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Nonlinear decoherence in quantum state preparation of a trapped ion

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We present a nonlinear decoherence model which models decoherence effects caused by various decohering sources in a quantum system through a nonlinear coupling between the system and its environment, and apply it to investigating decoherence in nonclassical motional states of a single trapped ion. We obtain an exactly analytic solution of the model and find very good agreement with experimental results for the population decay rate of a single trapped ion observed in the NIST experiments by Meekhof and co-workers [D. M. Meekhof *et al.*, Phys. Rev. Lett. **76**, 1796 (1996)]. [S1050-2947(99)05310-X]

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In recent years, much progress has been made in preparation, manipulation, and measurement of quantum states of the center-of-mass vibrational motion of a single trapped ion experimentally [1–8] and theoretically [9–16], which are not only of fundamental physical interest but also of practical use for sensitive detection of weak signals [17] and quantum computation in an ion trap [3,9]. In particular, the NIST group [4] has experimentally created and observed nonclassical motional states of a single trapped ion. In the NIST experiments [4], an anti-Jaynes-Cummings-model (JCM) interaction between the internal and motional states of a trapped ion is realized through stimulated Raman transitions, which couple internal states of the trapped ion to its motional states, when the Lamb-Dicke limit is satisfied and the driving laser fields are tuned to the first blue sideband. Detection of motional states is carried out by observing the evolution characteristics of quantum dynamics of internal levels of the trapped ion under the influence of the anti-JCM-type interaction. The NIST experiments revealed the fact that the population of the low atomic state ($P_\downarrow$) evolves according to the following phenomenological expression:

$$P_\downarrow(t)=\frac{1}{2}\left(1+\sum_n p_n\cos(2gt\sqrt{n+1})e^{-\gamma_n t}\right), \tag{1}$$

where p_n is the initial probability distribution of motional states of the trapped ion in the Fock representation, g is a coupling constant between the atomic internal and motional states, and γ_n is a decay rate. The experimentally observed decay rate is of the form

$$\gamma_n=\gamma_0(n+1)^\nu, \tag{2}$$

where the observed value of ν is $\nu\simeq 0.7$.

A question that naturally arises is, how do we explain the above experimentally observed decay rate? It is generally accepted that the appearance of the decay factor γ_n in the evolution of internal states is a consequence of decoherence. It is of practical significance for a good understanding of decoherence for preparation of nonclassical states and quantum computation in ion traps. There are various sources of decoherence [1], such as ion vibrational decoherence, ion internal-state decoherence, decoherence caused by nonideal external fields, and so on. Recently, Schneider and Milburn [18] have investigated decoherence due to laser intensity and phase fluctuations and obtained the power ν in Eq. (2) with $\nu\doteq 0.5$ instead of the experimentally observed value 0.7. More recently, Murao and Knight [19], using the master equation method, have studied decoherence due to the imperfect dipole transitions and fluctuation of vibrational potential in the NIST experiments. In spite of these efforts, the problem of decoherence in quantum state preparation of a trapped ion has not been satisfactorily solved, and its character and microscopic origin still call for further attention. In particular, it should be pointed out that the experimentally observed decay rate indicated in Eq. (2) is a collective effect caused by various decohering sources, not by a specific decohering source. Nevertheless, authors in Refs. [18,19] investigated the decay rate caused only by a specific source of decoherence, not by various sources of decoherence. So how to model the experimentally observed decay rate caused by various decohering sources is an interesting subject in quantum state preparation and manipulation of a trapped ion. In this paper, we present a nonlinear decoherence model to model decoherence effects caused by various decohering sources in a quantum system. We shall show that our theoretical model can describe the experimentally observed decay rate in the NIST experiments [4] well.

We consider a single trapped ion with mass m and laser cooled to the Lamb-Dicke limit. Following the notation of Ref. [19], we denote three related internal states and motional states of the ion by $|i\rangle$ $(i=\underline{0},\downarrow,\uparrow)$ and $|n\rangle$ $(n=0,1,2,\dots)$, respectively. The free Hamiltonian of the trapped ion is given by $\hat{H}_0=\hbar\omega_x\hat{a}^\dagger\hat{a}-\hbar\omega_{01}|\downarrow\rangle\langle\downarrow|-\hbar\omega_{02}|\uparrow\rangle\langle\uparrow|$, where ω_{01} (ω_{02}) is the transition frequency between states $|\downarrow\rangle$ ($|\uparrow\rangle$) and $|\underline{0}\rangle$, and $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator of the motional states with the corresponding frequency ω_x. Two driving laser beams with detuning Δ, wave vector $\vec{k}_1$ ($\vec{k}_2$), and frequency ω_1 (ω_2) are used to cause dipole transitions between the level $|\downarrow\rangle(|\uparrow\rangle)$ and $|\underline{0}\rangle$.

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With the dipole and rotating wave approximations, under large detuning condition the intermediate level $|\underline{0}\rangle$ can be adiabatically eliminated when the Lamb-Dicke limit is met and the driving laser beam is tuned to the first blue sideband. Then, in the interaction picture of $\hat{H}_0$, the effective Hamiltonian of the system has the anti-JCM-type form

$$\hat{H}_S=\hbar g(\hat{a}^\dagger\sigma_+ +\hat{a}\sigma_-), \tag{3}$$

where g is a coupling constant, which depends on the coupling strength between internal and motional states of the trapped ion and the Lamb-Dicke parameter defined by $\eta=\delta k x_0$, where δk is the wave-vector difference of the two Raman beams along x, and $x_0=\sqrt{\hbar/2m\omega_x}$. For simplicity, we set $\hbar=1$ throughout this paper.

The Hamiltonian (3) is diagonal in the dressed-state representation with the following basis:

$$|\varphi(n,i)\rangle=\frac{1}{\sqrt{2}}[|\downarrow,n\rangle-(-1)^i|\uparrow,n+1\rangle], \quad i=1,2 \tag{4}$$

$$|\varphi(0,3)\rangle=|\uparrow,0\rangle. \tag{5}$$

And we have $\hat{H}_S|\varphi(n,i)\rangle=E_{ni}|\varphi(n,i)\rangle$ with eigenvalues $E_{ni}=(-1)^{i+1}g\sqrt{n+1}$ for $i=1,2$, and $E_{03}=0$.

Before going to our model, let us briefly recall a few basic facts about the interaction between a quantum system and its environment. The interaction between the system and its environment may create two types of effects [20–34]: decoherence and dissipation, which can be mathematically described by decaying of the off-diagonal and diagonal elements of the reduced density operator of the system, respectively. These two effects have been paid much attention in various areas, for instance, quantum measurement [20,25–28], condensed matter physics [21–23], quantum computation [29–31], and so on. The decoherence effect makes the states of the system continuously decohere to approach classical states [20,27]. The dissipation effect dissipates energy of the system to the environment [21–23]. The two effects can be understood in terms of Hamiltonian formalism [32–34]. If we assume the total Hamiltonian of the system plus environment to be $\hat{H}_T=\hat{H}_S+\hat{H}_R+\hat{H}_I$, where $\hat{H}_S$ and $\hat{H}_R$ are Hamiltonians of the system and environment, respectively, and $\hat{H}_I$ is the interaction Hamiltonian between them, when the Hamiltonian of the system commutes with that of the interaction between the system and environment, i.e., $[\hat{H}_S,\hat{H}_I]=0$, which means that there is no energy transfer between the system and the environment, energy of the system is conservative, so that what interaction between the system and environment describes is the decoherence effect. When $[\hat{H}_S,\hat{H}_I]\neq 0$, there is energy transfer between the system and environment, so that what interaction between the system and environment describes is the dissipation effect. It should be pointed out that the decoherence and dissipation happen at different time scales [29,30]. The dissipation effect occurs at the relaxation time τ_{rel}, while the decoherence time scale τ_d is much shorter than τ_{rel} with the time evolution of a quantum system. Hence, we here restrict our attention on decoherence effect.

We now present our model. We use a reservoir consisting of an infinite set of harmonic oscillators to model the environment of the single trapped ion in the NIST experiments, and assume that in the interaction picture of $\hat{H}_0$ the total Hamiltonian is of the following phenomenological form:

$$\hat{H}_T=\hat{H}_S+\sum_k \omega_k\hat{b}_k^\dagger\hat{b}_k+F(\{\hat{O}_S\}) \times\sum_k c_k(\hat{b}_k^\dagger+\hat{b}_k)+F^2(\{\hat{O}_S\})\sum_k \frac{c_k^2}{\omega_k^2}. \tag{6}$$

Here the first term is the Hamiltonian of the system in the interaction picture given by Eq. (3); the second term is the Hamiltonian of the reservoir; the third one represents the interaction between the system and the reservoir with a coupling constant c_k, where $\{\hat{O}_S\}$ is a set of linear operators of the system or their linear combinations in the same picture as that of $\hat{H}_S$, $F(\{\hat{O}_S\})$ is an operator function of $\{\hat{O}_S\}$. In order to enable what the interaction between the system and the reservoir describes in Eq. (6) is decoherence, not dissipation, we require that the linear operator $\hat{O}_S$ commutes with the Hamiltonian of the system, i.e., $[\hat{O}_S,\hat{H}_S]=0$. It is well known that the decohering process can indeed be considered as a quantum measurement process. The conventional definition of a quantum measurement involves any form of interaction between a quantum object and a classical system. Therefore, the interaction function $F(\{\hat{O}_S\})$ in the model (6) can involve any form of interaction between the system and environment. This enables it to model the collective decohering behavior caused by various decohering sources. The concrete form of the function $F(\{\hat{O}_S\})$ may be regarded as an experimentally determined quantity. The last term in Eq. (6) is a renormalization term, which is discussed in Ref. [21]. When $F(\{\hat{O}_S\})$ is a linear and nonlinear function of the linear operator $\hat{O}_S$, we call decoherence described by the interaction between the system and the reservoir linear and nonlinear decoherence, respectively, in the similar sense of the linear and nonlinear dissipation implied in Ref. [21]. In this sense, decoherence investigated in Ref. [19] is a kind of linear decoherence. In what follows we shall show that nonlinear decoherence can better describe the decay rate in the NIST experiments.

The Hamiltonian (6) can be exactly solved by making use of the unitary transformation

$$\hat{U}=\exp\left(F(\{\hat{O}_S\})\sum_k \frac{c_k}{\omega_k}(\hat{b}_k^\dagger-\hat{b}_k)\right). \tag{7}$$

After applying the unitary transformation (7) to the total Hamiltonian (6), we get a decoupled Hamiltonian $\hat{H}_T'=\hat{H}_S+\Sigma_k\omega_k\hat{n}_k$, where $\hat{n}_k=\hat{b}_k^\dagger\hat{b}_k$. The density operator associated with the decoplued Hamiltonian is given by

$$\hat{\rho}_T'(t)=e^{-i\hat{H}_T't}\hat{\rho}_T'(0)e^{i\hat{H}_T't}, \tag{8}$$

where $\hat{\rho}_T'(0)=\hat{U}\hat{\rho}_T(0)\hat{U}^{-1}$, with $\hat{\rho}_T(0)$ being the initial total density operator. Through a converse transformation of

Eq. (7), it is straightforward to obtain the total density operator associated with the original Hamiltonian (6) with the expression

$$\hat{\rho}_T(t)=e^{-i\hat{H}_S t}\hat{U}^{-1}e^{-it\sum_k \omega_k\hat{n}_k}\hat{U}\hat{\rho}_T(0)\hat{U}^{-1}e^{it\sum_k \omega_k\hat{n}_k}\hat{U}e^{i\hat{H}_S t}. \tag{9}$$

We assume that the system and reservoir are initially in thermal equilibrium and uncorrelated, so that $\hat{\rho}_T(0)=\hat{\rho}_S(0)\otimes\hat{\rho}_R(0)$, where $\hat{\rho}_S(0)$ and $\hat{\rho}_R(0)$ are the initial density operator of the system and the reservoir, respectively. $\hat{\rho}_R(0)$ can be expressed as $\hat{\rho}_R=\Pi_k\hat{\rho}_k(0)$ where $\hat{\rho}_k(0)=(1-e^{-\beta\omega_k})e^{-\beta\omega_k\hat{n}_k}$ is the density operator of the kth harmonic oscillator in thermal equilibrium, where $\beta=1/k_BT$, k_B and T being the Boltzmann constant and temperature, respectively. After taking the trace over the reservoir, from Eq. (9) we can get the reduced density operator of the system, denoted by $\hat{\rho}(t)=\mathrm{tr}_R\hat{\rho}_T(t)$; its matrix elements in the dressed-state representation are explicitly written as

$$\rho_{(m',i')(m,i)}(t)=\rho_{(m',i')(m,i)}(0)R_{m'i'mi}(t)e^{-i\phi_{m'i'mi}(t)}. \tag{10}$$

Here the phase is defined by

$$\phi_{m'i'mi}(t)=[E_{m'i'}-E_{mi}], \tag{11}$$

and $R_{m'i'mi}(t)$ is a reservoir-dependent quantity given by

$$\begin{aligned}R_{m'i'mi}(t)=\prod_k\ &\mathrm{Tr}_R\{D(-\alpha_{mik})\\ &\times e^{-it\omega_k\hat{n}_k}D(-\alpha_{mik})D(-\alpha_{m'i'k})\\ &\times e^{-it\omega_k\hat{n}_k}D(-\alpha_{m'i'k})\hat{\rho}_k(0)\},\end{aligned} \tag{12}$$

where $\alpha_{mik}=f(\{O_{mi}\})c_k/\omega_k$ with O_{mi} being an eigenvalue of the linear operator $\hat{O}_S$ in a dressed state, i.e., $\hat{O}_S|\varphi(m,i)\rangle=O_{mi}|\varphi(m,i)\rangle$, and $D(\alpha)=\exp(\alpha\hat{b}_k^+-\alpha^*\hat{b}_k)$ is a displacement operator.

Making use of properties of the displacement operator:

$$D(\alpha)D(\beta)=D(\alpha+\beta)\exp[i\ \mathrm{Im}(\alpha\beta^*)], \tag{13}$$

$$\exp(x\hat{n}_k)D(\alpha)\exp(-x\hat{n}_k)=\exp(\alpha e^x\hat{b}_k^+-\alpha^*e^{-x}\hat{b}_k), \tag{14}$$

and the following formula [35]:

$$\mathrm{Tr}_R[D(\alpha)\hat{\rho}_k(0)]=\exp\left[-\frac{1}{2}|\alpha|^2\coth\left(\frac{\beta\omega_k}{2}\right)\right], \tag{15}$$

we find that the reservoir-dependent quantity $R_{m'i'mi}(t)$ can be written as the following factorized form:

$$R_{m'i'mi}(t)=e^{-i\delta\phi_{m'i'mi}(t)}e^{-\Gamma_{m'i'mi}(t)}, \tag{16}$$

with the following phase shift and damping factor:

$$\delta\phi_{m'i'mi}(t)=[F^2(\{O_{m'i'}\})-F^2(\{O_{mi}\})]Q_1(t), \tag{17}$$

$$\Gamma_{m'i'mi}(t)=[F(\{O_{m'i'}\})-F(\{O_{mi}\})]^2Q_2(t). \tag{18}$$

Here the two reservoir-dependent functions are given by

$$Q_1(t)=\int_0^\infty d\omega J(\omega)\frac{c^2(\omega)}{\omega^2}\sin(\omega t), \tag{19}$$

$$Q_2(t)=2\int_0^\infty d\omega J(\omega)\frac{c^2(\omega)}{\omega^2}\sin^2\left(\frac{\omega t}{2}\right)\coth\left(\frac{\beta\omega}{2}\right), \tag{20}$$

where we have taken the continuum limit of the reservoir modes: $\Sigma_k\rightarrow\int_0^\infty d\omega J(\omega)$, where $J(\omega)$ is the spectral density of the reservoir, and $c(\omega)$ is the corresponding continuum expression for c_k.

We assume that the system is initially in a state $\hat{\rho}(0)=|\downarrow\rangle\langle\downarrow|\otimes\Sigma_np_n|n\rangle\langle n|$. Then, from Eqs. (10)–(16) we find that at time t the population of the lower atomic state is given by

$$P_\downarrow(t)=\frac{1}{2}\left(1+\sum_n\ p_n\cos[\phi_{n1n2}(t)+\delta\phi_{n1n2}(t)]e^{-\Gamma_{n1n2}(t)}\right), \tag{21}$$

which indicates that the interaction between the system and reservoir induces a phase shift $\delta\phi_{n1n2}(t)$ and a damping factor $\Gamma_{n1n2}(t)$ in the time evolution of the atomic population.

Taking into account the experimental expression (1), we choose the following linear operator and interaction function:

$$\hat{O}_S=\hat{a}^\dagger\sigma_++\hat{a}\sigma_-, \tag{22}$$

$$F(\{\hat{O}_S\})=\hat{O}_S^{2d+1}, \tag{23}$$

where d is an adjustable parameter to describe the nonlinearity in the interaction, which reflects the deviation degree of the nonlinearity of $F(\{\hat{O}_S\})$ with respect to the linear operator $\hat{O}_S$. The value of the parameter d is determined by the experimental results. With these choices, it is easy to find that

$$F^2(O_{n1})-F^2(O_{n2})=0, \tag{24}$$

$$F(O_{n1})-F(O_{n2})=2(\sqrt{n+1})^{2d+1}. \tag{25}$$

Then the phase shift in Eq. (21) naturally vanishes, and the damping factor becomes

$$\Gamma_{n1n2}=4(n+1)^{2d+1}Q_2(t), \tag{26}$$

so that we can find from Eq. (21) that

$$P_\downarrow(t)=\frac{1}{2}\left(1+\sum_n\ p_n\cos(2gt\sqrt{n+1})e^{-4(n+1)^\nu Q_2(t)}\right), \tag{27}$$

where $\nu=2d+1$ and $Q_2(t)$ is given by Eq. (20). From Eq. (27) we see that the argument of the cosine function on the right-hand side of Eq. (27) does have the same form as that in the experimental expression (1). Comparing the theoretical expression (27) with the experimental result (1), we find that when the nonlinear deviation $d\doteq-0.15$, the n dependence of the damping factor in Eq. (27) is completely in agreement with that seen in the experimental expression (1). The final step is to determine the time dependence of the damping factor in Eq. (27). From Eqs. (19), (20), and (27), we see that all necessary information about the effects of the environment is contained in the spectral density of the reservoir. Equation. (27) indicates that the time dependence of the damping factor is completely determined by the spectral density of the reservoir. The experimental expression (1) requires that the time dependence of the damping factor must be linear, so that if we choose the spectral density such that

$$Q_2(t)=\frac{1}{4}\gamma_0 t, \tag{28}$$

where γ_0 is a characteristic parameter, then we can get an expression of $P_{\downarrow}(t)$, which has exactly the same form as the experimental result (1). It is possible to find a spectral density of the reservoir to satisfy the condition (28). For instance, for the case of zero temperature, if we take the spectral density $J(\omega)=\gamma_0/[2\pi c^2(\omega)]$, substituting it in Eq. (20) we can realize Eq. (28).

In conclusion, we have presented a nonlinear decoherence model, and obtained its exactly analytic solution. It has been shown that our model can give precisely the same expression of the population decay rate of the single trapped ion as that observed in the NIST experiments [4]. The nonlinear decoherence model can describe the NIST experiments very well. This indicates that the reservoir and the nonlinear coupling between the system and the reservoir, which we design, properly model the real environment of the single trapped ion in the experiments. It is worthwhile to emphasize that the nonlinearity in the coupling describes a collective contribution of various decohering sources to the decay rate. Hence, what the nonlinear decoherence describes is a collective decoherence effect caused by various decohering sources, not a specific decoherence source. We have noted that authors in Ref. [19] obtained the decay rate in Eq. (1), but decoherence which they considered is a specific decoherence caused by the imperfect dipole transitions and fluctuation of vibrational potential, so their results cannot cover the contribution of other decohering sources to the decay rate in the NIST experiment. It can be expected that the nonlinear decoherence model proposed in the present paper can describe decoherence behaviors of a wide variety of quantum systems.

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