Probing colored noise from the index of refraction of strongly driven two-level atoms

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(Received 8 September 1998; revised manuscript received 9 February 1999)

We analyze the effects of a reservoir having a generic spectrum (colored noise) on the index of refraction of a system of strongly driven two-level atoms (gas) probed by a weak field. We show that for high Rabi frequencies, Ω, a simple analytic expression results for the susceptibility function (with respect to the probe) when $\nu \approx \Omega$, where ν is the detuning between driving and probe fields frequencies. Several distinct features are revealed in that function when compared to the one resulting from a reservoir with a white-noise spectrum (atom-vacuum coupling). We also show that the Mach-Zehnder interferometry could permit distinguishing between the two kinds of spectra and we point out the sensitive dependence of the index of refraction of the atomic gas on Ω and find a signature for the non-Markovian evolution. [S1050-2947(99)01811-9]

PACS number(s): 42.50.Ct, 32.70.Jz, 42.50.Hz

I. INTRODUCTION

In many quantum systems interacting with an environment the evolution of the density operator can be described in the framework of a master equation

$$i d\rho(t)/dt = [H,\rho(t)] + \int_0^t K(t-t')\rho(t')dt', \qquad (1)$$

where H is a Hamiltonian operator responsible for the intrinsic properties of the system and its interaction with applied external fields, and $K(t-t')$ is a superoperator taking into account an interaction with some infinite reservoir. An effective width of the two-time kernel $K(t-t')$ is called memory time; it is the mean interval of time that separates a variable at time t' from its derivative at a later time t, such that the former influences significantly the latter. Frequently enough, sufficiently good results can be obtained under the simplest assumption of a zero memory time, when $K(t-t') \propto \delta(t-t')$, leading to what is known as a Markovian evolution. In such a case, the spectrum of the operator $K(\tau)$ does not depend on the frequency, representing a white noise. Although any real evolution is, strictly speaking, non-Markovian, the corrections due to a finite memory time are very small in many cases [1].

However, there exist examples of physical systems when a nonzero memory time or a frequency-dependent ''colored''-noise spectrum lead to significant changes in the dynamical properties of the system. Such a situation can take place, e.g., for atoms inside high-Q electromagnetic cavities when the discrete structure of the cavity modes becomes essential [2]. In particular, when an atom in a cavity is strongly driven by an external radiation (the pumping field or simply the pump) the phenomenon of dynamical supression of spontaneous emission occurs [3–6]. Also, it was shown that if an atom is strongly pumped at a frequency near a two-level transition frequency and is probed by a weak field (called the probe), then depending on the functional form of the cavity-reservoir spectrum, the form of the atomic response function or the absorption spectrum significantly change comparing with the Markovian case [7,8]. Yet as another instance, for a low-density atomic gas confined into a cell or a cavity with closed ends, when the coupling between the atoms and the cell modes prevails over the vacuum-atom coupling (the white-noise spectrum being shadowed by the colored spectrum), the absorption line-shape function changes from one- to two-bump shape beyond some critical temperature [9,10]. More recently it was verified [11] that ''memory'' effects on the atomic absorption line shape function (the imaginary part of the susceptibility with respect to the probe) are significantly enhanced when the atoms are strongly driven and then the colored noise leaves a signature, which is characterized by a linear increase as a function of Rabi frequency of the heights of the peaks of the absorption line shape function.

In this paper we concentrate on the real part of the atomic susceptibility function, χ', which could be determined, e.g., via measuring the dispersive part of index of refraction, n', of a gas of strongly pumped atoms with the help of a Mach-Zehnder interferometer. We show that by analyzing the dependence of the function χ' on the Rabi frequency Ω and the detuning between probe and pump fields ν one could infer about the nature of the coupling between atoms and the colored-noise reservoir created by the cavity (or cell). We also show that for high values of Ω (compared to the natural atomic decay constant of the upper level) and $\Omega \approx \nu$, the measurement of the modified atomic decay rate and the dy-

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namic frequency shift for several values of Ω and ν could permit us to determine the functional dependence of the cell-reservoir spectrum on frequencies.

The paper is organized as follows. In Sec. II we present a stationary solution to the generalized master equation for a density matrix of a two-level atom driven by a strong pumping field and probed by a weak monochromatic probe field in the presence of a reservoir with an arbitrary spectrum. In Sec. III we present an analytical expression for the dispersive part of the atomic susceptibility function for high Rabi frequencies under the resonant condition, and establish characteristic features of the non-Markovian evolution. A possible experimental scheme and the connection between the susceptibility function and the photocurrent output of the Mach-Zehnder interferometer is discussed in Sec. IV. Finally, Sec. V presents our conclusions.

II. THE STATIONARY SOLUTION OF THE GENERALIZED MASTER EQUATION

Let us specify the concrete structure of the generalized master equation (1). An atom is modeled by a two-level system, so its density matrix is a 2×2 Hermitian matrix

$$\rho = W_1|1\rangle\langle 1| + W_2|2\rangle\langle 2| + W_3|1\rangle\langle 2| + W_4|2\rangle\langle 1|. \quad (2)$$

We assume a rotating wave coupling between the atomic variables and the external (pump and probe) fields, so the Hamiltonian H reads

$$H = \frac{1}{2}\omega_0\sigma_0 + (F_1 e^{-i\omega_1 t}\sigma_+ + F_2 e^{-i\omega_2 t}\sigma_+ + \text{H.c.}), \quad (3)$$

where ω_0 is the atomic transition frequency, F_1 (F_2) and ω_1 (ω_2) are the coupling constant and frequency of the driving (probe) field and we consider $|F_1| \gg |F_2|$. The coupling constants can be expressed in terms of the vector dipole-matrix element between the excited and ground atomic states $\vec{\mu}_{12}$ and the electric-field strengths $\vec{E}_i$ as $F_i = -\vec{\mu}_{12}\cdot\vec{E}_i$, $i=1,2$.

The reservoir is assumed being made of an infinite number of oscillator modes interacting resonantly with the driven two-level atom (this is the most common model adopted to describe formally a reservoir although other ones were proposed [12]). Then, the superoperator kernel $K(t-t')$ can be written as [11]

$$K(t-t')\cdot = \text{Tr}_R[V_{SR}, e^{-iL_0(t-t')}[V_{SR}, \rho_R \cdot]], \quad (4)$$

where $L_0\cdot \equiv [H_0, \cdot]$ is the Liouvillian operator, H_0 is the free (atom plus reservoir) Hamiltonian,

$$V_{SR} = \int \sqrt{g(\omega-\omega_0)}(b_\omega^\dagger \sigma_- + \text{H.c.})\, d\omega \quad (5)$$

is the atom-reservoir resonant interaction term, the function $g(\omega-\omega_0) = D(\omega-\omega_0)|\kappa(\omega-\omega_0)|^2$ combines the reservoir spectrum $D(\omega-\omega_0)$ and the coupling constant $\kappa(\omega-\omega_0)$ that may be frequency-dependent, and ρ_R is the density operator of the reservoir at thermal equilibrium. For more than one reservoir interacting with an atom more terms in Eq. (5) should be considered in Eq. (4). Each term has its own spectrum and operators of one reservoir commute with the operators of the other.

The solution to Eq. (1) with Hamiltonian (3) can be written as $\rho(t) = \rho_0(t) + \Delta\rho(t)$, where $\Delta\rho(t)$ is a small correction term to the density matrix $\rho_0(t)$ of the driven atom due to the weak probe field; thus, the functions $W_i(t)$, $i = 1,2,3,4$, at the stationary regime read

$$W_1(t) = W_1^\infty + \Delta W_1(t), \quad (6)$$

$$W_2(t) = W_2^\infty + \Delta W_2(t), \quad (7)$$

$$W_3(t) = \overline{W}_3^\infty e^{-i\omega_1 t} + \Delta W_3(t), \quad (8)$$

$$W_4(t) = \overline{W}_4^\infty e^{i\omega_1 t} + \Delta W_4(t), \quad (9)$$

where W_1^∞ (W_2^∞) is the unperturbed (by the probe) population of the upper (lower) atomic level, and $\overline{W}_3^\infty$ and $\overline{W}_4^\infty$ are the coherence coefficients; the terms with the prefix Δ correspond to the corrections due to the probe. Taking into account the first Floquet harmonics, the terms $\Delta W_i(t)$ can be written as [13]

$$\Delta W_1(t) = \delta W_1 + \eta e^{-i\nu t} + \eta^* e^{i\nu t}, \quad (10)$$

$$\Delta W_2(t) = -\delta W_1 - \eta e^{-i\nu t} - \eta^* e^{i\nu t}, \quad (11)$$

$$\Delta W_3(t) = e^{-i\omega_1 t}(\delta W_0 + \delta W_+ e^{-i\nu t} + \delta W_-^* e^{i\nu t}), \quad (12)$$

$$\Delta W_4(t) = \Delta W_3^*(t), \quad (13)$$

where $\nu = \omega_2 - \omega_1$ is the detuning of the probe from the driving-field frequency. Inserting Eq. (2) and Eqs. (6)–(13) into Eq. (1) leads to a set of algebraic equations determining the coefficients δW_1, η, δW_0, δW_+, and δW_-. In the special case of a zero temperature of the reservoir (this simplification is quite adequate for optical transitions) and the exact resonance between the driving field and atomic transition frequency, $\omega_1 = \omega_0$, these algebraic equations read

$$Q(\nu)\,\eta - iF_1\delta W_+ + iF_1^*\delta W_- = -iF_2^*(\overline{W}_3^\infty)^*, \quad (14)$$

$$Q(0)\,\delta W_1 + iF_1^*\delta W_0^* - iF_1\delta W_0 = -(iF_2^*\delta W_+^* - iF_2\delta W_+), \quad (15)$$

$$Z(\nu)\,\delta W_+ + i2F_1^*\eta = -iF_2^*(W_1^\infty - W_2^\infty), \quad (16)$$

$$Z(0)\,\delta W_0 + i2F_1^*\delta W_1 + i2F_2^*\eta^* = -Z(0)\overline{W}_3^\infty - iF_1^*(W_1^\infty - W_2^\infty), \quad (17)$$

$$Z(-\nu)\,\delta W_-^* + i2F_1^*\eta^* = 0. \quad (18)$$

We defined

$$Z(x) \equiv ix - \Gamma(x), \quad (19)$$

$$Q(x) \equiv -ix + \Gamma(x) + \Gamma^*(-x), \quad (20)$$

$$\Gamma(x)=\int_0^\infty \frac{ig(\omega-\omega_0)}{x+\omega_0-\omega+i\epsilon}d\omega$$
$$=\int_0^\infty g(\omega-\omega_0)\left[\pi\delta(x+\omega_0-\omega)+i\mathrm{P}\left(\frac{1}{x+\omega_0-\omega}\right)\right]d\omega$$
$$\approx \pi g(x)-i\mathrm{P}\int_{-\infty}^{\infty}\frac{g(\omega)}{\omega-x}d\omega, \tag{21}$$

where P stands for the principal value. We consider that $g(\omega-\omega_0)$ is nearly an even function of its argument; this assumption is quite reasonable since the atom and reservoir exchange energy resonantly. Then $\Gamma(0)=\pi g(0)$ is real up to small corrections. We also note from the right-hand side of the first equality in Eq. (21) that $|\Gamma(x)|\leqslant|\Gamma(0)|$. The other quantities that enter Eqs. (14)–(18) are the coherence coefficient of the unprobed system [9],

$$\overline{W}_3^\infty=\frac{-iF_1^*\Gamma(0)}{\Gamma^2(0)+2|F_1|^2}, \tag{22}$$

and the upper energy level population,

$$W_1^\infty=\frac{|F_1|^2}{\Gamma^2(0)+2|F_1|^2}. \tag{23}$$

The population of the lower energy level is promptly obtained since $W_2^\infty=1-W_1^\infty$.

III. THE ATOMIC SUSCEPTIBILITY FUNCTION

The delayed induced polarization of an atom that results after the application of an external field $\vec{E}(t')$ is

$$\vec{P}(t)=\int_0^t \boldsymbol{\chi}(t-t')\cdot\vec{E}(t')dt', \tag{24}$$

$\boldsymbol{\chi}(t-t')$ being the dyadic susceptibility. For a monochromatic field, $\vec{E}(t)=\vec{E}_1e^{-i\omega t}+\text{c.c.}$, and Eq. (24) in the stationary limit becomes

$$\vec{P}(t)=2\,\mathrm{Re}[e^{-i\omega t}\boldsymbol{\chi}(\omega)\cdot\vec{E}_1]. \tag{25}$$

where

$$\boldsymbol{\chi}(\omega)=\int_0^\infty \boldsymbol{\chi}(t)e^{-i\omega t}dt \tag{26}$$

is the Fourier transform having the property $\boldsymbol{\chi}^*(\omega)=\boldsymbol{\chi}(-\omega)$. Writing the polarization vector as an average value of the atomic dipole moment $\vec{\mu}$,

$$\vec{P}(t)=\langle\vec{\mu}(t)\rangle=\mathrm{Tr}[\rho(t)\vec{\mu}(t)], \tag{27}$$

and using explicit expressions for the elements of matrix $\rho(t)$ given in the preceding section we obtain

$$\vec{P}(t)=2\,\mathrm{Re}[W_3(t)\vec{\mu}_{12}]. \tag{28}$$

Averaging the vectorial dipole-matrix elements over angles $\vec{\mu}_{12}\vec{\mu}_{21}=\frac{1}{3}|\vec{\mu}_{12}|^2\boldsymbol{I}$ ($\boldsymbol{I}$ is the unit matrix), we obtain the scalar susceptibility function with respect to the probe field for N atoms per unit volume [11],

$$\chi(\nu,\Omega)=-\frac{N}{3}|\vec{\mu}_{12}|^2\delta W_+/F_2=-\frac{N\lambda_0^3}{32\pi^3}\delta\widetilde{W}_+(\nu,\Omega), \tag{29}$$

where $\lambda_0=2\pi c/\omega_0$ is the wavelength of the atomic transition and $\delta\widetilde{W}_+(\nu,\Omega)\equiv\hbar\gamma\delta W_+(\nu,\Omega)/F_2$ is the dimensionless coherence function (it does not depend on the probe field and its modulus is less or of the order of unity), which depends on two frequencies: the detuning between the pump and probe frequencies ν and the Rabi frequency $\Omega=2|F_1|$. The square of the matrix dipole element $|\vec{\mu}_{12}|^2$ is replaced by the natural (vacuum) atomic decay constant

$$\gamma=(4\omega_0^3|\vec{\mu}_{12}|^2)/(3\hbar c^3). \tag{30}$$

Solving Eqs. (14)–(18) one gets

$$\delta\widetilde{W}_+(\nu,\Omega)$$
$$=\frac{i(W_2^\infty-W_1^\infty)\left[\Omega^2\left(1-\frac{Z^*(-\nu)}{Z^*(0)}\right)-2Q(\nu)Z^*(-\nu)\right]}{\Omega^2[Z(\nu)+Z^*(-\nu)]-2Q(\nu)Z(\nu)Z^*(-\nu)}, \tag{31}$$

$$W_2^\infty-W_1^\infty=\frac{2\Gamma^2(0)}{2\Gamma^2(0)+\Omega^2}. \tag{32}$$

The standard (Markovian) case corresponds to the frequency-independent function $g(\omega-\omega_0)=g_0$. Then, $\Gamma(x)=\Gamma(0)=\pi g_0$, so for $\Omega=0$ (no pumping) function (31) assumes the canonical form $\delta\widetilde{W}_+(\nu,0)=(\nu+i\gamma/2)^{-1}$ provided the constants g_0 and γ [Eq. (30)] are related as follows:

$$\gamma=2\pi g_0. \tag{33}$$

For the sake of simplicity we shall express hereafter all the quantities having the dimension of frequency [like $\Gamma(x)$, ν, Ω] in units of γ, unless it is stated explicitly that all the parameters are dimensional. Then, in particular, $\Gamma(0)=1/2$ in the Markovian case.

Formula (31) can be simplified in the case of a strong pumping field $\Omega\gg1$. One can verify that under this condition, the real part of Eq. (31) attains its maximum value at a frequency $\nu=\nu_m$, slightly shifted from the Rabi frequency

$$\nu_m=\Omega+\frac{3}{2}\Gamma_I(\Omega)+O\left(\frac{1}{\Omega}\right). \tag{34}$$

The subscripts (R,I) stand for real and imaginary parts. For the frequencies close to ν_m, the real part of the susceptibility function changes its behavior when compared to the usual situation of a two-level atom probed without pumping, namely, the real part acquires the Lorentzian shape,

$$\mathrm{Re}\,\delta\tilde{W}_+(\nu,\Omega)=\left\{12\Omega\Gamma(0)\Gamma_R(\Omega)\left[1+\frac{4(\nu-\nu_m)^2}{9\Gamma_R^2(\Omega)}\right]\right\}^{-1} \tag{35}$$

(again up to small corrections of the order of Ω^{-1}). Now one can see that by varying Ω it should be possible to determine experimentally the real part of the function $\Gamma(\Omega)$ since the width of the Lorentzian shape is $\Delta\nu=3\Gamma_R(\Omega)$. The shift of ν_m from Ω, $3\Gamma_I(\Omega)/2$, could also be determined. Thus, by varying Ω it should be possible to determine the complex function $\Gamma(\Omega)$.

There is, however, a delicate point. As was emphasized in [2], in the case when the spectral function $g(\omega-\omega_0)$ significantly differs from the white-noise constant, this function depends actually not only on the frequencies but it is very sensitive to the spatial and angular coordinates of the atom too. Due to the discrete structure of the resonance modes in the cavity, the characteristic space scale of changes of $g(\omega-\omega_0)$ is of the order of the wavelength corresponding to the maximal coupling between the atom and the cavity. At each point inside the cavity this function has distinct sharp maxima. Since we have no possibility to control the position of the atom with high precision, the observable effects are determined by the *averaged* spectral function $\overline{g}(\omega-\omega_0)$, where averaging is performed over spatial and angular coordinates in a volume whose dimensions are much greater than the optical wavelength. Evidently, the dependence of the averaged spectral function on its argument is rather smooth. To illustrate the general results given above, we suppose an effective spectral function

$$\overline{g}(\omega-\omega_0)=g_0+g_c[1+\tau^2(\omega-\omega_0)^2]^{-1}, \tag{36}$$

where the first term represents the vacuum white noise whereas the second is for the cavity modes and a Lorentzian shape is assumed as is frequently done [2], although other functional forms are also possible depending on the type of the cavity [17]. In Eq. (36), τ is a correlation time for the colored reservoir variables and g_c is an effective coupling constant of the atom with the cavity-colored spectrum, whereas g_0 is the coupling constant with the high-frequency ''white''-vacuum modes, which do not feel the presence of a cavity; note that for $\tau=0$ one gets the vacuum+cavity white noise. An advantage of function (36) is that it permits a simple calculation of integral (21) (provided the lower limit of integration is extended to $-\infty$):

$$\Gamma(x)=\frac{1}{2}\left[\gamma+\gamma_c\frac{1+ix\tau}{1+(x\tau)^2}\right],\quad \gamma_c=2\pi g_c \tag{37}$$

noting that γ_c could be positive or negative. With this form of $\Gamma(x)$, the point of maximum of $\mathrm{Re}\,\delta\tilde{W}_+(\nu,\Omega)$ is

$$\nu_m=\Omega+\frac{3\gamma_c\tau\Omega}{4(1+\tau^2\Omega^2)}+O\left(\frac{1}{\Omega}\right), \tag{38}$$

thus due to the non-Markovian character of the evolution; the shift $|\nu_m-\Omega|$ attains its largest value, $3|\gamma_c|/8$, at $\tau\Omega=1$; so, by varying Ω until the shift attains its higher value, it should be possible to determine experimentally γ_c and τ. It is worth noting that by disregarding terms $O(1/\Omega)$ in Eq. (38), $\nu_m-\Omega>0$ ($\nu_m-\Omega<0$) means a positive (negative) γ_c. The width of the line is

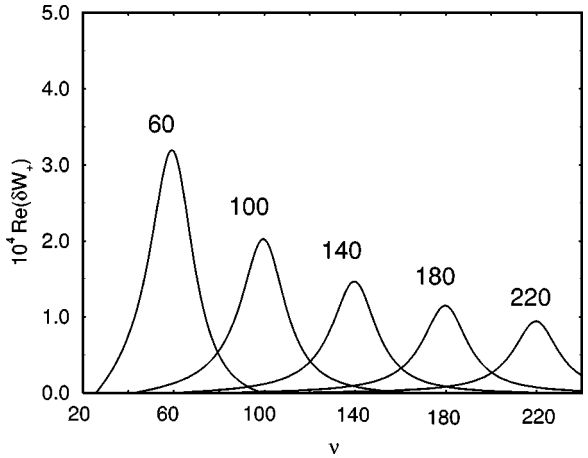

FIG. 1. The function Re δW_+, Eq. 35, versus ν and $\gamma_c=15$ in units of γ for $\tau=0$. The numbers at each curve stand for the Rabi frequency in units of γ.

$$\Delta\nu=\tfrac{3}{2}[\gamma+\gamma_c(1+\tau^2\Omega^2)^{-1}], \tag{39}$$

and it shows that γ_c can assume values only in the open interval $(-\gamma,\infty)$; thus, two different situations may happen for an atom in a cavity ($\gamma_c\neq0$): (i) For $\gamma_c<0$ the linewidth $\Delta\nu$ is always narrower than the natural linewidth $(3\gamma/2)$ and it becomes broader with increasing Ω although $\Delta\nu<3\gamma/2$. (ii) For $\gamma_c>0$ the linewidth is always broader than $3\gamma/2$ and it becomes narrower with increasing Ω. We judge that these effects are worth being verified experimentally as discussed in the next section. The height of $\mathrm{Re}\,\delta\tilde{W}_+(\nu,\Omega)$ is (all the frequencies, τ and γ_c in units of γ)

$$h\equiv\mathrm{Re}\,\delta\tilde{W}_+(\Omega,\nu_m,\tau)=\frac{1+\tau^2\Omega^2}{3\Omega(1+\gamma_c)(1+\gamma_c+\tau^2\Omega^2)}. \tag{40}$$

If $\tau=0$, or $\tau\neq0$ and $\gamma_c\leqslant8$ the height h decreases monotonically as $1/\Omega$ as Ω is increased; however, for $\tau\neq0$ and $\gamma_c>8$ (depending on the geometry of a cavity, it is possible to reach the value $\gamma_c\sim100$ [2]), then h increases for values of $\Omega\tau$ in the open interval $(0.5[(\gamma_c-2)-\sqrt{\gamma_c(\gamma_c-8)}]^{1/2}$ and $0.5[(\gamma_c-2)+\sqrt{\gamma_c(\gamma_c-8)}]^{1/2})$, attaining a maximum and afterward decreasing monotonically for values of $\Omega\tau$ larger than $0.5[(\gamma_c-2)+\sqrt{\gamma_c(\gamma_c-8)}]$.

In Figs. 1–4 we plotted $\mathrm{Re}\,\delta\tilde{W}_+(\nu)$ for $\nu\geqslant20$ (this function is antisymmetric with respect to ν). The numbers at each curve correspond to Ω and we considered $\tau=0$ (Fig. 1) and $\tau=0.01$ (Figs. 2 and 3). In Fig. 1 for high Rabi frequencies ($\Omega\geqslant60$) and $\gamma_c=15$, $\mathrm{Re}\,\delta\tilde{W}_+(\nu)$ has the maxima at $\nu\approx\Omega$, and their heights decrease as $1/\Omega$. For $\tau=0.01$ and $\gamma_c=15$, $\mathrm{Re}\,\delta\tilde{W}_+(\nu)$ suffers a drastic change (see Fig. 2); contrary to the behavior for $\tau=0$, now the maxima are located at ν_m, shifted to the right from Ω, and the peaks decrease, pass through a minimum, then increase again monotonically and the linewidths decrease. For $\tau=0.01$ and

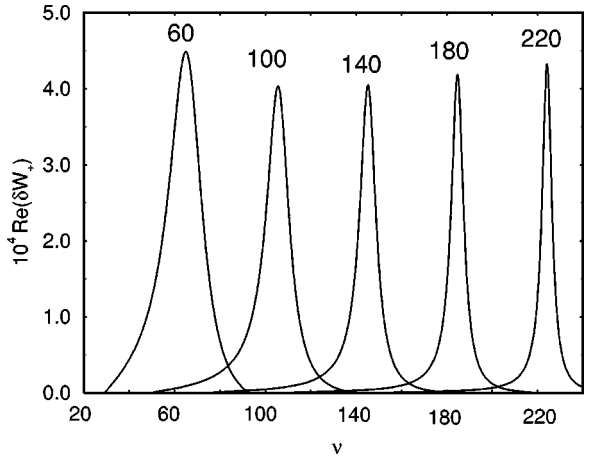

FIG. 2. The same as Fig. 1, for $\tau=0.01$.

$\gamma_c=-0.5$ (Fig. 3) the lines are much more narrow than in the previous cases and even narrower than in the case of the atom interacting with vacuum modes only, $\gamma_c=0$; however, the heights of the peaks decrease monotonically with increasing Ω.

In order to compare the shapes of the curves for Markovian ($\gamma_c=0$) and non-Markovian evolutions ($\gamma_c\neq 0$, $\tau=0.01$), but the same Ω, we picked the curves labeled as 100 in Figs. 2 and 3 and plotted them in Fig. 4, in a zoom-in of the abcissa coordinate. The solid line is for $\gamma_c=0$; the others are for $\tau=0.01$ and the small (large) dashed lines correspond to $\gamma_c=-0.5$ ($\gamma_c=15$). A finite τ and $\gamma_c<0$ ($\gamma_c>0$) modifies the shape of the solid curve in three aspects: (a) a shift of the point of maximum to the left (right) of $\nu=\Omega$, (b) a narrowing (broadening) of the width $\Delta\nu$ (although it is wider the larger Ω is) and (c) a decrease (increase) in the height of the curve.

IV. A POSSIBLE EXPERIMENTAL SCHEME

We have established that the space-averaged spectral function of the colored vacuum in a cavity can be determined from the real part of the averaged atomic susceptibility, $\overline{\chi}=\overline{\chi}'+i\overline{\chi}''$, with respect to the probe field, provided the pumping is strong enough. In turn, the averaged susceptibility can be easily recovered from the real and imaginary parts

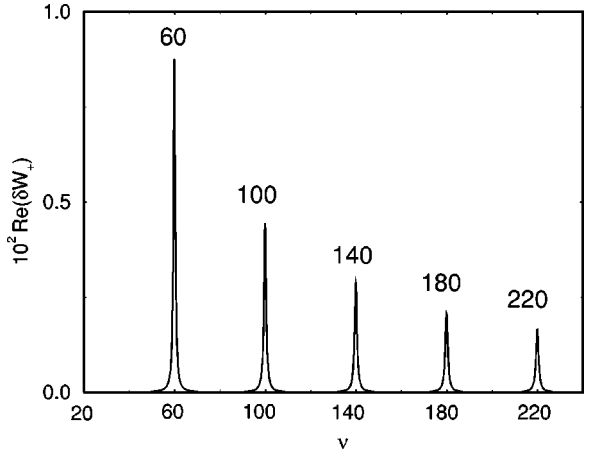

FIG. 3. The same as Fig. 1, for $\gamma_c=-0.5$ and $\tau=0.01$.

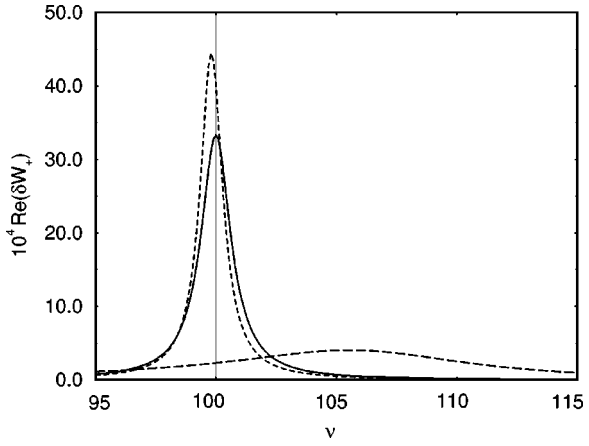

FIG. 4. The function Re δW_+, Eq. (35), versus ν in units of γ, $\Omega=100$; $\gamma_c=0$ (solid line), $\tau=0.01$ and $\gamma_c=-0.5$ (small dashes), and $\tau=0.01$ and $\gamma_c=15$ (large dashes).

of the index of refraction, $n=n'+in''$, of a gas inside the cavity, due to the known formula

$$4\pi\overline{\chi}'=(n')^2-1-(n'')^2. \tag{41}$$

The index of refraction can be measured, e.g., with the aid of a balanced two-port Mach-Zehnder interferometer [14]. The devised experimental setup can be seen in Fig. 5. Beam *a* emerges through the cell where its original intensity is attenuated due to the atomic absorption; besides, it acquires a supplementary phase due to the index of refraction of the gas. The phase shifter compensates the spurious phases introduced by the beam splitters, effects of the faces of an ''empty'' cell, etc., such that the difference in photocurrents j_c and j_d be zero. For typical lasers the phase fluctuations can be neglected because the phase angle diffuses away from its initial values at a rate inversely proportional to the mean number of laser photons [15]. Also, atomic-collision effects can be neglected since we consider a low density gas [15], and Doppler broadening can be substantially reduced quite below radiation linewidth by working at mK temperatures or

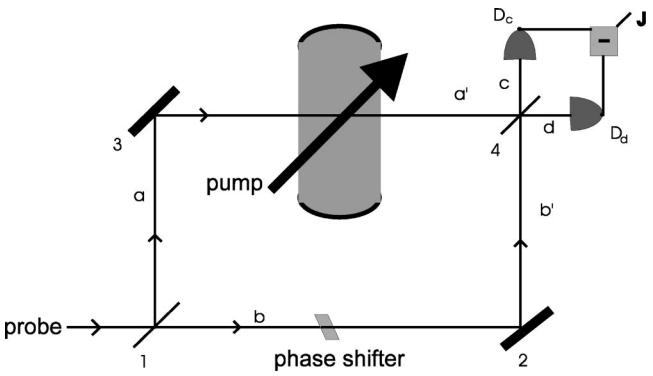

FIG. 5. Mach-Zehnder interferometric scheme: a probe beam interferes with itself after being splitted (1 and 4 are 50:50 beam splitters, 2 and 3 are perfect reflectors); beam *a* goes through a cell (closed optical cavity) filled with a two-level atoms gas at density *N* getting out as a' while beam *b* passes through a phase shifter becoming beam b'. Beams *c* and *d* are collected at the photocurrent detectors D_c and D_d and the current (or photocount) difference *j* is measured.

by using experimental techniques of laser beams as reported in [16] for compensation of the atomic recoil.

The amplitudes of beams c and d are given in terms of amplitudes of those of a and b,

$$c=\sqrt{\eta}e^{i\chi_0}b+i\sqrt{1-\eta}e^{-ik_0L(1-n)}a, \tag{42}$$

$$d=\sqrt{\eta}e^{-ik_0L(1-n)}a+i\sqrt{1-\eta}e^{i\chi_0}b, \tag{43}$$

where η is the transmitivity coefficient of the beam splitter, χ_0 is a phase introduced by a phase shifter, k_0 is the free-space wave number, L is the length of the driven-atoms gas crossed by the probe, and n is the (complex) index of refraction in that region. Thus, in the presence of atoms driven by a strong field, the difference in photocurrent $\delta j_0 \equiv j_d - j_c \propto \langle c^+ c\rangle - \langle d^+ d\rangle$, (for $\chi_0=0$) is

$$\delta j_0 \equiv j_d - j_c = j_{inc}\kappa \sin(\Delta\phi), \tag{44}$$

where j_c and j_d are the photocurrents detected at ports c and d; j_{inc} is the photocurrent associated to the probe without cell or interferometer.

$$\kappa=\exp(-2\pi n''L/\lambda_p), \tag{45}$$

$$\Delta\phi=2\pi(n'-1)L/\lambda_p, \tag{46}$$

are, respectively, the attenuation factor and the phase difference between beams c and d, λ_p is the wavelength of the probe, and n' and n'' are the real and imaginary parts of the index of refraction. Introducing a phase $\chi_0=\pi/2$ in beam b', the photocurrent difference becomes shifted by this same phase, such that

$$\delta j_{\pi/2}=j_{inc}\kappa\cos(\Delta\phi). \tag{47}$$

Squaring and summing the difference in photocurrents (44) and (47) and then dividing the result by the squared photocurrent j_{inc} one gets the square of the attenuation factor κ from which n'' can be obtained. Thus, it becomes possible to isolate the pure oscillatory part of the photocurrent, $\sin(\Delta\phi)$, related to n'. For $2\pi|\chi'|\ll 1$ one can write $\Delta\phi \approx N\lambda_0^2 L\ \mathrm{Re}\,\delta\tilde{W}_+(\nu,\Omega)$; then for appropriate values of λ_0, L, and N, such that $(\Delta\phi)\ll 1$, it could be possible to extract the phase difference $\Delta\phi$ from the photocurrent, thus the real part of the index of refraction would be obtained immediately. For $\gamma_c \geqslant 10$ and $20\leqslant\Omega\leqslant 220$ one verifies that $h \sim 10^{-5}$, which is an upper limit for $\mathrm{Re}\,\delta\tilde{W}_+(\nu,\Omega)$.

For instance, one could consider the 852-nm line of the $6S_{1/2}\rightarrow 6P_{3/2}$ transition in Cs atoms with natural width $\gamma = 3.3\times 10^7\ \mathrm{s}^{-1}$. Equation $I/\Omega^2 = I_s/\gamma^2$ [18] relates the intensity of the driving field to the Rabi frequency, where $I_s = (2\pi^2/3)(\hbar c/\lambda^3)\gamma$ is the saturation intensity and $I_s = 10^{-3}\ \mathrm{W/cm^2}$ for that transition. Thus in order to attain $\Omega/\gamma \approx 100$, the intensity of the driving field should be of the order of 10 $\mathrm{W/cm^2}$; a laser beam focused on a small region within the cell could permit attaining such high values for the Rabi frequency.

V. DISCUSSION AND CONCLUSIONS

Here we discussed the effects of the cavity modes on a driven atom by analyzing the dispersive part of the susceptibility with respect to a probe field, much weaker than the driving one. We considered that the cavity modes participate with a colored spectrum additionaly to the white-noise spectrum of the vacuum. We showed that for high values of the Rabi frequency Ω (measured in units of the natural linewidth) and for detunings $\nu\approx\Omega$ the function χ' takes a Lorentzian shape with width, shift, and height depending on Ω. Thus, by varying the values of Ω these quantities could be determined from the experiment. We discussed a possible experiment based on the Mach-Zender interferometry, the measurement of the index of refraction of the driven atoms gas in the cell cavity could permit us to infer about the nature of the cavity colored-noise spectrum; however, we do not exclude the possibility of a better scheme of measurement. It is worth commenting about the role of the Rabi frequency on the peculiar behavior of enhancing the effects of τ; qualitatively, it can be understood by looking at the reservoir frequency spectrum in the dressed-atom formalism [19]: At on resonance $(\Delta\omega=0)$, the driving field splits the atomic-transition frequency from ω_0 to $\omega_0\pm\Omega$; thus, assuming the instance of spectrum (36) one gets $g(\Omega\tau)=g_0 + g_c/[1+(\Omega\tau)^2]$. For $\tau\neq 0$ it is possible to reduce the effect of γ_c around the shifted transition frequency by increasing the Rabi frequency; thus by diminishing the influence of the cavity modes on the atom, the modified linewidth should approach the natural one, verifying a narrowing (broadening) of the linewidth for $g_c>0$ $(g_c<0)$. The driven atom presenting a lower sensitivity to the cavity modes at the new transition frequencies, $\omega_0\pm\Omega$, is reflected by the probe that responds showing sharper and higher peaks with increasing Ω when $\gamma_c>8$ with a new damping constant $\gamma\rightarrow\gamma + \gamma_c/(1+\tau^2\Omega^2)$. For small driving fields the line shapes of Figs. 1–3 are replaced by the usual dispersionlike shapes around the origin. In conclusion, we showed that by strongly driving two-level atoms and using the above-described interferometric setup, the effects of the cavity could be put in relief and the function $\Gamma(\nu)$ could be estimated from experiment.

ACKNOWLEDGMENTS

G.A.P. thanks CNPq for total financial support. S.S.M. thanks CNPq and FINEP for partial financial support.

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