Quantum computation with quantum dots and terahertz cavity quantum electrodynamics

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A quantum computer is proposed in which information is stored in the two lowest electronic states of doped quantum dots (QD's). Many QD's are located in a microcavity. A pair of gates controls the energy levels in each QD. A controlled-NOT (C-NOT) operation involving any pair of QD's can be effected by a sequence of gate-voltage pulses which tune the QD energy levels into resonance with frequencies of the cavity or a laser. The duration of a C-NOT operation is estimated to be much shorter than the time for an electron to decohere by emitting an acoustic phonon. [S1050-2947(99)00311-X]

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I. INTRODUCTION

A quantum computer processes quantum information that is stored in ''quantum bits'' (qubits) [1]. If a small set of fundamental operations, or ''universal quantum logic gates,'' can be performed on the qubits, then a quantum computer can be programmed to solve an arbitrary problem [2]. The explosion of interest in quantum computation can be traced to Shor's demonstration in 1994 that a quantum computer could efficiently factorize large integers [3]. Further boosts came in 1996, with the proof that quantum error correcting codes exist [4,5]. It has since been shown that if the quantum error rate is below an accuracy threshold, quantum information can be stored indefinitely [6].

The implementation of a large-scale quantum computer is recognized to be a technological challenge of unprecendented proportions. The qubits must be well-isolated from the decohering influence of the environment, but must also be manipulated individually to initialize the computer, perform quantum logic operations, and measure the result of the computation [7].

Implementations of universal quantum logic gates and quantum computers have been proposed using atomic beams [8], trapped atoms [9] and ions [10], bulk nuclear magnetic resonance [11], nanostructured semiconductors [12–15], and Josephson junctions [16,17]. In schemes based on trapped atoms and ions, qubits couple with collective excitations or cavity photons. Such long-range coupling enables two-bit gates involving an arbitrary pair of qubits, which makes programming straightforward. However, in the atomic and ionic schemes [9,10], the gates must be performed serially, whereas existing error correcting schemes require some degree of parallelism. In semiconductor and superconductor schemes which have been proposed [12–17], only nearest-neighbor qubits can be coupled, and significant overhead is involved in coupling distant qubits. However, some of these schemes have the important advantage that gate operations can be performed in parallel.

It is widely agreed that a solid-state quantum computer, if it can be realized, will be the only way to produce a quantum computer containing, for example, 10^3 qubits. The remainder of this paper describes what is, to our knowledge, the first proposal for a semiconductor-based quantum computer in which quantum gates can be effected between an arbitrary pair of qubits. The qubits consist of the lowest electronic states of specially engineered quantum dots (QD's) and are coupled by terahertz cavity photons. The proposal combines ideas from the atomic and ionic implementations described above with recent developments in the spectroscopy of doped semiconductor nanostructures at terahertz frequencies [18–20].

II. QUANTUM BITS AND FUNDAMENTAL QUANTUM LOGIC OPERATIONS

The fundamental building blocks of the proposed computer are the nanostructures shown in Fig. 1. Three disks of a semiconductor (e.g., GaAs) are embedded in a semiconductor with a larger band gap (e.g., $Al_xGa_{1-x}As$). The central disk is taller than the outer two. The barriers between the disks are sufficiently thin to allow an electron to rapidly tunnel between them. A structure consisting of a set of three disks and the two intervening barriers is hereafter called a quantum dot (QD). Each QD which is to participate in the quantum computation must have one and only one electron.

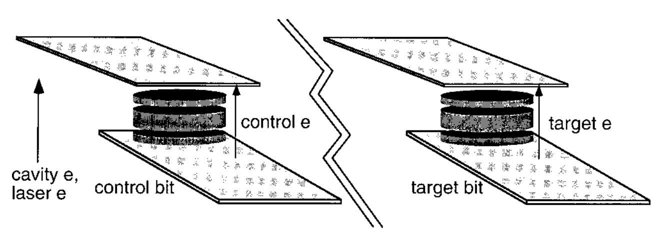

FIG. 1. Fundamental elements of the proposed quantum computer. Each set of quantum dots (QD's) contains one electron, and is individually addressable by a pair of gate electrodes. One QD is chosen to be a control bit, the other a target bit for a controlled-not (CNOT) operation. Many fundamental elements are embedded in a single-mode cavity.

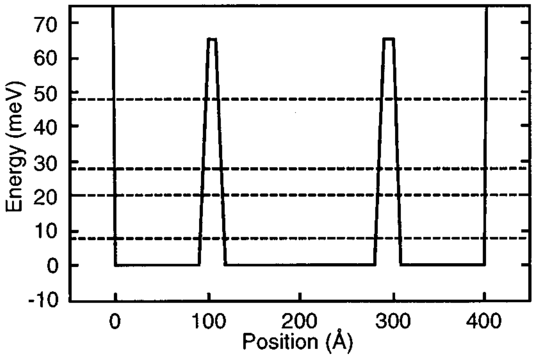

FIG. 2. Potential and energy level diagram for the lowest energy levels of a set of coupled QD's, which is suitable for a qubit. The ground ($|0\rangle$) and first excited ($|1\rangle$) states are used to store quantum information. The second excited state ($|2\rangle$) is an auxiliary state, which is used to effect a controlled-not operation, but does not store quantum information. The height of the QD is 41 nm, and the potential inside is 0 except for two 2-nm barriers with 65-meV potential which separate the central 17-nm well from the outer 10-nm wells.

The potential and four lowest electronic energy levels for a particular realization of a QD are shown in Fig. 2. The lowest two energy levels, denoted $|0\rangle$ and $|1\rangle$, will form the qubits which store quantum information. The third energy level, labeled $|2\rangle$, will serve as an auxiliary state to perform conditional rotations of the state vector of the qubit, much like the auxiliary state in the ion trap computer [10]. Below and above each QD is an electrical gate. Voltages applied to these gates are used to control the spacing between and absolute values of the energy levels of the QD's via the Stark effect. A large number of individually gated QD's is contained in a three-dimensional (3D) microcavity whose fundamental resonance has a wavelength λ_c much longer than a QD. A continuous-wave laser with a fixed wavelength different from λ_c is introduced through one side of the cavity.

Figure 3(a) shows the energies E_{10} and E_{20} of the 0-1 and 0-2 transitions in a QD as a function of the electric field e applied via the gates. Also shown in Fig. 3(a) are the energies of a cavity mode photon $\hbar\omega_c$, a laser photon $\hbar\omega_l$, and the sum $\hbar\omega_l+\hbar\omega_c$. The state of an electron in a QD can be coherently manipulated by tuning E_{10} and E_{20} into and out of resonance with $\hbar\omega_c$, $\hbar\omega_l$, and the sum $\hbar\omega_l+\hbar\omega_c$.

A general Hamiltonian describing a QD interacting with cavity photons and the laser field is given by

$$\begin{aligned}\hat{H}=&\hbar\omega_c\hat{a}_c^+\hat{a}_c+E_{10}(e)\hat{\sigma}_{11}+E_{20}(e)\hat{\sigma}_{22}+\hbar g_{01}(e)\\&\times\{\hat{a}_c^+\hat{\sigma}_{01}+\hat{\sigma}_{10}\hat{a}_c\}+\hbar\Omega_{l,01}(e)\{\hat{\sigma}_{01}\exp(i\omega_l t)\\&+\hat{\sigma}_{10}\exp(-i\omega_l t)\}+\hbar g_{12}(e)\{\hat{a}_c^+\hat{\sigma}_{12}+\hat{\sigma}_{12}\hat{a}_c\}\\&+\hbar\Omega_{l,12}(e)\{\hat{a}_c^+\hat{\sigma}_{12}\exp(i\omega_l t)+\hat{\sigma}_{21}\hat{a}_c\exp(-i\omega_l t)\}.\end{aligned}\tag{1}$$

Here, $\hat{a}_c$ denotes the cavity-mode annilihation operator, and $\hat{\sigma}_{ij}=|i\rangle\langle j|$ is the projection operator from the QD state $|j\rangle$ to state $|i\rangle$. The vacuum Rabi frequencies are $g_{ij}=qz_{ij}e_{\mathrm{vac}}$, where

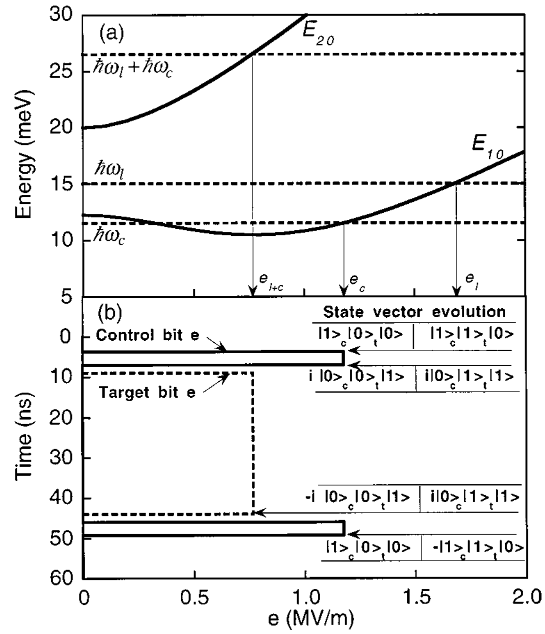

FIG. 3. (a) Transition energies between states $|0\rangle$ and $|1\rangle$ (E_{10}) and between $|0\rangle$ and $|2\rangle$ (E_{20}) vs applied electric field, and photon energies of a cavity mode ($\hbar\omega_c$), a laser ($\hbar\omega_l$), and the sum $\hbar\omega_l+\hbar\omega_c$. The E_{10} transition resonates with $\hbar\omega_c$ and $\hbar\omega_l$ at electric fields e_c and e_l, respectively. The E_{20} transition resonates with the two-photon transition with energy $\hbar\omega_l+\hbar\omega_c$ at electric field e_{l+c}. (b) A sequence of electric field pulses to a control and a target bit which are used in a C-NOT gate. First, a "π" pulse is applied to the control bit, transferring a photon to the cavity and multiplying the state vector by i if and only if the control bit is 1. Then, a "2π" pulse is applied to the target bit, multiplying the state vector by -1 if and only if there is a photon in the cavity and the target bit is in its ground state. Finally, a second "π" pulse is applied to the control bit, removing the photon from the cavity, returning the control bit to the excited state, and again multiplying the state vector by i. The state vectors in which the control bit is 0 are unaffected by the sequence of electric field pulses, and thus are not shown. One-bit rotations can be effected by applying an appropriately timed pulse with amplitude e_l. As shown by Cirac and Zoller [10], the gate shown here, together with one-bit rotations on the target bit, results in a C-NOT operation.

$$e_{\mathrm{vac}}=\left(\frac{\hbar\omega_c}{2\varepsilon_0\varepsilon V}\right)^{1/2}\tag{2}$$

is the amplitude of the vacuum electric field in the cavity and ϵ and V are the dielectric constant and volume of the cavity, respectively, q is the electronic charge, and z_{ij} is the dipole matrix element of the $|i\rangle\rightarrow|j\rangle$ transition. One step in the controlled-not (C-NOT) operation will be a Rabi oscillation between states $|0\rangle$ and $|2\rangle$ involving both cavity and laser photons at $e=e_{l+c}$. An effective Hamiltonian describing these two-photon processes is given by replacing the last two terms of Eq. (1) with

$$H_{\text{two-photon}}=\hbar\tilde{\Omega}(e)\{\hat{a}_c^+\hat{\sigma}_{02}\exp(i\omega_l t)+\hat{\sigma}_{20}\hat{a}_c\exp(-i\omega_l t)\},\tag{3}$$

where the two-photon effective Rabi frequency $\tilde{\Omega}$ is given by

$$\hat{\Omega}(e)=\frac{g_{01}(e)\Omega_{l,12}(e)}{\omega_{21}(e)-\omega_l}+\frac{g_{12}(e)\Omega_{l,01}(e)}{\omega_{21}(e)-\omega_c}. \quad (4)$$

The effective two-photon Hamiltonian neglects ac Stark shifts and terms which do not satisfy resonance conditions. In addition, we envision a scenario in which the first term of Eq. (4) dominates $[\omega_{21}(e_{l+c})-\omega_l \ll \omega_{21}(e_{l+c})-\omega_c]$, and the conditional phase shift dominates over phase shifts induced by the cavity field alone $[\Omega_{l,12}(e_{l+c}) \gg g_{01}(e_{l+c})]$.

During the operation of this quantum computer, a qubit which is simply storing quantum information is in state $|0\rangle$ or state $|1\rangle$, and the electric field across it is held at a fiducial value at which the energy levels of the qubit are not resonant with $\hbar\omega_c$, $\hbar\omega_l$, or $\hbar\omega_l+\hbar\omega_c$. For simplicity, we choose this fiducial field to be zero. For $e\approx e_c$, the first interaction term dominates as $|\omega_{10}(e_c)-\omega_c| \ll |\omega_{10}(e_c)-\omega_l|$. If the cavity contains one photon *or* the qubit state vector is in state $|1\rangle$, then the qubit will execute vacuum Rabi oscillations with frequency g_{01}, in which the probability of finding the electron in the excited state oscillates 90° out of phase with the probability of finding one photon in the cavity. For $e\approx e_l$, the second interaction term has the resonant contribution. Here, the state vector of the qubit rotates between states $|0\rangle$ and $|1\rangle$ with laser Rabi frequency $\Omega_{l,01}$. Finally, for $e \approx e_{l+c}$, the $H_{\text{two-photon}}$ dominates. If the cavity contains one photon *and* the qubit state vector begins in state $|0\rangle$, then it rotates between states $|0\rangle$ and the auxiliary $|2\rangle$ with frequency $\tilde{\Omega}(e_{l+c})$. If either the qubit is in state $|1\rangle$ *or* the cavity does not contain a photon, then the qubit state vector is not rotated for $e\approx e_{l+c}$.

A C-NOT operation is effected by a series of voltage pulses applied across the gates of a pair of qubits. The pulses always begin and end with the qubit at the fiducial electric field ($e=0$), and rise to a target value e_c, e_l, or e_{l+c}. Figure 3(b) shows a sequence of voltage pulses which effects a two-qubit gate, which is equivalent to a C-NOT operation [10]. The cavity always begins with no photons. First, a "π" pulse with height e_c and duration $\pi/(2g_{01})$ is applied to the control bit. If the control bit is in state $|0\rangle$, it is unaffected. If it is in state $|1\rangle$, it rotates into state $|0\rangle$ and acquires a phase *i*, and the cavity acquires a single photon. Next, a "2π" pulse with height e_{l+c} and duration $\pi/\tilde{\Omega}(e_{l+c})$ is applied to the target bit. If the target bit is in state $|1\rangle$, it is unaffected. If it is in state $|0\rangle$ *and* the cavity contains one photon, it acquires a phase -1. Finally, a pulse with height e_c, identical to the first pulse, is again applied to the control bit. If there is a photon in the cavity, it is absorbed by the control bit, returning it to state $|1\rangle$ while the control bit acquires another phase *i*. The end result is a gate in which the state vector of the two-qubit system acquires a phase -1 if and only if both control and target bits are initially 1. The sequence of state-vector rotations which is effected by the series of electric field pulses is identical to the sequence effected by a series of laser pulses applied to cold trapped ions [see Eq. (3) of Ref. [10]]. In order to effect a C-NOT operation (inversion of the target bit if and only if the control bit is 1), it is necessary to apply to the target bit "$\pi/2$" and "$3\pi/2$" pulses with height e_L and durations $\pi/(4\Omega_{L,01})$ and $3\pi/(4\Omega_{L,01})$, respectively, before and after the sequence shown in Fig. 3(b) [10].

A few additional conditions are required to ensure the fidelity of C-NOT operations. To ensure that nearly all of the state-vector rotation occurs while the electric field is at its target value, the rise and fall times δt of the pulses must be short compared to the period of the Rabi oscillation at the target *e*. At the same time, in order to minimize the probability of a transition between $|0\rangle$ and $|1\rangle$ induced by the ramping electric field, one requires the changes to the Hamiltonian to be adiabatic ($\delta t \gg \hbar/E_{10}$). As with other schemes for quantum computation, the timing between the successive pulses in the C-NOT operation must be carefully adjusted to compensate for the quantum-mechanical phases $\exp(-iE_{10}t/\hbar)$ accumulated by inactive qubits in their excited states. In order to achieve the sort of fidelity which is required for a C-NOT operation in a quantum computer, it may be necessary to adjust the heights and durations of the electric field pulses to account for ac Stark shifts in the energy levels of the QD's which are induced by the laser field. These important details will be addressed in a future publication.

III. REQUIREMENTS FOR QUANTUM COMPUTATION

The ability to effect a C-NOT operation is one of several requirements for a universal quantum computer. Other requirements include the following.

(i) *Initializing the computer*. Before a quantum computation begins, each qubit must be in a well-defined state. In the proposed computer, it suffices to wait, with all gate voltages at the fiducial voltage ($e=0$) and at a temperature $T \ll E_{10}/k_B$, until each qubit relaxes to its ground state. For $E_{10}\approx 10\,\text{meV}$, one requires a temperature $T \ll 120\,\text{K}$. From calculations detailed below of the predicted energy relaxation times in QD's, a wait of less than 1 s will certainly ensure that all qubits are in state $|0\rangle$.

(ii) *Inputting initial data*. At the beginning of a quantum computation, arbitrary rotations of the state vectors of qubits are required to load data into the qubit registers. Arbitrary one-bit rotations are effected using Rabi oscillations induced by the laser field, by applying pulses with height e_l and duration between 0 and $2\pi/(\Omega_{l,01})$.

(iii) *Readout*. At the end of a quantum computation, the state of each qubit must be measured. One of us has proposed a tunable antenna-coupled intersubband terahertz (TACIT) quantum-well-based detector [21,22]. This device efficiently absorbs and detects terahertz photons, but only in a narrow bandwidth centered on the intersubband absorption frequency. This frequency is Stark tunable with application of moderate electric fields, such as are used to tune the transition frequencies of the qubits in this proposal. We propose to integrate such a detector into the cavity. During the computation phase, the detector would be tuned far off of the cavity resonance frequency ω_c, minimally affecting the *Q* of the cavity. During the read-out phase, the TACIT detector would be tuned to the cavity resonance frequency. Calculations indicate that TACIT detectors can have very high quantum efficiency, and thus will spoil the *Q* of the cavity when it is tuned to the cavity resonance. The qubits can then be read out sequentially by tuning them to ω_c. If the qubit is in state $|1\rangle$, it will emit a photon in the time required for a π pulse $[\pi/(2g_{01})]$ which will be detected virtually immediately. For the parameters discussed below, the rate at which qubits can be read out will then be roughly $g_{01}=3\times10^{8}$ Hz, enabling 1000 qubits to be read out in roughly 3 μs. A detector is required which has a noise equivalent power less than $(E_{10}/g_{01})^{1/2}\approx10^{-17}\mathrm{W/Hz}^{1/2}$ with a bandwidth greater than g_{01}. It is likely that such a combination of sensitivity and speed could be achieved with careful engineering of a TACIT detector [23].

(iv) *Error correction*. Existing schemes for error correction require the execution of quantum logic gates in parallel. One can imagine parallelizing the scheme proposed here by enlarging the cavity to create several cavity modes in the frequency range over which QD energy level spacings are tunable. This would come at the cost of slowing down gate operations by reducing the vacuum electric field and hence the vacuum Rabi frequency $g_{01}\alpha V^{-1/2}$. A more intriguing possibility is to somehow marry a nearest-neighbor-coupled semiconductor scheme for quantum computation like [12–14] with a nonlocal scheme like the one proposed here. In this case, logic gates would be effected in parallel in clusters of qubits coupled with nearest-neighbor interactions, while qubits in distant clusters could communicate serially via long-range interactions mediated by cavity photons.

(v) *Decoherence*. This is the most problematic issue pertaining to most quantum computers. In the computer proposed here, decoherence of the electronic state of the QD as well as of the cavity photons must be considered.

There are no experimental data on the decoherence of electronic intraband excitations in isolated QD's loaded with a single electron. Dephasing in ''open'' quantum dots defined by gate electrodes in a two-dimensional electron gas (2DEG) has been studied, yielding dephasing times $t_{\phi}\leqslant2$ ns [24]. The studied dots have $E_{10}\leqslant20\ \mu$eV. The times are consistent with those predicted for disordered 2D systems. The rate of spontaneous emission of acoustic phonons in $\approx$200-nm double QD devices containing 15–25 electrons has recently been deduced. From the transport currents of order $I=10^{-12}A$, one deduces an energy relaxation time of order $q/I=10^{-7}$ s for transitions with energies near 50 μeV [25]. However, the transition energies, QD geometry, and number of electrons in the experiments [24,25] are very different from those of Fig. 1, and it is thus impossible to draw conclusions about decoherence in the QD's envisioned in this proposal.

Many interactions will potentially cause decoherence of electrons in the computer proposed here. Some can be mitigated by clever engineering, including the following.

(a) The emission of freely propagating photons which is eliminated because the QD's are in a 3D cavity with a very high quality factor.

(b) The interaction with fluctuations in the potentials of the two gate electrodes associated with each QD. Both crosstalk from switching voltages on distant QD's and thermal fluctuations (i.e., Johnson noise) on a QD's gate electrodes can contribute to fluctuating gate potentials with frequencies much lower than $E_{10}/\hbar$ [26,27]. Such low-frequency noise causes adiabatic changes $\delta E_n(t)$ to the energy of levels E_n, which lead phase errors $\delta\phi_n(t)=-1/\hbar\int\delta E_n(e_N(t')dt')$ (here, $n=0,1$). Such phase errors can be restricted to occur only during the time of a logic operation. The gate electrodes are made out of a superconductor. When a QD is not involved in a logic operation, its two gates are connected to a superconducting ground by a superconducting path. Since there is no dissipation, there are no thermal fluctuations. Furthermore, electric fields generated far from the QD are screened by the gate electrodes. While a QD is being switched, the connection to the superconducting ground must be broken. Low-frequency noise which occurs during this time will contribute to an error in the accuracy of a C-NOT operation. These and other possible errors in the C-NOT operation, as well as possible ways to correct them, will be analyzed in a future publication.

(c) The interaction with metastable traps in the semiconductor. Metastable traps in the semiconductor are a source of extremely slow time-varying electric fields ($f<100$ Hz). If the fluctuating traps are far from a given gate electrode, they will cause a slow fluctuating potential on that electrode, which is screened as described in (b). Hence, it is only the traps which are fluctuating in the tiny volume between the two gate electrodes that pose a serious problem. The density of such traps in semiconductor nanostructures is constantly being reduced with advances in processing.

(d) Inhomogeneity of quantum dots. Different dots will vary slightly in their energy levels and matrix elements. This inhomogeneity can arise from geometrical variations between the quantum dots, and also from the presence of quenched disorder (static charged defects). Inhomogeneity and static disorder do not contribute to decoherence of quantum bits. However, to perform accurate one- and two-bit operations in an inhomogeneous population of quantum dots, each quantum dot in the quantum computer will need to be calibrated (by performing a C-NOT operation, for example) before a quantum computation is run. Note that all solid-state implementations of quantum computers will share the need to calibrate in order to overcome disorder.

The lifetime of a cavity photon must be sufficiently long to enable many C-NOT operations with high fidelity. This will require the development of few-mode THz cavities with extremely low loss. The expected cavity losses cannot be analyzed without details of the quantum computer's architecture, which are beyond the scope of this paper, and materials properties which have not yet been measured. It is likely that cavities made from conventional metals will introduce losses which are unacceptable. One attractive possibility is dielectric cavities, made, for example, from ultrapure Si. Existing measurements of optical loss in Si at THz frequencies are dominated by free-carrier absorption [28], which can be eliminated by purifying and cooling the Si. Residual losses in Si are due to a process in which a THz photon creates two phonons [29]. These minuscule THz losses have not been measured or realistically computed to our knowledge. A second promising possibility is to use quantum dots with E_{01} smaller than the energy gap of an *s*-wave superconductor (3 meV for niobium, 9 meV for Rb_3C_{60} [30]). A cavity with volume $\ll\lambda^3$ and extremely low loss could then be made of a segment of superconducting transmission line.

IV. TIMES TO DECOHERE AND PERFORM C-NOT

Consider now a specific GaAs/Al_xGa_{1-x}As QD and lossless dielectric cavity designed to minimize the time required for a C-NOT operation, while at the same time avoiding the emission of longitudinal optical (LO) phonons ($\hbar\omega_{\mathrm{LO}}$

$\approx$36 meV in GaAs) and also minimizing the rate of acoustic-phonon emission. Cavity and laser photon energies are chosen to be 11.5 and 15 meV. These energies are sufficiently large to enable an adequate vacuum electric field e_{vac} while their sum is still comfortably smaller than $\hbar\omega_{\mathrm{LO}}$. Assuming perfect cylindrical symmetry, the states are labeled with quantum numbers $|l,m,n\rangle$, associated with the radial, azimuthal, and axial degrees of freedom, respectively. The potential along the cylindrical axis of the QD (z direction) and the numerically computed four lowest energy levels are depicted in Fig. 2. Figure 3(a) shows the transitions E_{10} and E_{20} versus electric field e. Assuming infinite walls in the radial direction, the radial wave functions are given by Bessel functions. The difference between the energy of the ground and first radial excited states is $\Delta E_r = 30$ meV for radius $a=13$ nm, assuming $m^* = m_e/15$. This is higher than the highest energy reached by an electron during a C-NOT operation ($26.5\,\mathrm{meV} = \hbar\omega_l + \hbar\omega_c$), eliminating decoherence arising from coupling between axial and radial excited states of the QD. The growth of QD's similar to those in Fig. 1 is currently being attempted. One method is to grow stacked self-assembled QD's [31]. A second method is to make QD's made by growing GaAs/$\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ quantum wells with the conduction-band profile tailored to give the desired potential in the z direction (for example, that shown in Fig. 2), depositing small islands on top of the quantum well to serve as an etch mask, etching through the quantum-well layers which are not protected by the islands, and then regrowing $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ [32].

A. Decoherence due to interactions with acoustic phonons

We now consider the decoherence of an excited electron associated with its interaction with acoustic phonons. We first calculate the time for an electron to relax from state $|1\rangle$ to state $|0\rangle$ by emission of a longitudinal acoustic (LA) phonon. In the language of NMR, this is a T_1 process. This is computed using the deformation-potential approximation, in which electrons scatter from potential fluctuations arising from local volume compressions and dilations induced by LA phonons. Piezoelectric coupling between electrons and transverse acoustic phonons exists in III-V semiconductors, but is thought to be weaker than deformation-potential coupling [33,34].

Following Bockelmann [35], assuming zero temperature, the rate at which electrons relax between QD states by emitting LA phonons is given by Fermi's Golden Rule,

$$\tau_{i\to f}^{-1} = \frac{2\pi}{\hbar}\sum_{\vec{k}} |\langle\psi_i|W|\psi_f\rangle|^2\,\delta(E_f - E_i - E_{\vec{k}}). \tag{5}$$

Here, the deformation-potential interaction W is given by

$$W = \left(\frac{\hbar K}{2\rho c_s}\right)^{1/2} D e^{i\vec{k}\cdot\vec{x}}, \tag{6}$$

where $K=|\vec{k}|$, $\rho = 5300\,\mathrm{kg/m^3}$, $c_s = 3700$ m/s, and the deformation potential $D=8.6$ eV. We approximate the eigenfunctions associated with motion in the z direction by those for an infinite square well of the width 40 nm, which fit the exact wave functions reasonably well. As the volume V of the crystal is taken to infinity, it can be shown that the expression for τ_{10} becomes

$$\tau_{10}^{-1} = \frac{D^2 K_{10}^3}{4\pi\hbar\rho c_s^2}\int_0^1 dq' \frac{q'}{\sqrt{1-q'^2}} \times \left| N\int_0^{x_{01}} dr' J_0(\alpha q' r') J_0^2(r') r' \right|^2 \times \left| \frac{2}{\pi}\int_{-x/2}^{x/2} dz' \cos(z')\sin(2z') e^{i\beta\sqrt{1-q'^2}z'} \right|^2. \tag{7}$$

Here, $K_{10} = E_{10}/\hbar c_s$, $q' = q/K_{10}$, where q is the radial phonon wave vector, $N^{-1} = \int_0^{x_{01}} dr' r' J_0^2(r') \approx 0.779$ is the normalization factor for the radial wave functions, $x_{01} \approx 2.404$ is the first zero of $J_0(a)$, $r' = r x_{01}/a$, where r is the radial distance, $z' = z\pi/h$, $h=40$ nm is the height of the QD, $\alpha = K_{10} a/x_{01}$, and $\beta = K_{10} h/\pi$. At $e=0$, one finds $E_{10} = 12.25$ meV, $K_{10} = 5.03\,\mathrm{nm}^{-1}$, $\alpha = 27$, and $\beta = 64$. The relatively large values of α and β indicate that the characteristic phonon wavelength is much shorter than the size of the QD, leading to very small values for the integral over q' [36]. The value of τ computed by evaluating Eq. (7) numerically at the above parameters is 150 μs.

Second, we briefly discuss "pure dephasing" of the electronic states of a QD by interaction with LA phonons. This is a "T_2" process in the language of NMR. Pure dephasing of quantum-confined excitons by acoustic phonons has been considered, experimentally and theoretically, by Fan *et al.* [37]. They find that at low temperatures ($\approx$10 K) the dephasing of the excitons is dominated by the radiative lifetime of the exciton (T_1), with the contribution of pure dephasing becoming significant as the temperature is increased. Theoretically, they follow treatments by Huang and Rhys [38] and by Duke and Mahan [39]. A polaronic coupling between the exciton and the phonons renormalizes the energy levels of the excitons and gives the associated peaks in the density of states nonzero widths.

We have not considered polaronic effects in the coupling between an electron in a QD and acoustic phonons. Polaronic effects on electrons in QD's will be more similar to those on hydrogenic donors than excitons. The work of Duke and Mahan finds that phonon-induced linewidths of the transitions of hydrogenic donors in CdTe are much smaller than those on excitons in the same material. We speculate that such phonon-induced linewidths in the GaAs QD's discussed here will be sufficiently small so as not to limit operation of the proposed quantum computer. Verifying this speculation requires a significant generalization of existing calculations, and is beyond the scope of this paper.

B. Time to execute C-NOT

The time required to execute a C-NOT operation for the particular QD structure is now estimated [40]. For a dielectric cavity resonating at $\hbar\omega_c = 11.5$ meV with index of refraction $n=3.6$, the maximum vacuum electric field $e_{\mathrm{vac}} \approx 49$ V/m is achieved for a cavity with minimal volume $(\lambda_c/2)^3$, where $\lambda = c/(n\omega_c) = 30\,\mu$m is the wavelength of the resonant radiation inside the cavity. This vacuum electric field, together with the matrix elements $z_{10}(e_c = 1.177\,\text{MV/m}) = 6.32\,\text{nm}$, $z_{10}(e_{l+c} = 0.7668\,\text{MV/m}) = 6.95\,\text{nm}$, $z_{10}(e_l = 1.682\,\text{MV/m}) = 4.97\,\text{nm}$, and $z_{21}(e_{l+c} = 0.7668\,\text{MV/m}) = 6.52\,\text{nm}$, and a laser electric field of 30.7 kV/m, enable one to compute the time required for a C-NOT operation. The 2π pulse applied to the target bit requires interaction with both a laser and a cavity photon, and hence is by far the longest operation, requiring 25 ns. The π pulses applied to the control bit require 3.3 ns each. Unconditional one-bit rotations which occur at $e = e_l$ take only a few ps for a laser electric field of 30 kV/m. It is likely the laser would need to be attenuated for these rotations, in order to satisfy the requirement that the transition time for the electrical pulse is much shorter than the period of the Rabi oscillation at the target electric field. *If* the dominant mechanism for decoherence is given by acoustic-phonon emission, then the above calculations suggest that several thousand C-NOT operations can be performed before the computer decoheres.

V. CONCLUSIONS

We have proposed a quantum computer in which quantum information is stored in the lowest electronic levels of doped quantum dots. The energy levels in each dot are controlled by dedicated gate electrodes. THz photons in a cavity act as a data bus which can couple an arbitrary pair of quantum dots. A sequence of adiabatic voltage pulses applied to individual quantum dots can effect a C-NOT operation involving any two quantum bits in the computer. We hope that this concrete proposal for a quantum information processor will stimulate theoretical and experimental activity.

As with all proposals for quantum computation, the obstacles to implementing this one are formidable. Among the most important challenges, new types of QD's must be constructed, gated and loaded with single electrons [41]; few-mode THz cavities with extremely high Q must be fabricated; and single THz photons must be detected. Although each of these worthy challenges is beyond today's state of the art, the rapid pace of progress in materials science and THz technology makes us optimistic that these obstacles will be overcome in the not too distant future.

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