Quantum noise and polarization properties of vertical-cavity surface-emitting lasers

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We present theoretical and experimental results on the quantum noise performances of vertical-cavity-surface-emitting-lasers (VCSELs). Using a semiclassical laser noise approach, we demonstrate that the ratio $M=\overline{S}/\overline{P}$ of the powers in the two polarization modes of the laser is one of the essential parameters governing the VCSELs quantum noise behavior. The generation of amplitude squeezed states of light is only possible for two well-selected cases which are the ideal two-polarization-mode laser ($M\approx 1$) and the ideal single-polarization-mode laser ($M\approx 0$), respectively. Furthermore, we show that gain suppression, as well as other relevant semiconductor parameters, have also to be taken into account to model realistically the VCSEL's quantum noise. These theoretical investigations are very well supported by our experimental results. We demonstrate experimentally that indeed there exists a direct link between the parameter M and the shot noise normalized amplitude noise. In a two-polarization-mode laser as, e.g., represented by a large diameter air-post VCSEL, squeezed states could not be generated because of insufficiently low values of M even though a strong anticorrelation between the two modes exists. With a single-polarization-mode laser, as realized with small diameter oxide confined VCSELs, we have been able to demonstrate the generation of single-mode squeezed light with a VCSEL. A squeezing level of 0.9 dB has been measured, which with the detection efficiency corrected, results in 1.3 dB at the VCSEL upper facet. [S1050-2947(99)08010-5]

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I. INTRODUCTION

Vertical-cavity-surface-emitting lasers (VCSELs) represent, for many photonic and optoelectronic application aspects, very interesting laser structures. They normally possess a very low threshold current, can be modulated very fast, and can be technologically highly integrated [1–3]. Due to their small size, even new quantum effects and phenomena may occur in such structures [4]. The modification of the spontaneous emission spectrum [5], a complex spatiotemporal dynamical behavior, as well as complex polarization properties and dynamics, have been studied theoretically [6–8] and have been experimentally demonstrated as well [9,10].

One characteristic of major importance is the VCSELs polarization behavior. In a circular symmetrical cavity, two orthogonal polarization modes coexist. These modes are always competing against each other, which can be described by their coupling to the same carrier reservoir, leading to very complex dynamics [8,11]. The competition between these two polarization modes influences their static performances as well as their dynamics and noise properties [12]. Experimental investigations have already shown the influence of this multimode behavior of VCSELs on their noise performances [13,14], but a combined comprehensive and systematic experimental and theoretical study of this problem is still needed.

The possibility of generating light with subshot intensity noise with a semiconductor laser was demonstrated theoretically [15] and experimentally [16] a few years ago. The important requirement is that the laser is quietly pumped, i.e., efficiently pumped with a constant current source so that the quiet stream of injected electrons is directly transformed into a quietly emitted photon stream. Up to now, most of the experimental results obtained on squeezing have been performed with edge emitting lasers. High levels of squeezing have been reached with a maximum of 8.3 dB using an asymmetric Fabry-Perot laser structure at 77 K [16]. The quantum noise properties of VCSELs and, even more, their performance as squeezed states generators, have only been experimentally studied in a few publications [17,18]. However, only two successful attempts of generating squeezed states of light with VCSELs have been reported [19,20] with a maximum measured level of squeezing of 1.3 dB [19]. Therefore, a variety of questions on the VCSELs quantum noise properties have yet to be answered.

From a theoretical first glance VCSELs seem indeed to possess a large potential as amplitude squeezed state generators [21,22]. These models have been rather simple and did not take into account the polarization properties of VCSELs. Up to now very few papers have considered the importance of the polarization properties on the VCSELs quantum noise properties [8,23–25]. Thus, we have extended our semiclassical model for the description of the semiconductor laser noise that already proved its validity [22,26,27] to gain better insight into these problems. The existence of two polarization modes is incorporated into the model and further semiconductor lasers specifics as, e.g., gain suppression are also considered. We find indeed that the polarization properties have a tremendous influence on the VCSELs quantum noise. We derive, in particular, conditions for which the emission of squeezed states of light is possible. These conditions and further conclusions coincide pretty well with all experimental results obtained thus far [19,20].

The paper is organized as follows. In Sec. II, the model is described and explained in detail. Section III presents the simulation results and is divided into two subsections. In the first part, we present our modeling results concerning the strong influence of the polarization properties of the VCSELs. From these results we demonstrate the two cases

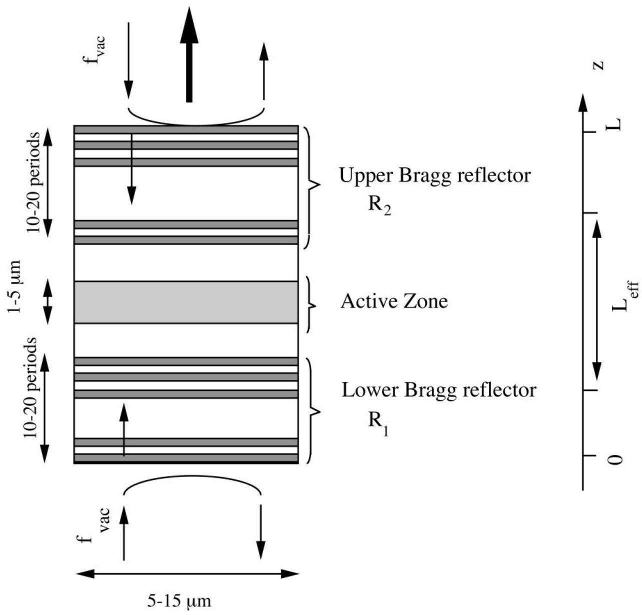

FIG. 1. Schematic depiction of a vertical cavity surface emitting laser (VCSEL) including the vacuum fluctuation forces at the laser facets.

for which the generation of amplitude squeezed light is possible. The second part of Sec. III then illustrates the different influence of self and cross gain saturation, and the photon lifetime on the amplitude noise characteristics of a two-mode VCSEL. A conclusion is finally drawn summarizing the required optimum operation and necessary device parameters to obtain amplitude squeezed states from a VCSEL. Section IV then presents the results of our experimental investigations of a single-polarization-mode ($M\approx 0$) and a two-polarization-mode ($M\approx 1$) VCSEL, and a comparison of their noise performances. Squeezing is not obtained with the two-polarization-mode VCSEL. On the contrary, squeezing is observed with the single-polarization-mode laser, with a measured noise level of 0.9 dB below the shot noise level. The last section finally summarizes the results of this work.

II. THEORETICAL DESCRIPTION

We have developed a semiclassical model based on Green's function method [26,27] in order to calculate and to simulate the noise performances of semiconductor lasers and especially of VCSELs [22,28]. Figure 1 shows the schematical depiction of a VCSEL structure. It can be simply considered a narrow active zone sandwiched in between two high reflecting Bragg mirrors. Vacuum fluctuations present at the laser facets as depicted in Fig. 1 are introduced into this model as added Langevin noise forces at the laser facets. This method has already proven its validity and strength, and has given equivalent results as existing quantum-mechanical and semiclassical theories [15]. Its unique advantage is to give analytical expressions for amplitude and phase noise even if gain suppression and spatial hole burning are considered. This is an important aspect of the model because these two effects have an influence on the spontaneous emission and the noise spectra for every kind of laser structure [26,27].

VCSELs seem to possess, as already demonstrated [22], a high potential as squeezed state emitters when described in a simple approximation as a small, high reflection coated Fabry-Perot laser. The necessary pumping level to obtain a specific amount of squeezing with a VCSEL is lower than for a normal edge emitting laser [22]. However, VCSELs suffer from their very low polarization selectivity, which is accompanied by instabilities, i.e., a complex polarization dynamics in general [11]. Experiments have shown that the resulting competition between the two polarization modes leads to an increase in the VCSELs intensity noise [12,17]. In order to study the influence of these effects of the polarization on the quantum noise, our one-mode model has been extended to a two-mode model [28], representing the two polarization modes of the VCSELs in a similar way as the spectral modes in Agrawal's former work [29]. Our model is based on the assumption that only two polarization modes exist. More detailed models of VCSELs polarization have already been presented [9,11]. These models enable us to calculate, knowing the parameters of the used materials and the VCSEL structure, the static and dynamic polarization properties of these lasers. With their descriptions of VCSEL physics they provide a deep knowledge of VCSELs polarization properties. Our more simple model cannot give such complex and precise results on polarization, but it is very appropriate for the calculations of the VCSELs noise properties and one obtains simple analytical solutions, which other models are not able to provide. Phenomenological equations are taken for the gain, the self and the cross coupling of the modes containing microscopic parameters such as the spin relaxation time. Only two photon population equations are used; no phase equation is used. For our purpose, introducing a phase equation would not add any supplementary result and would not influence the calculation of amplitude noise properties. This implies that the linewidth enhancement factor α is not directly taken into account, but indirectly, through the noise terms and their statistics.

In reality, a VCSEL may possess many transverse modes which give a contribution to the two existing polarization directions. The modes that we have used in our calculations may be interpreted as super modes or pseudomodes. They correspond to the superposition of the different contributions of each transverse mode. A more precise modeling of such a complex system with several modes is possible but would give results difficult to analyze and, indeed, no analytical solutions. Our simplified approach, in contrast, has given very interesting results which are in excellent agreement with experimentally obtained data.

The calculations of the total amplitude noise, the amplitude noise in each mode, and the correlations between the noise in the two modes are based on the following set of rate equations for the photon densities of the P (primary) and S (secondary) mode and for the carrier density N inside the laser cavity:

$$\frac{dP}{dt}=v_g g_1 P-\frac{P}{\tau_{p1}}+F_P, \tag{1}$$

$$\frac{dS}{dt}=v_g g_2 S-\frac{S}{\tau_{p2}}+F_S, \tag{2}$$

$$\frac{dN}{dt}=\frac{I}{eV}-\frac{N}{\tau_e}-v_g(g_1P+g_2S)+F_N, \qquad (3)$$

where v_g is the group velocity, τ_{p1} and τ_{p2} the photon lifetime in each polarization mode, I the pumping current, τ_e the carrier lifetime, and F_P, F_S, and F_N the Langevin noise forces associated with the photon densities of the two polarization modes and with the carriers.

The gain of each polarization mode g_1 and g_2 is given by

$$g_1=g_d(N-N_0)-\beta_{11}P-\theta_{12}S, \qquad (4)$$

$$g_2=g_d(N-N_0)-\beta_{22}S-\theta_{21}P, \qquad (5)$$

where g_d is the differential gain, N_0 the transparency carrier density, β_{11} and β_{22} are the gain suppression coefficients of each polarization mode and θ_{12} and θ_{21} are the cross saturation coefficients, respectively. The gain suppression term β has already been found to strongly modify the laser's amplitude noise in the single-mode case [22] and, therefore, both self and cross saturation effects have to also be considered in our extended two-mode model.

The statistics of the different Langevin noise forces must be known to calculate the quantum noise properties of VCSELs. Vacuum fluctuations and pump noise are the major contributions in our model. The influence of gain suppression and spatial hole burning on the different Langevin noise forces statistics can also be considered [26,27]. The following statistics are obtained in a simple description (for further details see [13,14]):

$$D_{PP}=R_{sp}\overline{P}+\frac{\overline{P}}{\tau_{P1}}, \qquad (6)$$

$$D_{SS}=R_{sp}\overline{S}+\frac{\overline{S}}{\tau_{P2}}, \qquad (7)$$

$$D_{NP}=-R_{sp}\overline{P}, \qquad (8)$$

$$D_{NS}=-R_{sp}\overline{S}, \qquad (9)$$

$$D_{SP}=\frac{\sqrt{\overline{SP}}}{\tau_{P1}\tau_{P2}}, \qquad (10)$$

where $\langle F_X(t)F_Y(t')\rangle=2D_{XY}\delta(t-t')$ with $\{X,Y\}\in\{P,S,N\}$, $\overline{P}$, $\overline{S}$, and $\overline{N}$ are the steady-state solutions of Eqs. (1)–(3), and R_{sp} is the spontaneous emission rate coupled to the lasing mode.

The statistics of the Langevin noise forces associated with the carriers is directly influenced by the noise of the pumping current. We give here only the diffusion coefficients in the case of normal pumping (pump noise at the shot noise level) and quiet pumping (no pump noise):

$$D_{NN}=\frac{\overline{N}}{2\tau_e}+\frac{3}{4}R_{sp}(\overline{P}+\overline{S}) \quad \text{(quiet pumping)}, \qquad (11)$$

$$D_{NN}=\frac{\overline{N}}{\tau_e}+R_{sp}(\overline{P}+\overline{S}) \quad \text{(normal pumping)}. \qquad (12)$$

The spectra of the amplitude fluctuations of P and S, $\tilde{p}(\omega)$ and $\tilde{s}(\omega)$, respectively, are obtained after linearization around the steady-state solution and a subsequent Fourier transform analysis:

$$\tilde{p}(\omega)=\frac{\{[(\Gamma_s+i\omega)(\Gamma_N+i\omega)+v_g^2g_dg_2\overline{S}]F_P-v_g^2g_dg_2\overline{P}F_S+v_gg_d\overline{P}F_S\}}{\Delta(\omega)}, \qquad (13)$$

$$\tilde{s}(\omega)=\frac{[D(\omega)F_S-v_g^2g_dg_2\overline{S}F_P+v_gg_d\overline{S}(\Gamma_P+i\omega)F_N]}{\Delta(\omega)}, \qquad (14)$$

with the following definitions:

$$\Delta(\omega)=D(\omega)(\Gamma_s+i\omega)+v_g^2g_dg_2\overline{S}(\Gamma_s+i\omega), \qquad (15)$$

$$\Gamma_P=\frac{R_{sp}}{\overline{P}}+\beta_{11}\overline{P}, \qquad (16)$$

$$\Gamma_S=\frac{R_{sp}}{\overline{S}}+\beta_{22}\overline{S}, \qquad (17)$$

$$\Gamma_N=\frac{1}{\tau_e}+v_gg_d(\overline{P}+\overline{S}). \qquad (18)$$

The amplitude noise spectra for each mode are obtained from the photon number fluctuations directly by

$$S_P(\omega)=\frac{\langle\tilde{p}(\omega)\tilde{p}(\omega)^*\rangle}{4\overline{P}}, \qquad (19)$$

$$S_S(\omega)=\frac{\langle\tilde{s}(\omega)\tilde{s}(\omega)^*\rangle}{4\overline{S}}, \qquad (20)$$

where the symbol $\langle\ \rangle$ represents the average operation for the Langevin noise terms.

Using these definitions, analytical formulas are obtained for the amplitude noise for each polarization mode P and S, $S_p(\omega)$, and $S_s(\omega)$, respectively,

$$4\overline{P}|\Delta(\omega)|^2 S_P(\omega)=|(\Gamma_N+i\omega)(\Gamma_S+i\omega)+v_g^2 g_d g_2 \overline{S}|^2 2D_{PP}+v_g^4 g_d^2 g_1^2 \overline{P}^2 2D_{SS}+v_g^2 g_d^2 \overline{P}^2(\Gamma_S^2+\omega^2)2D_{NN}-2\Gamma_S v_g^3 g_d^2 g_1 \overline{P}^2 2D_{SN} \\ +2v_g g_d \overline{P}(v_g^2 g_d g_2 \overline{S}+\Gamma_N(\Gamma_S^2+\omega^2))2D_{PN}-2v_g^2 g_d g_1 \overline{P}(v_g^2 g_d g_2 \overline{S}+\Gamma_N\Gamma_S-\omega^2)2D_{SP} \tag{21}$$

and

$$4\overline{S}|\Delta(\omega)|^2 S_S(\omega)=v_g^4 g_d^2 g_2^2 \overline{S}^2 2D_{PP}+|D(\omega)|^2 2D_{SS}+v_g^2 g_d^2 \overline{S}^2(\Gamma_P^2+\omega^2)2D_{NN}-2v_g g_d \overline{S}\,\mathrm{Re}[D(\omega)(\Gamma_P-i\omega]2D_{SN} \\ -2\Gamma_{PP} v_g^3 g_d^2 g_2 \overline{S}^2 2D_{PN}-2v_g^2 g_d g_1\,\mathrm{Re}[D(\omega)]2D_{SP}. \tag{22}$$

The total amplitude noise $S_{S+P}(\omega)$, consisting of the two polarization modes, is also obtained in the same manner:

$$\begin{aligned}4\overline{S}|\Delta(\omega)|^2 S_{S+P}(\omega)\\ =&(\Gamma_S^2+\omega^2)(\Gamma_P^2+\omega^2)2D_{PP}+|D(\omega)-v_g^2 g_d g_2 \overline{S}|^2 2D_{SS}\\ &+|v_g g_d \overline{P}(\Gamma_S+i\omega)+v_g g_d \overline{S}(\Gamma_P+i\omega)|2D_{NN}\\ &-2v_g g_d \overline{S}\,\mathrm{Re}[(D(\omega)^*-v_g^2 g_d g_1 \overline{P})(v_g g_d \overline{P}(\Gamma_S+i\omega)\\ &+v_g g_d \overline{S}(\Gamma_P+i\omega))]2D_{SN}-2\,\mathrm{Re}[(v_g g_d \overline{P}(\Gamma_S+i\omega)\\ &+v_g g_d \overline{S}(\Gamma_P+i\omega))(\Gamma_S-i\omega)(\Gamma_P-i\omega)]2D_{PN}\\ &+2\,\mathrm{Re}[(\Gamma_S+i\omega)(\Gamma_P+i\omega)\\ &\times(D(\omega)^*-v_g g_d g_1 \overline{P})]2D_{SP}.\end{aligned} \tag{23}$$

The spectrum of the correlation $\mathcal{C}(\omega)$ between the amplitude fluctuations of the two mode defined as $\langle\tilde{p}(\omega)\tilde{s}^*(\omega)\rangle$ is then given by

$$\begin{aligned}|\Delta(\omega)|^2\mathcal{C}(\omega)=&-v_g^2 g_d g_2 \overline{S}(v_g^2 g_d g_2 \overline{S}+\Gamma_S\Gamma_N-\omega^2)2D_{PP}\\ &-v_g^2 g_d g_1 \overline{P}2\,\mathrm{Re}[D(\omega)]2D_{SS}\\ &+\{2v_g^4 g_d^2 g_2 g_1 \overline{SP}+2\,\mathrm{Re}(D(\omega)[(\Gamma_S+i\omega)\\ &\times(\Gamma_P+i\omega)+v_g^2 g_d g_2 \overline{S}])\}2D_{SP}.\end{aligned} \tag{24}$$

It is important to note that only the external noise spectra are measurable quantities and not the noise spectra corresponding to the internal field. Internal and external noise spectra, as well as external and internal normalized correlation spectra, are completely different as has been already demonstrated [15,26]. The external noise spectra are obtained through the boundary conditions for the field at the laser facets. The amplitude fluctuations of the emitted polarization modes P_{ext} and S_{ext} can be approximated by the following equations with the reflection coefficient of the VCSELs upper facet equal to R:

$$\delta A_{P_{ext}}=\sqrt{\frac{(1-R)}{\tau}}\frac{\tilde{p}}{2\sqrt{\overline{P}}}-\sqrt{R}f_{vac}, \tag{25}$$

$$\delta A_{S_{ext}}=\sqrt{\frac{(1-R)}{\tau}}\frac{\tilde{s}}{2\sqrt{\overline{S}}}-\sqrt{R}f_{vac}, \tag{26}$$

where $A_{P_{ext}}$ and $A_{S_{ext}}$ are the amplitudes of the emitted fields P_{ext} and S_{ext}, τ the light round-trip time in the cavity, and f_{vac} the vacuum fluctuations.

The noise spectra of the emitted P and S modes are obtained using the same methods as for the internal amplitude noise spectra [Eqs. (21) and (22)] and are given by

$$S_{P_{ext}}(\omega)=\frac{(1-R)}{\tau}S_P(\omega)+\frac{R}{4}-\frac{R\sqrt{1-R}}{4\tau}\left[2\,\mathrm{Re}\left(\frac{v_g^2 g_d g_2 \overline{S}-D(\omega)}{\Delta(\omega)}\right)\right], \tag{27}$$

$$S_{S_{ext}}(\omega)=\frac{(1-R)}{\tau}S_S(\omega)+\frac{R}{4}-\frac{R\sqrt{1-R}}{4\tau}\left[2\,\mathrm{Re}\left(\frac{v_g^2 g_d(g_2\overline{S}-g_1\overline{P})+(\Gamma_S+i\omega)(\Gamma_N+i\omega)}{\Delta(\omega)}\right)\right]. \tag{28}$$

The amplitude noise of the total emitted field, as well as the correlation between the amplitude fluctuations of the two modes, can be written for these conditions in a simple way:

$$S_{(S+P)_{ext}}(\omega)=\frac{(1-R)}{\tau}S_{S+P}(\omega)+\frac{R}{4}-\frac{R\sqrt{1-R}}{4\tau}\left[2\,\mathrm{Re}\left(\frac{(\Gamma_S+i\omega)(\Gamma_N+i\omega)+D(\omega)-v_g^2 g_d g_1\overline{P}}{\Delta(\omega)}\right)\right] \tag{29}$$

and

$$\mathcal{C}_{ext}(\omega)=\frac{(1-R)}{4\tau\sqrt{\overline{PS}}}\mathcal{C}_{int}(\omega)-\frac{R\sqrt{1-R}}{4\tau}\left[2\,\mathrm{Re}\left(\frac{D(\omega)-v_g^2 g_d g_2\overline{S}}{\Delta(\omega)}\right)\right]$$
$$-\frac{R\sqrt{1-R}}{4\tau}\left[2\,\mathrm{Re}\left(\frac{v_g^2 g_d(g_2\overline{S}-g_1\overline{P})+(\Gamma_S+i\omega)(\Gamma_N+i\omega)}{\Delta(\omega)}\right]+\frac{R}{4}. \tag{30}$$

These analytical formulas can be used to investigate the quantum noise properties of VCSELs and to determine the influence of the polarization properties on the emitted noise, as well as the correlation between the amplitude fluctuations of the two polarization modes.

III. MODELING RESULTS

A. Influence of pump rate and polarization mode ratio on the amplitude noise

In this section we study the influence of the existence of two orthogonal polarization modes S and P on the quantum noise properties of VCSELs. The main result of our calculations is that the parameter M, defined as the ratio $\overline{S}/\overline{P}$ of the power in the S mode by the power in the P mode can be considered an important parameter for the noise of VCSELs [30]. The values of all parameters used in the simulation are given in Table I.

For most practical applications, the amplitude of the total emitted field or its intensity fluctuations are the most interesting values. Figure 2 shows the amplitude noise of the total emitted power at low frequency ($\omega=0$) [31] normalized by the shot noise as a function of M for different values of the total photon number inside the laser cavity. The influence of M depends strongly on the photon number; no dependence on M is noticeable close to threshold ($P_0=10^3$ or $R_P=0.015$) and the noise is far above the shot-noise level. In this case, spontaneous emission is dominant, which is totally unpolarized. On the contrary, the parameter M has a strong impact on the total emitted amplitude noise high above threshold. At any high pump rate, a pronounced maximum for the amplitude noise appears for intermediate values for M. This maximum noise is far above the shot noise level. This maximum is governed by the counterinfluence of mode partition noise and of the correlations that exist between the fluctuations of the two polarization modes. For M close to 0, mode partition noise strongly increases with M, inducing a direct increase of the total amplitude noise. On the contrary, for M close to 1, a strong anticorrelation between the noise in the two modes builds up which, with further increasing M, leads to a decrease of the total amplitude noise. Therefore, a maximum of the total amplitude noise as a function of M for intermediate values of M is quite obvious.

We have to emphasize how the results of Fig. 2 have to be read. Not all data points in Fig. 2 can be identified with real world lasers, which means that arbitrary dependences within the noise vs M phase space may not be accessible because they are irreal. This is the case, e.g., for the ideal single-mode laser ($M=10^{-6}$) at threshold. However, if one knows for one particular laser its pumping rate and the appropriate M value, then Fig. 2 gives the respective emitted amplitude noise. It must also be noted that for a given VCSEL, knowing all its material parameters and its structure, the parameter M is a well-defined function of the pump level R_P. The parameter M can be influenced e.g., by changing the laser material parameters through stress. This can be mechanical stress or thermal stress, such as in the so-called hot-spot technique [32]. In the calculations shown in this section, the parameter M has been considered independent from R_P to be able to study only its influence for different pumping currents or total photon densities. Being aware of these restrictions, it is easy to recognize in Fig. 2 that there exist two

TABLE I. Definition and values of several parameters used in the simulation.

Parameters (units)	Symbol	Value
Diameter of the laser cavity (μm)	D	7 and 12
Length of the VCSEL active zone (μm)	L	$3\lambda/2$
Differential gain (cm^2)	g_d	1.5×10^{-16}
Carrier lifetime (ns)	τ_e	1
Spontaneous emission rate (1/s)	R_{sp}	1.35×10^{12}
Gain suppression coefficient (cm^2)	β	16×10^{-20} to 16×10^{-17}
Cross gain suppression coefficient (cm^2)	θ	16×10^{-20} to 16×10^{-17}
Photon lifetime (ps)	τ_p	2
Volume (cm^3)	V	4.0×10^{-11}

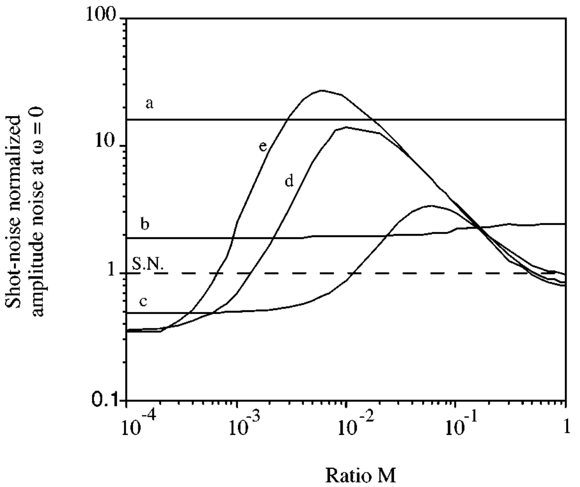

FIG. 2. Shot noise normalized total emitted amplitude noise at low frequency ($\omega=0$) as a function of M for different total photon numbers in the cavity $P_0=10^3$ (a), 10^4 (b), 10^5 (c), 5×10^5 (d), and 10^6 (e).

regimes where the emission of amplitude squeezed light is possible.

The first domain where squeezing is possible is for M close to 1. This case corresponds to the first published experimental results on amplitude noise squeezing with VCSELs [19]. The high anticorrelation and the equal noise in the two modes associated with the equality of power in the two polarization modes lead to the generation of squeezed states of light. The amount of squeezing that can be obtained is rather small. The parameter M has to be very close to 1 and the correlation close to -1, as well in order to realize the generation of squeezed states as already mentioned before.

The other domain is for M close to 0. In this case, the laser is nearly single mode and for small enough M the secondary mode cannot contribute to the noise because of its strongly reduced intensity. The decrease of the noise with increasing pumping level corresponds to the usual decrease of a single-mode laser and, under these conditions, a VCSEL can be considered a single-polarization-mode laser. We conclude from our model that better squeezing performances are obtained for M close to 0 than for M close to 1. At the same time, the range of M where squeezing occurs is also wider by far.

The same results are depicted in a different representation in Fig. 3, where the dependence of the normalized noise on the pump rate is plotted for different values of M. We find for extreme values of M ($M=0.0001$ and $M=1$), i.e., both for the single-mode and the two-mode laser, a continuous decrease of the noise level with increasing R_P. As already mentioned above, the achievable amount of squeezing, as well as the range of M for which squeezing is possible, is smaller for the two-mode than for the single-mode laser, the pump rate being kept constant. For intermediate values of M, a minimum of the noise appears at values of R_P between 0.1 and 1 which, depending on the exact value of M, can either be above or slightly below the shot-noise level, expressing once more again the delicate counterbalance between individual mode noise, correlation, and total noise. However, this noise minimum is of no practical relevance, because these intermediate values of M are normally difficult to be realized.

As already mentioned during the discussion of Fig. 2, the noise of a two-polarization VCSEL is strongly determined by the correlation between the modes, which again is strongly dependent on M. Figure 4 presents the normalized correlation at low frequencies ($\omega=0$) as a function of M for four different photon numbers inside in the cavity. All curves show a similar dependence of the normalized correlation on the parameter M. A normalized correlation close to -1 is obtained when the power is equally split between the S and P modes ($M=1$). For M close to 0, the normalized correlation increases and becomes, in most cases, positive. Only for these two extreme conditions ($M\approx0$ and $M\approx1$) the generation of amplitude squeezed states of light is possible. The strong negative value of the correlation for a wide range of M is directly linked to the strong coupling of the two photon distributions through the carrier system. Values close to 0 or positive values of the correlation can only be obtained when the secondary mode can be neglected.

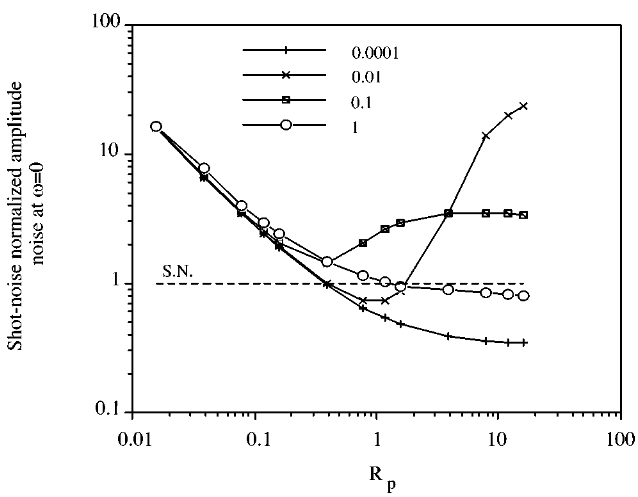

FIG. 3. Amplitude noise normalized by the shot noise at low frequency ($\omega=0$) as a function of the pump rate for various values of M.

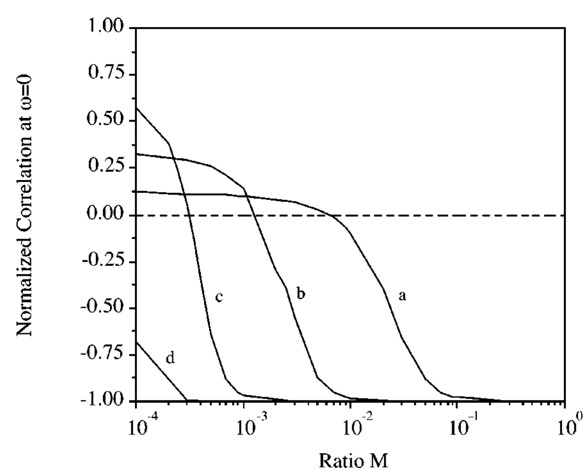

FIG. 4. Normalized correlation between the amplitude fluctuations of the two polarization modes P and S at low frequency ($\omega=0$) as a function of M for several photon numbers inside the cavity $P_0=10^3$ (a), 10^4 (b), 10^5 (c), and 10^6 (d).

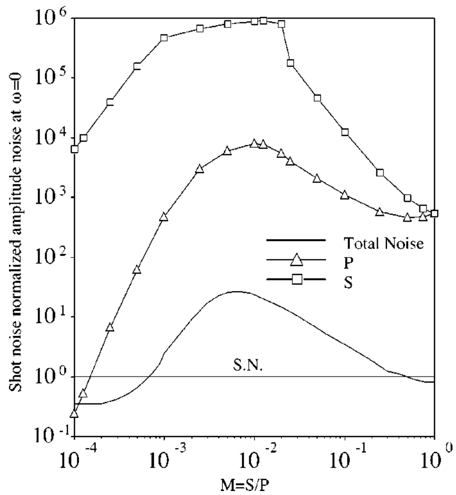

FIG. 5. Shot-noise normalized amplitude noise of the two polarization modes P and S and the total amplitude noise at low frequency $(\omega=0)$ as a function of M for $P_0=10^6$.

The strong influence of M on the noise in each single mode is clearly illustrated in Fig. 5 where the amplitude noise at low frequency $(\omega=0)$ of the individual P and S modes as well as the total amplitude noise, all of them normalized to the shot noise, are presented as a function of M for a photon number $P_0=10^6$ inside the cavity, corresponding to a pump rate $R_P=15.7$. The amplitude noise values at low frequency for both S and P modes are strongly influenced by the ratio M. The weaker secondary S mode shows a weaker dependence on M than the stronger primary P mode, which is the only one to exhibit squeezing at all. The amplitude noise of the P mode normalized to the shot-noise level is always smaller than that of the S mode.

The polarization modes in a VCSEL are different compared to the longitudinal modes in an edge emitting laser because a conventional edge emitter may emit many longitudinal modes that are more or less suppressed according to the laser structure. The fluctuations of the different longitudinal sidemodes are strongly anticorrelated with respect to the main mode and these side modes have a strong influence on the laser noise if the gain of the laser is homogeneously broadened [33,34]. Squeezing may be obtained benefiting from this strong anticorrelation but in a reduced amount. The only possibility of generating strongly squeezed states is to have a laser as purely single mode as possible, i.e., to have its side mode suppression ratio as high as possible. This has been successfully performed with different techniques using an external cavity [35], dispersive feedback [36], or injection locking [37,38]. The VCSELs have the great advantage of having two regimes where squeezing may be obtained and, moreover, they have the potential to reach a superior performance than edge emitters. They also show the possibility of single-mode squeezed light which is of strong interest for several application domains, such as interferometry, spectroscopy, as well as optical communications.

B. Influence of gain saturation and carrier lifetime on the amplitude noise

Many parameters of a laser structure have an influence on its noise and squeezing properties as already demonstrated for edge emitting lasers [15,26,27]. An important parameter is the efficiency of the laser which depends on the losses inside the cavity and which can even be limited by nonlinear saturation effects. We have therefore studied the influence of the laser's internal losses on the VCSELs quantum noise via the photon lifetime in the cavity and the gain saturation effect considered via the gain suppression coefficients.

Two total photon numbers in the cavity are considered, $P_0=10^5$ and $P_0=10^6$, respectively. These values represent two interesting cases for the laser. The first one corresponds to a laser close to threshold and the second one to a laser high above its threshold. In a single-mode laser, the influence of the gain suppression has been already proven to be different close to and far above the laser threshold.

Let us first concentrate on the influence of gain suppression. Gain suppression or any phenomenon inducing a saturation of the emitted power by the laser limits the amount of amplitude squeezing that can be obtained with a given laser structure [15,26]. We assume here for the sake of simplicity that all the nonlinear parameters (β_{11}, β_{22}, θ_{12}, and θ_{21}) are equal to a constant value β.

Figure 6 presents the total amplitude noise at low frequency $(\omega=0)$ normalized by the shot-noise level as a function of M for two photon numbers inside the cavity $P_0=10^5$ (top) and 10^6 (bottom) with four values of the gain suppression coefficient β as parameter. As expected, the total amplitude noise is influenced by the amount of gain suppression and a quantitatively different behavior is found, according to the polarization properties. The shape of the different curves plotted for the two pump rates, in general, is qualitatively the same but the noise level and the position of the noise maximum are different. The normalized noise changes very slightly with increasing gain suppression for low values $(M\approx 0)$ and high values $(M\approx 1)$ of M. Gain suppression mostly influences the position and value of the maximum of the noise for intermediate values of M. In addition, the range of M for which the laser generates squeezed light, decreases with increasing gain suppression coefficient β, and the range of M within which squeezing may occur is far wider for $P_0=10^5$ than for 10^6. This behavior has been also observed for a single-mode-laser model beyond the optimum biasing point of the laser [22]. Therefore, we conclude that, similar to a single-mode laser, a gain suppression as small as possible is an important and necessary condition to generate highly squeezed light with a VCSEL.

Another important parameter of the laser is the photon lifetime τ_P. It is determined by the value of the internal loss, the facets' reflectivities being held constant. In a single-mode description [2,15], internal loss sets a limitation to the achievable amount of squeezing high above the laser threshold due to the reduction of the efficiency of the laser. Figure 7 presents the total amplitude noise normalized by the shot noise at low frequency $(\omega=0)$ as a function of M for two values of the photon number inside the cavity $P_0=10^5$ (top) and 10^6 (bottom), and in each case for four different values of the photon lifetime. The shape of the different curves presented here is the same. However, a strong difference in the amount of squeezing achievable for low values of M as well as in the sensitivity against changes in the photon lifetime, appears between the two pump rates. The range of M for which squeezing is obtained is not much modified by

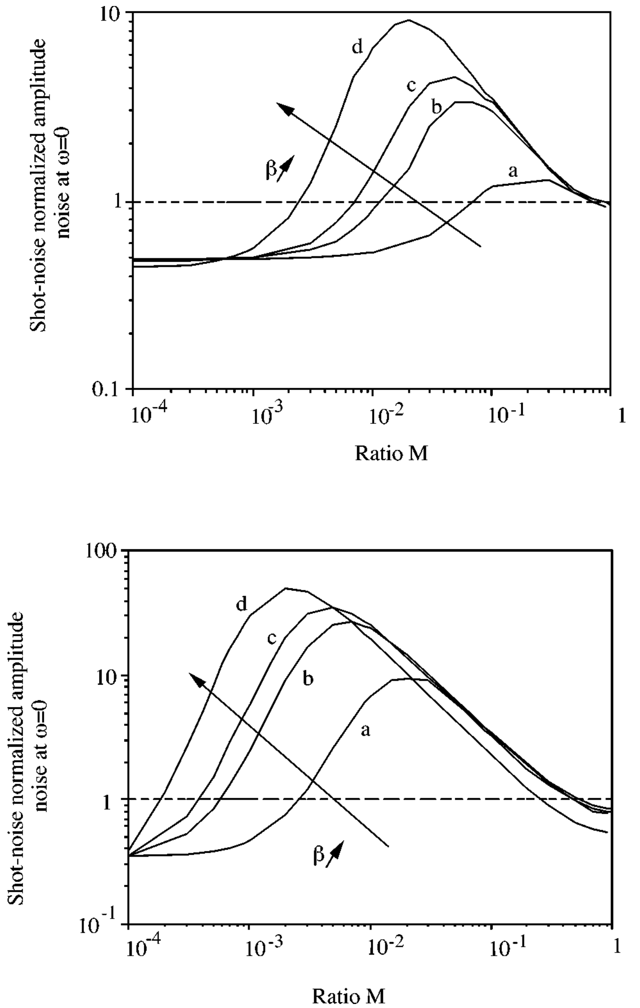

FIG. 6. Shot-noise normalized amplitude noise at low frequency $(\omega=0)$ as a function of M for different gain suppression coefficients $\beta=5\times10^3$ (a), 5×10^4 (b), 10^5 (c), and $5\times10^5\ \mathrm{cm}^{-1}$ (d) for two photon numbers inside the cavity $P_0=10^5$ (top) and 10^6 (bottom).

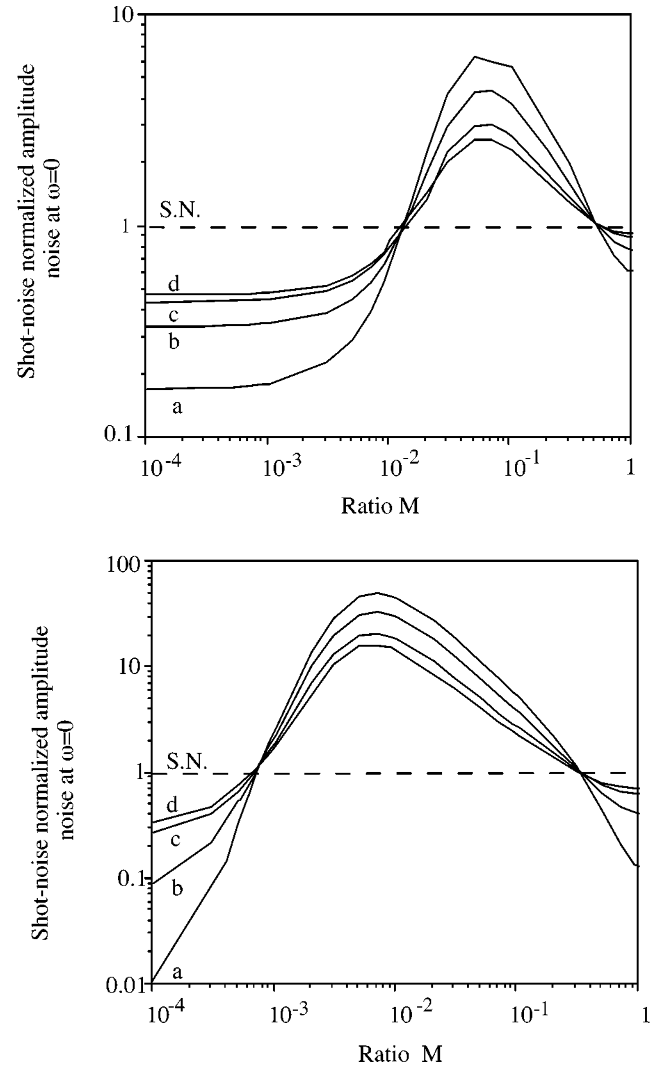

FIG. 7. Shot-noise normalized amplitude noise at low frequency $(\omega=0)$ as a function of M for different photon lifetime $\tau_p=5\times10^{-12}$ (a), 10^{-12} (b), 5×10^{-13} (c), and 10^{-13} s (d) for two photon numbers inside the cavity $P_0=10^5$ (top) and 10^6 (bottom).

changing τ_P, but the maximum achievable squeezing that can be obtained is reduced with increasing internal loss. In addition, the maximum noise (above the shot-noise level) for intermediate values of M is strongly reduced, which is the normal influence of losses on a ''super shot-noise'' laser noise. Finally, it is interesting to note, that the position of the noise maximum in dependence of M is not changing varying the photon lifetime, which is in contrast to the effect that has been observed for variation of the nonlinear gain saturation in the laser as shown in Fig. 6.

According to the theoretical results presented in the last two subsections, we have subsequently performed various experimental investigations of the quantum noise of two different types of VCSEL structures. We have especially studied the influence of the polarization properties of these lasers on their noise with particular emphasis on the conditions under which squeezing may be obtained.

IV. EXPERIMENTAL RESULTS

We have investigated two different types of lasers corresponding to the two classes of lasers that are interesting according to our theoretical results, i.e., the two-polarization-mode VCSEL $(M\approx1)$ and the single-polarization-mode VCSEL $(M\approx0)$. Figure 8 depicts our experimental setup for investigations of the quantum noise and the polarization properties of our two-polarization-mode VCSELs.

The two-polarization-mode VCSELs have been realized by large aperture air-post VCSELs and have been produced

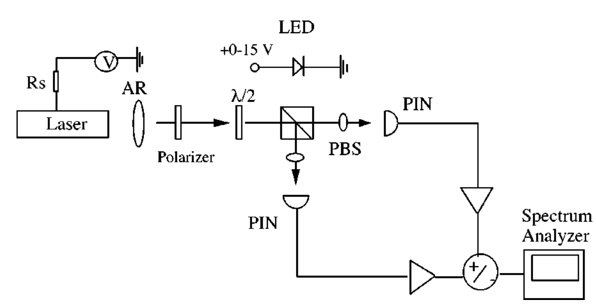

FIG. 8. Experimental setup for the measurement of the VCSEL amplitude noise.

in the former Paul Scherrer Institute in Zurich (now CSEM). They have a threshold current of 3–6 mA, a maximum emitted power of 2 mW, and very interesting polarization properties [39]. Their active zone consists of three quantum wells that are sandwiched between two Bragg reflectors. They emit in the wavelength range 770–785 nm. A balanced detection scheme is used to measure the amplitude noise of the VCSEL. A high-efficiency light-emitting diode whose light is injected into the fourth port of the polarizing beam splitter is used to calibrate the shot-noise level. In every branch, a large diameter photodiode (EG&G FND100) is used, which is directly connected to a Bias-Tee (PPL 5555), where the ac and dc parts of the photocurrent are separated. The two ac photocurrents, after being amplified through amplifiers (MITEQ AM-3A), are then added via a magic tee. An electrical spectrum analyzer measures the amplitude noise spectra. A Glan-Thompson polarizer in front of the VCSEL upper facet is used to separate the two polarization modes. The two polarization modes are obtained by finding the maxima of the photocurrent as a function of the position of the polarizer. This method enables us also to measure the noise spectra of the two polarization modes and then to calculate the correlation between their amplitude fluctuations. The noise spectra of the total emitted light is obtained without the polarizer.

The normalized correlation between the amplitude fluctuations of the two polarization modes is obtained after a measurement of the noise of the total emitted field, as well as the noise of each polarization mode. The normalized correlation is then calculated using the following equation:

$$\mathcal{C}(\omega)=\frac{S_{S+P}(\omega)-S_P(\omega)-S_S(\omega)}{2\sqrt{S_P(\omega)}\sqrt{S_S(\omega)}}, \tag{31}$$

where $S_P(\omega)$ and $S_S(\omega)$ are the noise of the S and P modes and $S_{S+P}(\omega)$ is the amplitude noise of the total field.

The first two interesting results of our measurement are the noise emitted by the laser and the ratio M as a function of the injection current. Figure 9 presents these experimental results for two VCSELs having different aperture diameters of 12 μm (top) and 7 μm (bottom), respectively. First, the parameter M is highly dependent on the pumping current. Second, a strong dependence of the shot-noise normalized amplitude noise on the pumping current is found as well. Low values of the amplitude noise only occur when the factor M is small enough. The most striking result is that the amplitude noise at low frequency seems to directly follow the evolution of the factor M with the pumping current. Such a strong link can be expected according to our theory.

Figure 10 shows the spectrum of the normalized correlation $\mathcal{C}(\omega)$ between the amplitude fluctuations of the two polarization modes, depicted on a logarithmic scale for $\mathcal{C}+1$ (left scale) and corresponding $\mathcal{C}(\omega)$ correlation (right scale) as a function of frequency for a pumping current close to threshold (5 mA), and for a pumping current high above threshold (15 mA).

An anticorrelation between the two modes always exists. Close to threshold, a moderate anticorrelation is found which is between -0.7 and -0.9. This is still due to the strong impact of spontaneous emission. On the other hand, high above threshold a very strong anticorrelation stronger than

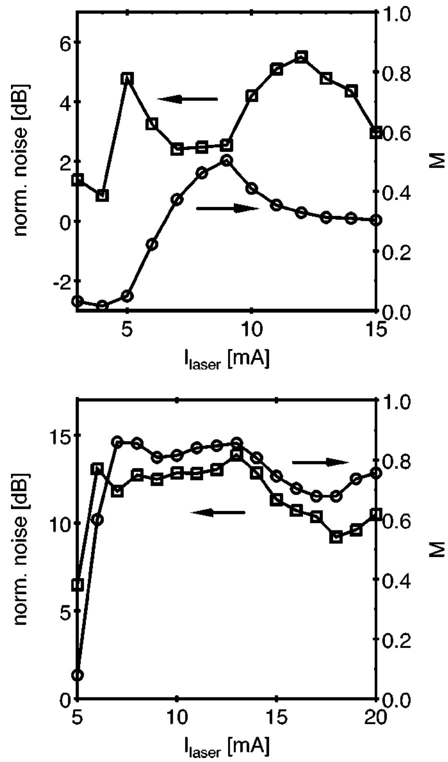

FIG. 9. Shot-noise normalized amplitude noise at low frequency (left scale) and ratio M (right scale) as a function of the pumping current for a VCSEL of 12 μm diameter (top) and 7 μm diameter (bottom).

-0.99 is found for all measured frequencies and even reaches for some frequencies the value of -0.999. This strong anticorrelation may enable the generation of squeezed states by the cancellation of the noise of the individual polarization modes.

In the following we will study this problem in more detail and investigate the relation between the laser structure and the correlation as a function of the applied pumping current. Figure 11 presents the measured normalized correlation for a measurement frequency of 600 MHz and the parameter M as functions of the injection current for a 12 μm (top) and 7 μm (bottom) diameter VCSEL. The bandwidth of the detector

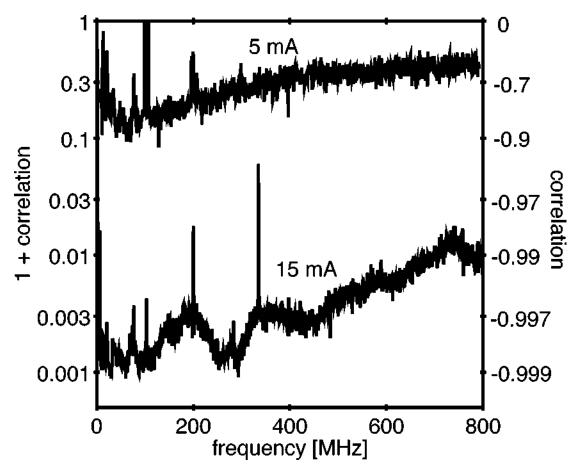

FIG. 10. Normalized correlation as a function of frequency for a 12 μm VCSEL for a pumping current of 5 mA and 15 mA.

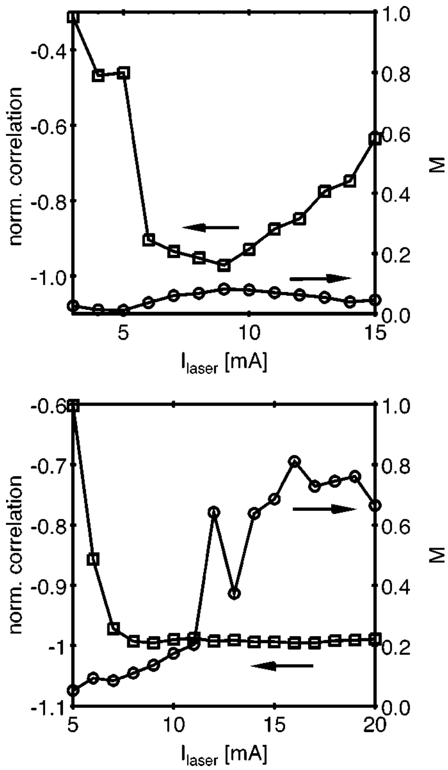

FIG. 11. Normalized correlation at low frequency (left scale) and parameter M (right scale) as a function of the pumping current for a VCSEL of 12 μm (top) and 7 μm diameter (bottom).

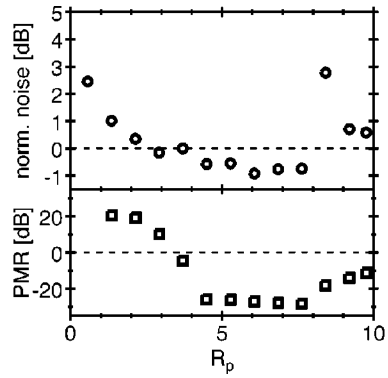

FIG. 12. Shot-noise normalized amplitude noise (top) and polarization mode ratio $\mathrm{PMR}=10\log_{10}(M)$ (bottom) as a function of the pumping level for a single-polarization mode VCSEL.

used in these measurements is 800 MHz. The normalized correlation is strongly dependent on the pumping current and roughly shows an inverse dependent behavior of the parameter M, in comparison to that of the correlation. This directly corresponds to the theoretical results presented in Fig. 4. The strong anticorrelation (lower picture) in principle should be sufficient enough to result in a strong reduction of the intensity noise of the total emitted field but the power is unfortunately not equally distributed among the two polarization modes, i.e., $M\neq 1$.

The first published demonstration of squeezing with a VCSEL has been with a two-polarization-mode VCSEL ($M\approx 1$). The obtained anticorrelation, as well as the total symmetry of the noise, enables the emission of amplitude squeezed light. Our theory has shown that the VCSEL must have M really very close to 1 to be able to reach squeezing, and this induces strong conditions on the laser used. On the contrary, the conditions on a single-polarization-mode VCSEL ($M\approx 0$) are not so strict. Therefore, we have decided to perform experiments with a single-polarization-mode VCSEL. The single-polarization-mode VCSELs have been produced by Professor Ebeling's group in the University of Ulm. These VCSELs have very good static performances as, for example, a very low threshold current (500 μA), a high quantum efficiency (0.95), and a high single-mode power level (typical 3 mW, maximum 4.8 mW) [40].

The lasers have been directly measured on wafer and the noise is measured using the direct detection technique. A large surface photodiode (EG&G FFD 200) was placed in front of the laser and connected through a bias tee (PPL 5590) to a MITEQ amplifier (AU-1332). The same polarizer is used as in the first experiment. Figure 12 presents our experimental results. The upper part shows the measured intensity noise normalized by the shot noise at a frequency of 63 MHz and the lower part shows the polarization mode ratio $\mathrm{PMR}=10\log_{10}(M)$ both as a function of the pump level R_P of the laser. A similar dependence of the shot-noise normalized amplitude noise was obtained on a wide range of frequency only limited by the bandwidth of our detector (150 MHz). The thermal noise of the system has been subtracted from the experimental data.

Starting from threshold, the PMR decreases and changes its sign at $R_p = 3.5$ because a polarization flip takes place. Subsequently, it decreases more and reaches its minimum value close to -20 dB for $R_p\approx 6$. However, with further increasing pump rate, the second polarization mode grows up. This is accompanied by an increase of the total laser noise because the noise contribution of the second polarization mode becomes relevant for the total noise.

All of these polarization features also express themselves in the dependence of the noise on the pumping level. For low values, the shot-noise normalized amplitude noise decreases and reaches its minimum at the same pumping rate for which the PMR reaches its minimum, too. A level of squeezing of 0.9 dB is obtained for a pumping level R_P equal to 6. This achieved level of squeezing corresponds to a level of 1.3 dB squeezing at the VCSEL upper facet when correcting for the detection efficiency of 0.82. With further increasing R_P first the noise starts to increase. This is directly linked to the increase of the PMR. With the PMR approaching 0 dB (two-mode laser case), the existing anticorrelation between the amplitude fluctuations of the two polarization modes balances the noise contributions and finally even a new decrease in the noise is observed ($R_p\approx 10$).

These results are in total agreement with our theoretical calculations and confirm the direct link between the ratio M and the quantum noise behavior of a VCSEL. In order to confirm this more quantitatively, we have calculated the dependence of M and the amplitude noise on the pump rate for a typical VCSEL in order to compare these results to the experimental results from Fig. 12. The upper part of Fig. 13 presents for a ''model'' VCSEL with particularly chosen parameters, the factor M as a function of the pumping current. A polarization flip to the orthogonal mode occurs close to threshold. Then M reaches its maximum close to 20 dB be-

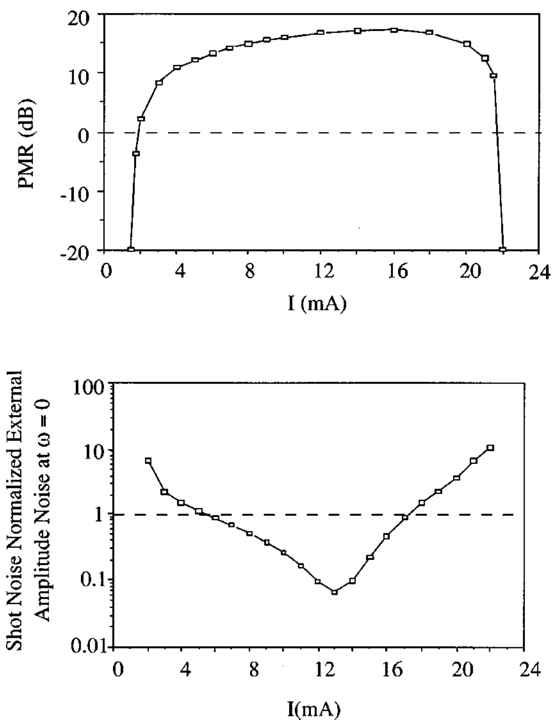

FIG. 13. Calculated dependence of the M parameter (top) and the shot-noise normalized amplitude noise at low frequency (bottom) on the pumping current I for a typical model VCSEL.

fore strongly decreasing with the occurence of a secondary polarization flip high above the laser threshold. The lower part of Fig. 13 presents, for the same conditions, the total amplitude noise normalized by the shot-noise level at low frequency also as a function of the pumping current. The shot-noise normalized amplitude noise decreases as long as M increases and reaches the noise minimum (with the occurence of squeezing), where M exhibits a maximum plateau. Then, the noise increases with decreasing M. At first sight this increase in the laser noise may seem unexpected because the absolute value of the PMR is in the range of 20 dB for all injection currents (except for the range, where the polarization flip occurs). However, the model already included saturation effects which may even lead to a rollover characteristic of the VCSEL, i.e., for high injection currents the slope efficiency is already decreasing and may even become negative. This strongly reduced efficiency then leads to the pronounced reincrease in the laser noise for $I \geqslant 13$ mA. The features of these calculations correspond very well to the obtained experimental results. However, a more precise comparison of experimental data with the theory needs the exact knowledge of all laser parameters, which are not always at our disposal. In conclusion, our performed modeling of the VCSEL polarization and noise properties is a good tool to estimate the quantum noise and squeezing properties even if a better knowledge of the VCSELs parameters is needed.

V. SUMMARY

We have studied the quantum noise as well as the squeezing performances of VCSELs. They theoretically possess a great potential for the generation of amplitude squeezed states of light. However, the polarization properties of these lasers have to be taken into account, as well as their technological structure, to determine their quantum noise properties. The competition between polarization modes induces additional noise and consequently limits the performances of VCSELs as squeezed states generators. The most important, but not sufficient parameter, is the ratio M between the power in the S mode and the power in the P mode. The optimum conditions for squeezing are either realized with a ratio M close to 1, where the strong anticorrelation between the modes plays the dominant role, or with a ratio M close to 0, where the VCSEL is the closest to a single-mode laser. We have obtained experimental results in good agreement with our theoretical predictions showing a strong influence of M on the quantum noise of the investigated VCSELs. Strong noise reduction was obtained with the two-polarization-mode VCSELs ($M \approx 1$) but not the generation of amplitude squeezed states of light. With a single-polarization VCSEL ($M \approx 0$), a measured degree of squeezing of 0.9 dB was obtained (estimated of 1.3 dB at the laser facet), the first demonstration of amplitude noise squeezing with a single lateral and polarization mode VCSEL. These kinds of nonclassical single-mode states of light are of great interest for many applications, such as spectroscopy, interferometry, and optical communications.

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