Chaotic nature of the stored current in an electron storage ring detected by a two-photon correlator for soft-x-ray synchrotron radiation

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A two-photon correlator has been constructed and successfully operated in measuring the true two-photon correlation of synchrotron radiation in the soft-x-ray region. The strong influence of the nonstationary features of the optical field, which is the so-called accidental correlation induced by the systematic bunched structure of the stored current in a storage ring, has been removed by a coherence time modulation technique. The explicit bunching effect of the measured two-photon correlation shows that synchrotron radiation is a chaotic rather than coherent radiation, indicating a chaotic nature of the stored current. This experimental method provides a way to measure the instantaneous emittance with the time scale of the coherence time τ_c, and to characterize the coherence property of incomplete free-electron lasers, such as self-amplified spontaneous emission. [S1050-2947(99)04710-1]

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I. INTRODUCTION

A classical prescribed electric current distribution always produces an electromagnetic field in a pure coherent state. This was first shown theoretically by Glauber in 1963 [1–3]. According to the quantum theory of optical coherence, such a radiation field is coherent to all higher orders [1,2,4]. Synchrotron radiation is spontaneous radiation emitted by a stored electric current, which is formed by a large amount of free electrons with almost the speed of light in a storage ring and has no thermal resistance as a conventional electric current [5]. Classical recoil effects due to the radiation loss per electron are negligible in the present high-energy storage ring. Therefore, for such a classical current, in which the quantum recoil effect is negligible in the process of radiation with a photon energy much smaller than the critical photon energy, it is worthwhile to measure the two-photon correlation for synchrotron radiation to check whether it is a totally coherent radiation or a chaotic radiation or any other exotic one.

Since the success of the measurement of two-photon correlation by Hanbury-Brown and Twiss (HBT) in 1956 [6], the behavior of the second-order coherence has been believed to be a good fingerprint to check whether light is in a coherent state or in an incoherent state, because it strongly reflects the photon statistics of the light [7,8]. The essential difference of photon statistics among three typical kinds of light, that is, a thermal, a coherent, and a photon-number squeezed light, induces the second-order coherence to behave as the bunching, the flat-response, and the antibunching effects, respectively.

It should be noted that there exist some other methods, as were reviewed by a recent paper [9], for the characterization of photon statistics. A simpler one is the photon-counting technique with a single photodetector, which has been long known and widely utilized. With this method, a negative-binomial distribution of the visible photons emitted from wiggler radiation has been observed [10]. However, when we go to the soft-x-ray region, where the degree-of-freedom parameter is much larger than unity, this method would exhibit some difficulty in distinguishing a thermal photon from a coherent photon, because in this case the negative-binomial distribution indeed turns into the Poisson distribution [4,11]. This is in fact the case in which the response time of the detector is much longer than the coherence time.

The ordinary interference experiments are related to the first-order coherence. By using the Young double-slit method, the first-order coherence of soft-x-ray synchrotron radiation has been measured at the Photon Factory of KEK by some of the authors [12]. As it is well known, because any optical field with single-mode excitation is first-order coherent, the first-order coherence cannot be utilized to distinguish the statistical property of light sources.

So far only very few experiments have been reported about the measurement of two-photon correlation in the x-ray region [13]. Synchrotron radiation is nonstationary, where the systematic temporal evolution of its density operator due to the bunched structure will contribute to a large amount of accidental correlation in the measurement of two-photon correlation by using a conventional correlator [7]. Therefore, the cancellation of this trivial correlation becomes a key in order to be able to get the true two-photon correlation for synchrotron radiation. Figure 1 shows how the accidental correlation is created due to the bunched structure. The point is that the resolving time T_R of the two photodetectors is much longer than the coherence time τ_c of a single emitted photon, with the latter determined by the resolution of the monochromator included in the measurement system. Obviously under this situation when one detector detects a photon, the other detector has more chances to detect a photon due to the bunched structure, causing the huge amount of accidental correlation. This accidental correlation is a positive correlation. It has nothing to do with the inherent photon statistics of the light source and will give a flat response just as a coherent light behaves. However, the strong background

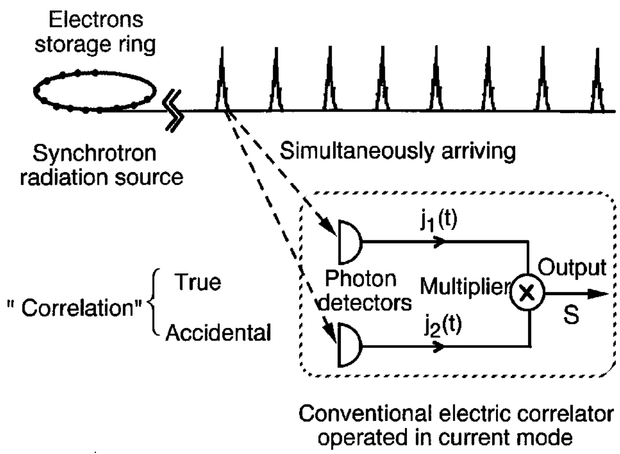

FIG. 1. Illustrating the creation of accidental correlation. $j_1(t)$ and $j_2(t)$ are the induced photoelectric currents and S is the correlation output completed by a linear multiplier.

formed by this accidental correlation would severely prevent us from observing the true two-photon bunching effect [15].

When a light source is stationary, such a difficulty does not arise in principle. Practically, however, even under this situation it is not a simple procedure to measure two-photon correlation for spontaneous radiation. The success of Hanbury-Brown and Twiss was helped by their very elaborate electronics to suppress the background noise [6,14].

To eliminate the accidental correlation, we first observe the following relation to express the observed two-photon correlation in Fig. 1 [16,17]:

$$S \propto \langle I_1 \rangle \langle I_2 \rangle \left(\tilde{A} + \kappa \frac{\tau_c}{T_R} \left| \gamma_{12} \right|^2 \right), \tag{1}$$

where chaotic radiation has been assumed and $1+|\gamma_{12}|^2$ is the degree of second-order coherence. $\tilde{A}$ is the so-called accidental correlation and has a magnitude of nearly unity. κ is the duty ratio of the light pulses. Apparently the first term is much larger than the second one in Eq. (1) due to existence of the reduction factor τ_c/T_R. Our idea is to modulate the coherence time τ_c with a frequency f by modulating the entrance slit width of the monochromator. However, two intensities detected by the two detectors are also unavoidably modulated with the same frequency f. Therefore, if the third-harmonic $3f$ component is detected, it includes only the true two-photon correlation.

II. A TWO-PHOTON CORRELATOR

Under the above consideration, an apparatus was constructed in which the coherence time of a photon is modulated by modulating the slit width of a grating monochromator. Figure 2 shows the schematic diagram of the optical system, which includes a precise Fraunhofer slit to diffract the incident beam, an entrance slit with modulated width, a spherical grating, an exit slit, a beam-splitting mirror, and two microchannel detectors.

Figure 3 shows the schematic diagram of the electric circuit to extract the true correlation signal, where multiplication is done by a double balanced mixer and a lock-in amplifier detects the $3f$ component of the signal. The time

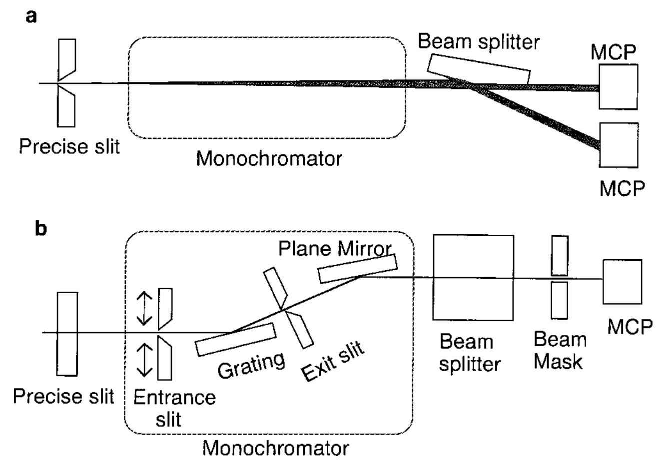

FIG. 2. Optical setup of the two-photon correlator for soft-x-ray synchrotron radiation. (a) is the top view and (b) is the side view. MCP's (microchannel plates) are the fast-response photon detectors with a response time of about 1 ns.

response of the detectors and the double balanced mixer is about 1 ns. Because the modulation of the width of the entrance slit is not exactly sinusoidal, there might be detected some possible accidental correlation signal caused by higher harmonics of the intensity modulation. To eliminate this possible trivial correlation, two identical band-pass filters are inserted to suppress the bunch repetition frequency 500 MHz, the revolution frequency 1.6 MHz, and some of their higher harmonics, of the 2.5-GeV electron storage ring. The detailed description will be given elsewhere [16].

III. EXPERIMENT

The whole experimental setup is shown in Fig. 4. The two-photon correlation was measured as a function of the width of the precise slit. When the width is large, the first-order coherence is decreased and consequently the higher-order coherence is also reduced because generally for chaotic light the lower-order coherence is a necessary condition for the higher-order coherence. Under the assumption of chaotic

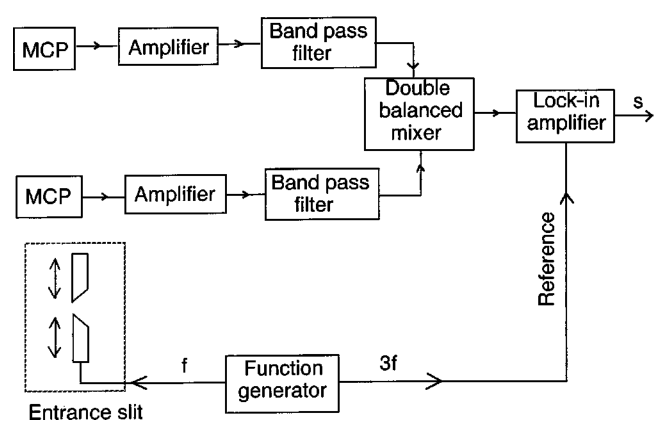

FIG. 3. An electric correlator for soft-x-ray synchrotron radiation. The entrance slit of the monochromator as is shown in Fig. 2(b) is modulated at a slow frequency f by a piezoelectric translator, and the $3f$ component of the output of the double balanced mixer, denoted by s, is detected with a lock-in amplifier.

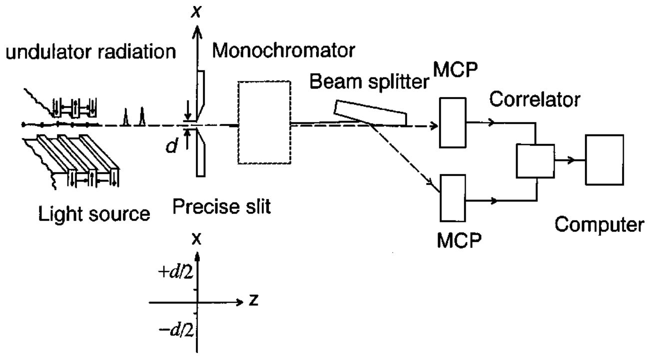

FIG. 4. Top view of the whole experiment system. The light source is the undulator radiation, and the correlation signal is measured as a function of the horizontal width d of the precise slit that plays a role of the diffraction of the incident synchrotron radiation.

radiation, the normalized output of the correlator is expressed as follows [17]:

$$S_0(d) \propto \tilde{A}_r + \Lambda\kappa \frac{\tau_c}{T_R}\left(\frac{d^2+24\Sigma^2}{(1+\Sigma^2/\sigma_c^2)d^2+24\Sigma^2}\right)^{1/2} \times \cos^{-1}\left(\frac{d^2}{(1+2\sigma_c^2/\Sigma^2)d^2+48\sigma_c^2}\right), \tag{2}$$

where σ_c is the transverse coherent length of the light falling on the precise slit and is expressed as follows [12]:

$$\sigma_c = \frac{\varepsilon_p \Sigma}{\sqrt{\varepsilon^2-\varepsilon_p^2}}. \tag{3}$$

$\tilde{A}_r$ is the residual accidental correlation, the influences of the electronics have been represented by a factor Λ, κ is the duty ratio as described in Eq. (1), ε is the total photon emittance which is given by the convolution of the electron beam emittance and a single-photon emittance ε_p, where $\varepsilon_p = \lambda/(4\pi)$, and λ is the photon wavelength. τ_c is the average coherence time, T_R is the average response time of the measuring system, Σ is the beam size at the precise slit, and d is the width of the precise slit.

The true two-photon correlation has been measured to investigate the statistical property of undulator radiation with photon energy 70 eV at the beamline BL16B connected to the 2.5-GeV electron storage ring of the Photon Factory, High Energy Accelerator Research Organization (KEK). The resolution $\lambda/(\Delta\lambda)$ of the monochromator is about 1000. Great care has been taken to eliminate the influence of coherent noises which oscillate at the same or some harmonic frequency as the reference input of the lock-in amplifier. We measured the two-photon correlation for different horizontal width d of the precise slit at 10, 20, 50, and 100 μm. For each value of the width d, the measuring time was about 2 h and the number of sampling points was about 10 000. The residual accidental correlation $\tilde{A}_r$ was also measured by changing the diffraction order of the grating monochromator from the first to the zeroth order. This is because the coherence time τ_c is negligibly small for the zeroth-order diffraction and only the first term in Eq. (2) is important. The results are listed in Table I, where $S(d)$ is the normalized average correlation output, σ is the standard deviation of $S(d)$, and θ is the average locked phase, which is in fact the phase difference between the signal and the $3f$ reference input of the lock-in amplifier. The measured residual accidental correlation A_r is listed in the final row in Table I. After subtracting this undesirable correlation A_r, the true two-photon correlation is drawn in Fig. 5. The dotted line is the theoretical curve according to Eq. (2) after measuring the horizontal beam size of about $\Sigma = 77.9\ \mu$m on the plane of the precise slit, and the error bars show the statistical confidence interval around the mean with 99% confident coefficient [18].

TABLE I. Experiment result.

d (μm)	$S(d)$ (units of 10^{-5})	σ (units of 10^{-5})	θ (deg)
10	1.799	0.023	−119.80
20	1.162	0.023	−120.59
50	0.998	0.025	−118.08
100	0.820	0.017	−123.80
A_r	0.348	0.0035	−121.26

IV. DISCUSSIONS

From Table I we can see that the problematic huge accidental correlation has been suppressed enough to reveal the small variation of the true two-photon correlation against the small spatial variation of observed points with satisfactory precision. For a field of totally coherent radiation, photon statistics should obey a Poisson distribution, and correspondingly the electrons excited from the detectors also obey the Poisson distribution, which makes the photoelectric currents of the two detectors behave as independent random fluctuations. The important consequence of the Poisson distribution is that the normalized correlation output, i.e., the second-

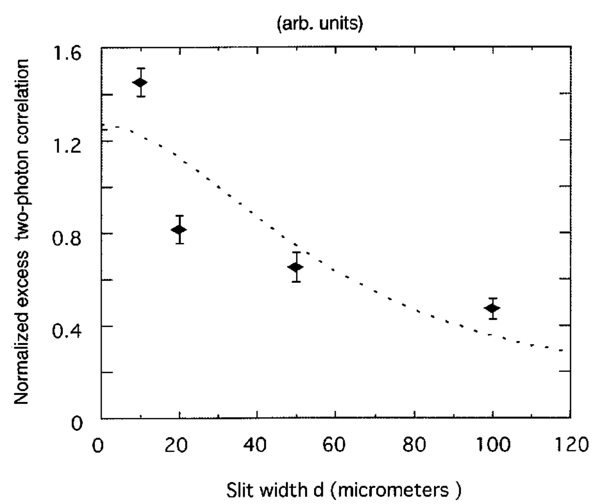

FIG. 5. Variation of the normalized true two-photon correlation vs the horizontal width d of the precise slit. The photon energy is 70 eV. The error bars show the statistical confidence interval around the mean with 99% confidence coefficient. The horizontal beam size Σ at the precise slit was measured to be about 77.9 μm. The dotted line is calculated from Eq. (2) after the least-square fitting with the experimental data, where the instantaneous total photon emittance ε is estimated to be about 40 nmrad.

order coherence, does not depend on any temporal or spatial delay of observed points. However, the explicit bunching effect observed in our experiment as is shown in Fig. 5 indicates that there exists a positive correlation, rather than independent fluctuations, between the two photoelectric currents. This implies that the photons in the field of undulator radiation have been severely different from the Poisson distribution. We have fitted the experimental data with a theoretical curve calculated from the absolute square of first-order coherence, as Eq. (2) shows, and found they agree well, although there is a slight discrepancy which is possibly caused by the theoretical approximation. The dependence of second-order coherence on first-order coherence is a basic characteristic of a chaotic optical field, because in such a field, higher-order coherence is completely determined by its first-order coherence.

The density operator for a field radiated by a classical current is given as follows according to Glauber's theory [1,3]:

$$\hat{\rho}(t)=\int p(\{\alpha_k\})|\{\alpha_k\}\rangle\langle\{\alpha_k\}|\prod_k d^2\alpha_k, \tag{4}$$

where $p(\{\alpha_k\})$ is the P representation for the multimode radiation field, and the time-dependent amplitudes $\alpha_k(t)$ are related with the electric current density distribution $\mathbf{j}(\mathbf{r},t')$ as follows:

$$\alpha_k(t)=\int_{-\infty}^{t}dt'\int d\mathbf{r}\,\mathbf{j}(\mathbf{r},t')e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\omega t'}. \tag{5}$$

Obviously any prescribed current distribution always leads to a pure coherent state, which means that $p(\{\alpha_k\})$ in Eq. (4) is simply a δ function.

The electron energy in the stored current is about 2.5 GeV, while that of the observed photon is 70 eV. The total radiation loss per turn is just 0.4 MeV, so the quantum recoil effect is almost negligible. The average distance between two neighboring electrons is of the order of 1 μm, which is much larger than the observed wavelength 17 nm. The reason why such a ''classical'' current does not produce coherent radiation is the stochastic ergodicity of the temporal and the spatial Fourier transformations of this stored current distribution just as Eq. (5) shows, which makes the radiation field behave as a statistical mixtures of pure coherent states with a weight $p(\{\alpha_k\})$ of Gaussian form. Temporally it arises from the discrete nature of electron charge and spatially from the high transverse freedom of such unbounded electrons, which makes the distance between electrons fluctuate randomly with time.

Experimentally, the two-photon correlation output is in fact an average within the response time T_R of the correlator. For a spontaneous radiation the coherence time τ_c of a single photon is usually much smaller than T_R. For our case the average T_R of the correlator is about 3 ns, while the average coherence time τ_c for the photon of 70 eV is about 0.1 ps. The coherence degree $\gamma_{12}(\tau)$ of first-order coherence could be decomposed into the product of the spatial part and the temporal part as follows under the condition of cross-spectral purity, which is usually satisfied by the experimental conditions [4,19]:

$$\gamma_{12}(\tau)=\gamma_{12}(\mathbf{r})\,\gamma(\tau), \tag{6}$$

where $\gamma_{12}(\mathbf{r})$ is pure spatial coherence which has been used as a simple notation γ_{12} in Eq. (1), and $\gamma(\tau)$ is pure temporal coherence. τ is the time delay. Although we had expected to detect only the spatial second-order coherence $|\gamma_{12}(\mathbf{r})|^2$ by changing the width d of the precise slit as Fig. 4 shows, the temporal coherence term $|\gamma(\tau)|^2$ would be inevitably induced due to the much slower time response of the correlator. However, it is averaged to give a stable output within the time scale of several times of the coherence time τ_c. In fact, it could be understood directly from Eq. (1) that the present modulation technique excludes the accidental ''long time'' correlation. Therefore, from the measurement of the present true two-photon correlation, the instantaneous information of the light source can be obtained with the time scale of coherence time τ_c. By fitting the experimental data as shown in Fig. 5, the instantaneous total photon emittance of the stored electron beam in the PF storage ring is estimated to be about 40 nmrad.

V. CONCLUSIONS

We have constructed a two-photon correlator based on an operating principle. This apparatus works well in extracting the true two-photon correlation for soft-x-ray synchrotron radiation. The dc drift has been canceled to almost zero compared with the signal output. The accidental correlation has also been efficiently suppressed to the magnitude of nearly $\frac{1}{10}$ of the true correlation, which enables us to observe the clear photon-bunching effect. The measured explicit bunching effect implies that synchrotron radiation, even high-brilliance undulator radiation, is chaotic radiation. This experimental method also provides a potential way to measure the instantaneous emittance with the time scale of the coherence time τ_c. The diagnosis of incomplete free-electron lasers, such as self-amplified spontaneous emission, could be performed by using this method, because if it is totally coherent light the normalized excess two-photon correlation would have a flat response, but would not show a photon-bunching effect.

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