the same upper bound on the absolute value of the quantity  $\Delta \bar{F}_f = (\rho_c^2 V^2/\tau) \int_{\tau - \Delta t}^{\tau} dt_1 Q(t_1) f(t_1)$ . The absolute values of both the ratios  $\Delta \bar{F}_{i,f}/\bar{F}_{\text{BR}}$  thus have an upper bound

$$\begin{aligned} \left| \frac{\Delta \bar{F}_{i,f}}{\bar{F}_{\text{BR}}} \right| &< \frac{3}{\tau R^3} \frac{v_{\text{max}}}{|\bar{A}_{xx}^{(1,1)}|} \frac{\Delta t}{\bar{v}} \frac{\Delta t}{\tau} \\ &= \frac{24}{(4 + \kappa)(2 - \kappa)^2 \Theta(2 - \kappa) + 8} \frac{v_{\text{max}}}{\bar{v}} \frac{\Delta t}{\tau}, \end{aligned} \quad (15)$$

where we used Eq. (11) for  $\bar{F}_{\text{BR}}$  with  $|Q| = \bar{v} \Delta t$  and the closed-form expression (12) for  $\bar{A}_{xx}^{(1,1)}$ . As both the speeds  $v_{\text{max}}$  and  $\bar{v}$  may be assumed to be independent of  $\Delta t$ , the upper bound (15) can be made arbitrarily small by letting  $\Delta t$  be sufficiently small, and thus  $\lim_{\Delta t \rightarrow 0} (\Delta \bar{F}_{i,f}/\bar{F}_{\text{BR}}) = 0$ . Using this result, the limit  $\Delta t \rightarrow 0$  in Eq. (13) is simply

$$\lim_{\Delta t \rightarrow 0} (\bar{F}/\bar{F}_{\text{BR}}) = 1, \quad (16)$$

as  $\int_0^{\tau} dt_1 f(t_1) = \tau^2 \bar{A}_{xx}^{(1,1)}$ . This means that while both  $\lim_{\Delta t \rightarrow 0} \bar{F} = 0$  and  $\lim_{\Delta t \rightarrow 0} \bar{F}_{\text{BR}} = 0$  [because  $\lim_{\Delta t \rightarrow 0} |Q| = \lim_{\Delta t \rightarrow 0} (\bar{v} \Delta t) = 0$ ], a “physical” average self-force  $\bar{F}$  obtained with a sufficiently small but finite  $\Delta t$  and accordingly small but finite displacement  $|Q| \ll c \Delta t$  is approximated arbitrarily closely by the BR self-force  $\bar{F}_{\text{BR}}$  of Eq. (11):

$$\bar{F} \approx \bar{F}_{\text{BR}} \quad \text{when } \Delta t \text{ is sufficiently small.} \quad (17)$$

We evaluate also an average force  $\bar{F}_Q$ , which is the time average of the force  $F_Q(t_2)$  defined by Eq. (40) of CP as the component of the self-force  $F(t_2)$  that is directly proportional to the displacement  $Q(t_2)$ . CP show that this force is canceled by a force that arises when the neutralizing body is removed temporarily for the duration of the field measurement. It is not clear whether a procedure could be devised for such a removal of the neutralizing body without introducing additional fields that affect the test body, but we shall leave this point aside. The average force  $\bar{F}_Q$  can be written as

$$\bar{F}_Q = \frac{\rho_c^2 V^2}{\tau} \int_0^{\tau} dt_2 Q(t_2) g(t_2), \quad (18)$$

where

$$\begin{aligned}
g(t_2) &= - \int \rho(r_1) dr_1 \int \rho(r_2) dr_2 \frac{\partial^2}{\partial x_1 \partial x_2} \frac{\Theta(t_2 - r)}{r} \\
&= - \frac{1}{2} f(\tau - t_2) - \frac{3}{2R^3} \\
&= \frac{1}{4R^3} (2 - \xi)(2 - 2\xi - \xi^2) \Theta(2 - \xi) - \frac{1}{R^3}, \quad (19)
\end{aligned}$$

with  $\xi = t_2/R$ . Here, the space integration is done simply by using the result (9) of the space integration in Eq. (8) on noting that the monopole component of the regularized function  $-\lim_{\epsilon \rightarrow 0} (\partial^2 / \partial x_1 \partial x_2) [\Theta(t_2 - r)/(r + \epsilon)]$  can be expressed in terms of the function  $\bar{A}_{xx0}^{(1,2)}(t_1, r)$  as

$$\begin{aligned}
\frac{1}{3} \frac{\delta'(t_2 - r)}{r} - \frac{2}{3} \lim_{\epsilon \rightarrow 0} \frac{\epsilon}{(r + \epsilon)^3} \frac{\Theta(t_2 - r)}{r} \\
= - \frac{1}{2} \bar{A}_{xx0}^{(1,2)}(\tau - t_2, r) - \lim_{\epsilon \rightarrow 0} \frac{\epsilon}{(r + \epsilon)^3} \frac{\Theta(t_2 - r)}{r}, \quad (20)
\end{aligned}$$

and that the contribution of the term  $-(2/3) \lim_{\epsilon \rightarrow 0} [\epsilon/(r + \epsilon)^3] \Theta(\tau - t_1 - r)/r$  in Eq. (7) to the function  $f(t_1)$  of Eq. (9) is  $-1/R^3$ .

According to Eq. (19), the function  $g(t_2)$  is related in a simple way to the function  $f(t_1)$ , and thus on the strength of the same argument as that leading to Eq. (17), but using the function  $g(t_2)$  instead of the function  $f(t_1)$ , it follows that

$$\bar{F}_Q \approx \bar{F}_{Q(\text{BR})} = - \frac{1}{2} \bar{F}_{\text{BR}} - \frac{3\rho_c^2 V^2 Q}{2R^3} \quad (21)$$

when  $\Delta t$  is sufficiently small,

where  $\bar{F}_{Q(\text{BR})} = (\rho_c^2 V^2 Q / \tau) \int_0^\tau dt_2 g(t_2)$  is the average force  $\bar{F}_Q$  obtained with the steplike trajectory  $Q_{\text{BR}}(t_2) = Q \Theta(t_2) \Theta(\tau - t_2)$ . Following CP, we now define an average force  $\bar{F}_{\text{RR}} = \bar{F} - \bar{F}_Q$ , which is the time average of what CP call the ‘‘radiation-reaction’’ component  $F_{\text{RR}}(t_2)$  [see Eq. (40) of CP] of the self-force  $F(t_2)$ . Using Eqs. (17) and (21), it is seen easily that

$$\begin{aligned}
\bar{F}_{\text{RR}} \approx (\bar{F}_{\text{BR}} - \bar{F}_{Q(\text{BR})}) &= \frac{3}{2} \left( \bar{F}_{\text{BR}} + \frac{\rho_c^2 V^2 Q}{R^3} \right) \\
&\equiv \bar{F}_{\text{RR}(\text{BR})} \quad \text{when } \Delta t \text{ is sufficiently small.} \quad (22)
\end{aligned}$$

This means that the BR limiting self-force  $\bar{F}_{\text{BR}}$  has a ‘‘radiation-reaction’’ component  $\bar{F}_{\text{RR}(\text{BR})}$ , given according to Eqs. (11), (12), and (22) by

$$\bar{F}_{\text{RR}(\text{BR})} = - \frac{3\rho_c^2 V^2 Q}{16R^3} (4 + \kappa)(2 - \kappa)^2 \Theta(2 - \kappa), \quad (23)$$

where  $\kappa = \tau/R$ . The average ‘‘radiation-reaction’’ force  $\bar{F}_{\text{RR}(\text{BR})}$  vanishes only for field measurement times  $\tau \geq 2R$ . Now, if the removal of the neutralizing body results in the

cancellation of the force  $\bar{F}_Q$ , then the limiting force  $\bar{F}_{Q(\text{BR})}$  must also be canceled, and a BR steplike trajectory would result in a net average self-force  $\bar{F}_{\text{RR}(\text{BR})}$  of Eq. (23), which, without a compensating spring, would lead to a minimum uncertainty  $\Delta \bar{E}_x \sim (\hbar / \tau R^3)^{1/2} (2 - \tau/R) \Theta(2 - \tau/R)$  in the measured field component. The absence of a neutralizing body would result, in the limit of a steplike trajectory, again in a time-averaged self-force that is independent of the details of the space-time course of the measurement procedure and, for a field measurement time  $\tau < 2R$ , the effect of which would have to be compensated by a BR spring when it is desired to measure the field to arbitrary accuracy. We note here that no use of any neutralizing body, instead of its possibly problematic temporary removal, would simply subtract from the average self-force  $\bar{F}_{\text{BR}}$  of Eq. (11) the force  $-\rho_c^2 V^2 Q / R^3$  of electrostatic attraction to the neutralizing body, resulting in a limiting average self-force that differs only by a factor of 2/3 from the average self-force  $\bar{F}_{\text{RR}(\text{BR})}$  of Eq. (23) that is obtained with the temporary removal.

The steplike character of the test body’s trajectory in the BR analysis is necessitated by the demands on the type of momentum measurements that have to be performed on the test body at the beginning and end of the field measurement period  $(0, \tau)$ . These momentum measurements are required for the determination of the momentum transfer along the given direction from the field to the test body, and are each allowed to have only a duration  $\Delta t \ll \tau$ . As BR have shown, the latter requirement is necessary in order to be able to neglect the radiation reaction on an extended test body during the time of the momentum measurement. Thus the momentum measurements are required to be of the ideal repeatable type, i.e., for a given precision, of arbitrarily short duration while at the same time not altering the momentum of the measured object. BR found in the course of their analysis a procedure for such repeatable momentum measurements; a similar procedure was found by Aharonov and Bohm independently some 30 years later [6,7]. A repeatable momentum measurement of accuracy  $\Delta p_x$  and duration  $\Delta t$  at the beginning of the field measurement period  $(0, \tau)$  still results in an unpredictable displacement  $Q$  of the test body such that  $|Q| \geq \hbar / \Delta p_x$ , occurring within the initial time interval  $(0, \Delta t)$ . The requirements that  $|Q| \ll a$  and  $|Q| \ll c \Delta t$  will be satisfied by having the mass of the test body sufficiently great, and this specification will also guarantee that the test body can be considered to be essentially at rest in the interval  $(\Delta t, \tau - \Delta t)$  in which it acquires momentum from the measured field [8].

The test body’s trajectory is thus necessarily of a steplike character, and so the ‘‘radiation-reaction’’ component  $\bar{F}_{\text{RR}}$  of its self-force can be approximated by the limiting ‘‘radiation-reaction’’ force  $\bar{F}_{\text{RR}(\text{BR})}$ , which is not affected by the removal of the neutralizing body. The removal or the absence of the neutralizing body would not open the possibility of an arbitrarily accurate measurement of a single field component, averaged over a time  $\tau < 2R$ , without a compensating spring.

The author gratefully acknowledges correspondence with F. Persico, whose searching questions helped the author to find correct expressions for some algebraic results used in the present Comment.

- [1] G. Compagno and F. Persico, Phys. Rev. A **57**, 1595 (1998).
- [2] N. Bohr and L. Rosenfeld, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **12**, No. 8 (1933); English translation in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983), p. 479.
- [3] V. Hnizdo, J. Phys. A **32**, 2427 (1999).
- [4] MATHEMATICA 3.0 (Wolfram Research, Inc., Champaign, IL, 1996).
- [5] For  $2R/c > \tau$  and thus  $V \sim R^3 \gtrsim c^3 \tau^3$ , the minimum uncertainty  $(\hbar/\tau V)^{1/2}$  is smaller than the ‘‘nondisturbing-measurement’’ minimum uncertainty  $2(\hbar/c^3 \tau^4)^{1/2}$  obtained for this regime by H.-H. von Borzeszkowski and M. B. Mensky, Phys. Lett. A **188**, 249 (1994).
- [6] Y. Aharonov and D. Bohm, Phys. Rev. **122**, 1649 (1961).
- [7] Y. Aharonov and A. Petersen, in *Quantum Theory and Beyond*, edited by T. Bastin (Cambridge University Press, Cambridge, 1971), p. 135.
- [8] The relation  $|Q| \gtrsim \hbar/\Delta p_x$  is not in conflict with the condition  $|Q| \ll c \Delta t$ , as  $\Delta p_x$  can be arbitrarily great without impairing the accuracy of the determination of the measured field component when the test body’s charge density is chosen sufficiently great [see Eq. (46) of BR].