Broadening of spectral lines due to dynamic multiple scattering and the Tully-Fisher relation

Sisir Roy* and Menas Kafatos†
Center for Earth Observing and Space Research, Institute for Computational Sciences and Informatics and Department of Physics, George Mason University, Fairfax, Virginia 22030

Suman Dutta‡
Physics and Applied Mathematics Unit, Indian Statistical Institute, Calcutta, India

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The frequency shift of spectral lines is most often explained by the Doppler effect in terms of relative motion, whereas the Doppler broadening of a particular line mainly depends on the absolute temperature. The Wolf effect, on the other hand, deals with the correlation-induced spectral change and explains both the broadening and shift of the spectral lines. In this framework a relation between the width of the spectral line is related to the redshift z for the line and hence with the distance. For smaller values of z a relation similar to the Tully-Fisher relation can be obtained and for larger values of z a more general relation can be constructed. The derivation of this kind of relation based on dynamic multiple-scattering theory may play a significant role in explaining the overall spectra of quasistellar objects. We emphasize that this mechanism is not applicable for nearby galaxies, $z \leqslant 1$. [S1050-2947(99)07407-7]

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I. INTRODUCTION

In studying the motion of astronomical objects, astrophysicists utilize the study of frequency shift of spectral lines. In general, an emphasis on relating the shift of a spectral line with its width has not been made so far. The Tully-Fisher (TF) relation [1] is an empirical correlation which finds that the luminosity L of a disk galaxy is proportional to its maxium rotational velocity $V_{\rm max}^{\alpha}$, where α has been observationally established to be (3-4) [2]. In spite of the frequent use of the TF relation as a distance indicator, the physical origin of this relationship is poorly understood, and it remains unclear whether all rotationally supported disk galaxies, including late-type spirals and irregulars, obey a single correlation of luminosity with linewidth. Recently, Matthews *et al.* [3] made an attempt to analyze this situation. It is generally argued that in order to obtain a measure of the maximum rotational velocity of a disk from its measured global $H-I$ profile width, some correction to the observed linewidth should be made for the effects of broadening due to turbulent (i.e., nonrotational) motions (cf. Roberts [4], Bottinelli *et al.* [5]). A linear summation of rotational and random motions adequately describes the observed profile widths of giant galaxies, while for dwarf galaxies (i.e., slowly rotating galaxies with Gaussian-like line profiles), a sum in the quadrature of the random and rotational terms is appropriate. However, Rhee [6] has pointed out the shortcomings of this type of addition for the type of objects considered here.

On the other hand, a dynamic multiple-scattering theory [7,8] derived from the field of statistical optics has been developed to account for the shift of a spectral line as well as the broadening of the line. It is shown that when light passes through a turbulent (or inhomogeneous) medium, due to multiple scattering the shift and the width can be calculated. Here, a sufficient condition for redshift has been derived and when applicable the shift is shown to be larger than broadening. The width of the spectral line can be calculated after multiple scatterings and a relation can be derived between the width and the shift z, which applies to any value of z. For small values of z this can be reduced to a Tully-Fisher-type relation. It would be interesting to estimate this width and make a comparison with the correction part of the line profile width considerd by Bottinelli *et al.* [5] applying to nonrotational motion. We want to emphasize that it is then possible to derive a relation for distance indicator (which reduces to a Tully-Fisher type relation as a special case) by studying the particular collisional mechanism which itself is a physical process. This kind of physical process can be identified even in the laboratory experiments and as such it is plausible that it applies to astronomical objects.

In this paper we shall start with a brief discussion of the various type of broadenings (Sec. II). In Sec. III we shall discuss the main results of our multiple scattering theory within the Wolf framework of redshifts [9]. Based on this approach a relation between the shift z and the width will be derived in Sec. IV. Finally, the possible implications for quasistellar object (QSO) observations will be discussed in Sec. V.

II. SPECTRAL BROADENING

If a spectral line is examined by means of a spectrograph, its width is dependent on the slit width employed. In general, the narrower the slit width the less broad the resulting spectral line as recorded on a photographic plate. Nevertheless, however narrow the slit width is, the sharpest line has a finite width even for the best optical system. Three causes producing the breadth of a spectral line are (1) its natural width, (2) Doppler effect, (3) external effect.

*Electronic address: sroy2@osf1.gmu.edu
†Electronic address: mkafatos@compton.gmu.edu
‡Electronic address: res9428@isical.ac.in

A. Natural linewidth

An atom can stay in two types of states. A state in which an atom free from external effects can remain for an arbitrarily long time is called a stationary state. Only the ground state is stationary since it corresponds to the minimum possible energy for a given atom. An excited state is nonstationary since spontaneous transitions to lower energy levels are possible for it. Such states are called transient states which are characterized by a finite lifetime. A quantitative measure of instability of an excited state is the time τ during which the number of atoms in a system at a given excited state decreases by a factor of e. This quantity τ is known as the *lifetime of the excited state* and coincides with the average time spent by the atoms in the excited state. In quantum mechanics, τ is associated with the probability of spontaneous radiative transitions from a given excited state to a lower energy level. Lifetimes of excited states of atoms generally lie in the range 10^{-9} to 10^{-8}. The finiteness of the lifetime of an atom in a transient state can easily be taken into account by introducing the damping factor into the expression for the wave function

$$\psi(\vec{r},t)=\exp^{[-(i/h)Et-\gamma t/2]}, \tag{1}$$

where γ is a positive constant. It should be noted that the wave function in stationary state is

$$\psi(\vec{r},t)=\exp^{[-(i/h)Et]}\psi(\vec{r}). \tag{2}$$

Therefore, the above function not only oscillates with a frequency $\omega=E/h$, but also attenuates with time due to the presence of the factor $\exp^{[-\gamma t/2]}$. The probability density of finding the atom in the state ψ is given by

$$|\psi(\vec{r},t)|^2=\exp[-\gamma t]|\psi(\vec{r})|^2. \tag{3}$$

The time during which the probability density diminishes to $1/e$th of its initial value is obviously the lifetime τ of the state. It then follows that

$$\gamma\tau=1. \tag{4}$$

The quantity γ defined by this expression is called the *damping constant*.

Since energy and time are canonically conjugate quantities, according to the Heisenberg uncertainty relation, the energy of an excited state is not exactly definite. The indeterminacy Γ in the energy of a transient state is connected with its lifetime τ through the relation

$$\Gamma\sim\frac{\hbar}{\tau}=\hbar\gamma. \tag{5}$$

Blurring of adjacent energy levels can generally be due to various reasons. The quantity Γ associated with the probability of spontaneous radiative transitions is called the natural width of the level. If an atom remains in the normal state for a long time the uncertainty of the energy value is small and the level is sharp. If the electron is excited to an upper level where it remains for some time, the uncertainty of the energy value is greater and the width of the level is greater. The natural linewidth is the minimum limit for the radiation linewidth.

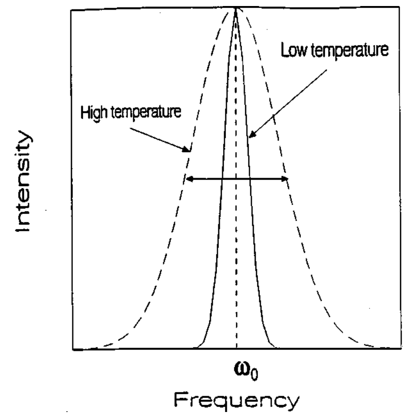

FIG. 1. Doppler broadening of spectral line.

B. Doppler broadening

The thermal motion of emitting atoms leads to the so-called Doppler broadening of spectral lines. Owing to thermal agitation, most of the atoms emitting light have high velocities. The random motion of the atoms and the molecules in a gas, however, produce a net broadening of the lines with no apparent shift in its central maximum. The frequency spread of this line is called the half-intensity breadth, and is given by

$$\delta=1.67\frac{\nu_0}{c}\sqrt{\frac{2RT}{m}}. \tag{6}$$

The half intensity breadth is defined as the interval between two points where the intensity drops to half its maximum vale. Here, R is the gas constant, T is the temperature of the gas in K, and m is the atomic weight. Therefore, the Doppler broadening is (1) proportional to the frequency ν_0, (2) proportional to the square root of the (absolute) temperature T; and (3) inversely proportional to the square root of the atomic weight m.

Experimental observations indicate that in keeping with the above equation, in order to produce sharper lines in any given spectrum, the temperature must be lowered. Furthermore, for a given temperature the lines produced by the lighter elements in the periodic table are in general broader than those produced by the heavy elements.

A comparison of the estimated values of natural width and Doppler width shows that the Doppler width at room temperatures is much larger than the natural width (Fig. 1).

C. Collisional broadening

The spectral broadenings described in the previous two subsections, can be classified as due to internal effects. There exist other reasons for spectral line broadening specifically an excited atom in a gas with a finite number density of particles undergoes collisions with neighboring atoms. Since the phase of radiation changes upon each collision, the monochromatic nature of the emission line is violated. This is effectively taken into account by introducing the total level width, which is equal to $\Gamma+\Gamma_{\rm col}$, where $\Gamma_{\rm col}=1/\pi\tau_0$ is the collision level width (τ_0 is the mean free time of the atom in gaseous medium). Another mechanism, which can explain the line broadening, is an application of statistical optics where the existence of a random dielectric susceptibility which fluctuates both spatially and temporally is assumed. The scattering of light induced by the random susceptibilty can produce both a shift and a broadening of the spectral line. This is called the Wolf effect [9], the main features of which are described as follows.

III. DYNAMIC MULTIPLE-SCATTERING THEORY

First, we briefly state the main results of Wolf's scattering mechanism. Let us consider a polychromatic electromagnetic field of light with central frequency ω_0 and width δ_0, incident on the scatterer. The incident spectrum is assumed to be of the form

$$S^{(i)}(\omega)=A_0 e^{[-(1/2\delta_0^2)(\omega-\omega_0)^2]}. \tag{7}$$

The spectrum of the scattered field is given by [10]

$$S^{(\infty)}(r\vec{u'},\omega')=A\omega'^4\int_{-\infty}^{\infty}K(\omega,\omega',\vec{u},\vec{u'})S^{(i)}(\omega)d\omega), \tag{8}$$

which is valid within the first-order Born approximation [11]. A can be determined in terms of A_0. Here $K(\omega,\omega')$ is the so-called scattering kernel and it plays the most important role in this mechanism. $\vec{u}$ and $\vec{u'}$ are the unit vectors in the direction of incident and scattered fields, respectively. Instead of studying $\mathcal{K}(\omega,\omega')$ in detail, we consider a particular case for the correlation function $G(\vec{R},T;\omega)$ of the generalized dielectric susceptibility $\eta(\vec{r},t;\omega)$ of the medium which is characterized by an anisotropic Gaussian function

$$\begin{aligned}G(\vec{R},T;\omega)&=\langle \eta^*(\vec{r}+\vec{R},t+T;\omega)\,\eta(\vec{r},t;\omega)\rangle\\&=G_0\exp\left[-\frac{1}{2}\left(\frac{X^2}{\sigma_x^2}+\frac{Y^2}{\sigma_y^2}+\frac{Z^2}{\sigma_z^2}+\frac{c^2T^2}{\sigma_\tau^2}\right)\right].\end{aligned} \tag{9}$$

Here G_0 is a positive constant, $\vec{R}=(X,\ Y,\ Z)$, and σ_x, σ_y, σ_z, σ_τ are correlation lengths. The anisotropy is indicated by the unequal correlation lengths in different spatial as well as temporal directions. $\mathcal{K}(\omega,\omega')$ can be obtained from the four-dimensional Fourier transform of the correlation function $G(\vec{R},T;\omega)$. In this case $\mathcal{K}(\omega,\omega')$ can be shown to be of the form

$$\mathcal{K}(\omega,\omega')=B\exp\left[-\frac{1}{2}(\alpha'\omega'^2-2\beta\omega\omega'+\alpha\omega^2)\right], \tag{10}$$

where

$$\alpha=\frac{\sigma_x^2}{c^2}u_x^2+\frac{\sigma_y^2}{c^2}u_y^2+\frac{\sigma_z^2}{c^2}u_z^2+\frac{\sigma_\tau^2}{c^2},$$

$$\alpha'=\frac{\sigma_x^2}{c^2}u_x'^2+\frac{\sigma_y^2}{c^2}u_y'^2+\frac{\sigma_z^2}{c^2}u_z'^2+\frac{\sigma_\tau^2}{c^2}, \tag{11}$$

and

$$\beta=\frac{\sigma_x^2}{c^2}u_xu_x'+\frac{\sigma_y^2}{c^2}u_yu_y'+\frac{\sigma_z^2}{c^2}u_zu_z'+\frac{\sigma_\tau^2}{c^2}.$$

Here $\hat{u}=(u_x,u_y,u_z)$ and $\widehat{u'}=(u_x',u_y',u_z')$ are the unit vectors in the directions of the incident and scattered fields, respectively.

Substituting Eqs. (1) and (4) in Eq. (2), we finally get

$$S^{(\infty)}(\omega')=A'e^{[-(1/2\delta_0'^2)(\omega'-\overline{\omega}_0)^2]}, \tag{12}$$

where

$$\begin{aligned}\overline{\omega}_0&=\frac{|\beta|\omega_0}{\alpha'+\delta_0^2(\alpha\alpha'-\beta^2)},\\ \delta_0'^2&=\frac{\alpha\delta_0^2+1}{\alpha'+\delta_0^2(\alpha\alpha'-\beta^2)},\end{aligned} \tag{13}$$

and

$$A'=\sqrt{\frac{\pi}{2(\alpha\delta_0^2+1)}}ABA_0\omega_0'^4\delta_0\exp\left[\frac{|\beta|\omega_0\overline{\omega}_0-\alpha\omega_0^2}{2(\alpha\delta_0^2+1)}\right].$$

Though A' depends on ω', it was approximated by James and Wolf [10] to be a constant so that $S^{(\infty)}(\omega')$ can be considered to be Gaussian.

The relative frequency shift is defined as

$$z=\frac{\omega_0-\overline{\omega}_0}{\overline{\omega}_0}, \tag{14}$$

where ω_0 and $\overline{\omega}_0$ denote the unshifted and shifted central frequencies, respectively. We say that the spectrum is redshifted or blueshifted according to whether $z>0$ or $z<0$, respectively. Here

$$z=\frac{\alpha'+\delta_0^2(\alpha\alpha'-\beta^2)}{|\beta|}-1. \tag{15}$$

It is important to note that this z number does not depend on the incident frequency, ω_0. This is a very important aspect if the mechanism is to apply in the astronomical domain. Expression (15) implies that the spectrum can be shifted to the blue or to the red, according to the sign of the term $\alpha'+\delta_0^2(\alpha\alpha'-\beta^2)>|\beta|$. To obtain the no-blueshift condition, we use Schwarz's inequality which implies that $\alpha\alpha'-\beta^2\geqslant 0$. Thus, we can take

$$\alpha'>|\beta|$$

as the condition sufficient to have only a redshift by that mechanism.

Let us now assume that the light, in its journey, encounters many such scatterers. What we observe at the end is the light scattered many times, with an effect as that stated above in every individual process. Let there be N scatterers between the source and the observer and z_n denote the relative frequency shift after the nth scattering of the incident light from the $(n-1)$th scatterer, with ω_n and ω_{n-1} being the central frequencies of the incident spectra at nth and $(n-1)$th scatterers. Then by definition,

$$z_n = \frac{\omega_{n-1} - \omega_n}{\omega_n}, \quad n = 1, \ 2, \ldots, \ N$$

or,

$$\frac{\omega_{n-1}}{\omega_n} = 1 + z_n, \quad n = 1, \ 2, \ldots, \ N.$$

Taking the product over n from $n=1$ to $n=N$, we get

$$\frac{\omega_0}{\omega_N} = (1+z_1)(1+z_2)\cdots(1+z_N).$$

The left-hand side of the above equation is nothing but the ratio of the source frequency and the final or observed frequency z_f. Hence,

$$1 + z_f = (1+z_1)(1+z_2)\cdots(1+z_N). \tag{16}$$

Since the z number due to such effect does not depend upon the central frequency of the incident spectrum, each z_i depends on δ_{i-1} only, not ω_{i-1} [here ω_j and δ_j denote the central frequency and the width of the incident spectrum at the $(j+1)$th scatterer]. To find the exact dependence we first calculate the broadening of the spectrum after N number of scatterings.

A. Effect of multiple scatterings on the spectral linewidth

From the second equation in Eq. (13), we can easily write,

$$\delta_{n+1}^2 = \frac{\alpha \delta_n^2 + 1}{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2} = \left(\frac{\alpha \delta_n^2 + 1}{\alpha'}\right)\left[1 + \delta_n^2\left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\right]^{-1}. \tag{17}$$

From Eq. (13), we can also write

$$\omega_{n+1} = \frac{\omega_n |\beta|}{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2}. \tag{18}$$

Then from Eqs. (14) and (15), we can write

$$z_{n+1} = \frac{\omega_n - \omega_{n+1}}{\omega_{n+1}} = \frac{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2}{|\beta|} - 1 = \frac{\alpha'}{|\beta|}\left\{1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\delta_n^2\right\} - 1. \tag{19}$$

Let us assume that the redshift per scattering process is very small, i.e.,

$$0 < \epsilon = z_{n+1} \ll 1$$

for all n. Then,

$$1 + \epsilon = \frac{\alpha'}{|\beta|}\left\{1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\delta_n^2\right\}$$

or,

$$(1+\epsilon)\frac{|\beta|}{\alpha'} = 1 + \left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\delta_n^2.$$

In order to satisfy this condition and in order to have a redshift only (or positive z), we see that the first factor $\alpha'/|\beta|$ in the right term cannot be much bigger than 1, and, more importantly,

$$\left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\delta_n^2 \ll 1. \tag{20}$$

In that case, from Eq. (17), after neglecting higher-order terms, the expression for δ_{n+1}^2 can be well approximated as

$$\delta_{n+1}^2 \approx \left(\frac{\alpha \delta_n^2 + 1}{\alpha'}\right)\left[1 - \delta_n^2\left(\frac{\alpha\alpha' - \beta^2}{\alpha'}\right)\right],$$

which, after carrying out a simplification, gives a very important recurrence relation:

$$\delta_{n+1}^2 = \frac{1}{\alpha'} + \frac{\beta^2}{\alpha'^2}\delta_n^2. \tag{21}$$

Therefore,

$$\begin{aligned}\delta_{n+1}^2 &= \frac{1}{\alpha'} + \frac{\beta^2}{\alpha'^2}\delta_n^2 = \frac{1}{\alpha'} + \frac{\beta^2}{\alpha'^2}\left[\frac{1}{\alpha'} + \frac{\beta^2}{\alpha'^2}\delta_{n-1}^2\right] \\ &= \left(\frac{\beta^2}{\alpha'^2}\right)^2 \delta_{n-1}^2 + \frac{1}{\alpha'}\left(1 + \frac{\beta^2}{\alpha'^2}\right)\cdots \\ &= \left(\frac{\beta^2}{\alpha'^2}\right)^{n+1}\delta_0^2 + \frac{1}{\alpha'}\left(1 + \frac{\beta^2}{\alpha'^2} + \cdots \frac{\beta^{2n}}{\alpha'^{2n}}\right).\end{aligned}$$

Thus,

$$\delta_{N+1}^2 = \left(\frac{\beta^2}{\alpha'^2}\right)^{N+1}\delta_0^2 + \frac{1}{\alpha'}\left(1 + \frac{\beta^2}{\alpha'^2} + \cdots \frac{\beta^{2N}}{\alpha'^{2N}}\right). \tag{22}$$

As the number of scattering increases, the width of the spectrum obviously increases and the most important topic to be considered is whether this width is below some tolerance limit or not, from the observational point of view. There may be several measures of that tolerance limit. One of them is the *sharpness ratio,* defined as

$$Q = \frac{\omega_f}{\delta_f},$$

where ω_f and δ_f are the mean frequency and the width of the observed spectrum.

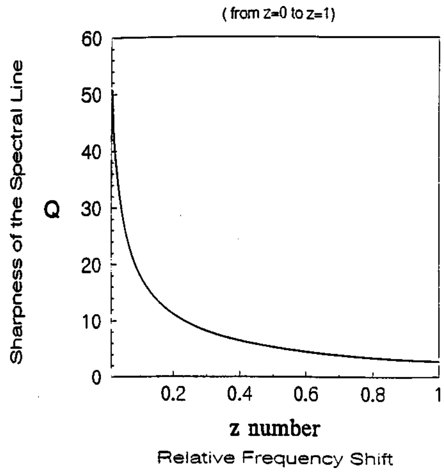

FIG. 2. Variation of sharpness of a spectral line with relative frequency shift (for low z).

After N number of scattering, this sharpness ratio, say Q_N, is given by the following recurrence relation:

$$Q_{N+1}=Q_N\sqrt{\frac{\alpha'}{\alpha'+(\alpha\alpha'-\beta^2)\delta_N^2}-\frac{1}{\alpha\delta_N^2+1}}.$$

It is easy to verify that the expression under the square root lies between 0 & 1. Therefore, $Q_{N+1}<Q_N$, and the line is broadened as the scattering process goes on (Figs. 2 and 3).

Under the sufficient condition of redshift [i.e., $|\beta|<\alpha'$] [12] it was shown that in the observed spectrum

$$\Delta\omega_{n+1}\gg\delta_n, \tag{23}$$

if the following condition holds:

$$\frac{\delta_n\omega_0(\alpha\alpha'-\beta^2)}{\alpha'+(\alpha\alpha'-\beta^2)\delta_n^2}\gg 1, \tag{24}$$

where ω_0 is the source frequency.

The relation (23) signifies that the shift is more prominent than the effective broadening so that the spectral lines are observable and can be analyzed. If, on the other hand, the broadening is higher than the shift of the spectral line, it will be impossible to detect the shift from the blurred spectrum. Hence we can take relation (24) to be one of the conditions necessary for the observed spectrum to be analyzable. For large N (i.e., $N\to\infty$), the series in the second term of the right-hand side of Eq. (22) converges to a finite sum and we get

$$\delta_{N+1}^2=\left(\frac{\beta^2}{\alpha'^2}\right)^{N+1}\delta_0^2+\frac{\alpha'}{\alpha'^2-\beta^2}.$$

If δ_0 is considered as arising out of Doppler broadening only, we can estimate $\delta_{\mathrm{Dop}}\sim 10^9$ for $T=10^4$ K. On the other hand, for the anisotropic medium, we can take $\sigma_x=\sigma_y=3.42\times10^{-1}$, $\sigma_z=8.73\times10^{-1}$, $\alpha'=8.68\times10^{-30}$, $\alpha=8.536\times10^{-30}$, and $\beta=8.607\times10^{-30}$ for $\theta=15^0$ [9]. Then the second term of the above expression will be much larger than the first term, and effectively, Doppler broadening can be neglected in comparison to that due to the multiple-scattering effect.

Now if we consider the other condition, i.e., $\alpha'<|\beta|$, the series in Eq. (22) will be a divergent one and δ_{N+1}^2 will be finitely large for large but finite N. However, if the condition (23) is to be satisfied, then the shift in frequency will be larger than the width of the spectral lines. In that case the condition $\alpha'<|\beta|$ indicates that blueshift may also be observed but the width of the spectral lines can be large enough depending on how large the number of collisions is. So in general, the blueshifted lines should be of larger widths than redshifted lines and may not be as easily observable.

B. Effect of source frequency on broadening

Rearranging Eq. (24) we get

$$\left(\delta_n-\frac{\omega_0}{2}\right)^2\leqslant\frac{\omega_0^2}{4}-\frac{\alpha'}{\alpha\alpha'-\beta^2}. \tag{25}$$

Since the left side is non-negative, the right side must be positive. Moreover, since the mean frequency of any source is always positive, i.e., $\omega_0\geqslant 0$, we must have

$$\omega_0\geqslant\sqrt{\frac{4\alpha'}{\alpha\alpha'-\beta^2}}. \tag{26}$$

We take the right side of this inequality to be the *critical source frequency* ω_c which is defined here as

$$\omega_c=\sqrt{\frac{4\alpha'}{\alpha\alpha'-\beta^2}}. \tag{27}$$

Thus for a particular medium between the source and the observer, the critical source frequency is the lower limit of the frequency of any source whose spectrum can be clearly analyzed. In other words, the shift of any spectral line from a source with frequency less than the critical source frequency for that particular medium cannot be detected due to its high broadening.

We now can classify the spectra of the different sources, from which light comes to us after passing through a scattering medium characterized by the parameters α, α', β. If we

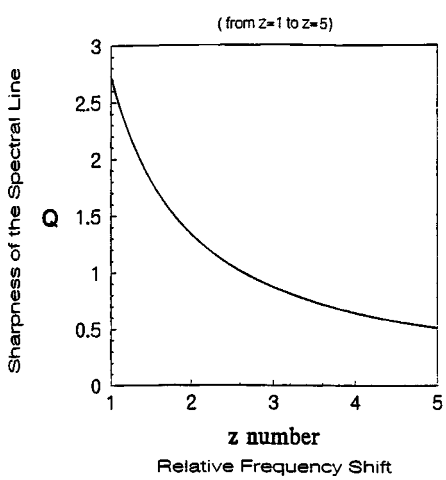

FIG. 3. Variation of sharpness for high (z).

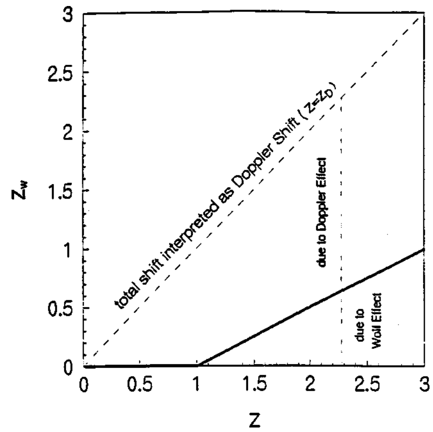

FIG. 4. Wolf contribution in relative frequency shift.

allow only the small-angle scattering in order to get prominent spectra, according to the Wolf mechanism, they will either be blueshifted or redshifted. The redshift of spectral lines may or may not be detected according to whether the condition (24) does or does not hold. In this way those sources whose spectra are redshifted, are classified in two cases, viz., $\omega_0 > \omega_c$ and $\omega_0 \leqslant \omega_c$. In the first case, the shifts of the spectral lines can be easily detected due to condition (23). But in the later case, the spectra will suffer from the resultant blurring.

C. Doppler shift vs Wolf shift

As we have seen in Sec. III A and Sec. III B, the mean frequency and width of a spectral line change after each scattering. The changes are given by the recurrence relations

$$\delta_{n+1}^2 = \frac{1}{\alpha'} + \frac{\beta^2}{\alpha'^2}\delta_n^2$$

and

$$\omega_{n+1} = \frac{\omega_n |\beta|}{\alpha' + (\alpha\alpha' - \beta^2)\delta_n^2}.$$

Using these, we can easily find the Wolf contribution to the observed shift assuming only that the source under consideration is quasimonochromatic. For this we rewrite Eq. (21) as

$$\delta_n^2 = \frac{\alpha'^2}{\beta^2}\delta_{n+1}^2 - \frac{\alpha'}{\beta^2}.$$

Now, according to Schrödinger [13], the width of the spectral lines must increase as a result of collisional processes. If this is the sole reason for the broadening, then we can estimate N, the number of collisions it undergoes in its way to us by comparing the width at each scatterer with a preassumed small δ (the spectral width of the quasimonochromatic source). We can easily calculate

$$z_W = (1 + z_{\mathrm{scat}})^N - 1, \tag{28}$$

where z_{scat} is the z number due to small angle scattering at each scatterer. The following graph (Fig. 4) illustrates the Wolf contribution in various observed shifts.

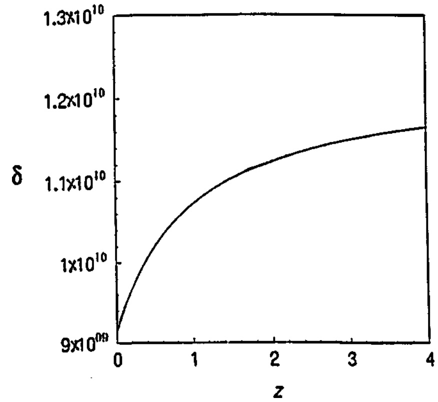

FIG. 5. Variation of width with shift.

The solid line curve represents the Wolf contribution, while the dotted line ($y = x$) represents the observed z number. Thus, the difference in height of these two curves provides the contribution due to the Doppler effect which up to the present day was assumed to be the total contribution to z.

IV. WIDTH OF THE SPECTRAL LINES AND THE TULLY-FISHER RELATION

According to our result (13), we conclude that both the shift and the width of a spectral line depend on the medium parameters. In Fig. 5, we show the variation of the spectral linewidth as a function of shift. We can write the relation between the width and the shift as

$$W = K(a^2 + \delta_0^2 z^2)^{1/2(1+z)}, \tag{29}$$

where K is a constant, a^2 is the minimum broadening, and δ_0^2 is the spectral width inherent to the source. The above relation is valid for $z > -1$. Taking the logarithm of both sides we can write

$$\ln W = \frac{1}{2(1+z)} \ln K(a^2 + \delta_0^2 z^2). \tag{30}$$

It is well known [14] that the distance modulus d can be written in terms of z as

$$d = m - M = 42.38 - 5\ln\left(\frac{H}{100}\right) + 5\ln z$$
$$+ (1.086)(1 - q_0)z + O(z^2), \tag{31}$$

where H is the Hubble constant in km/s/Mpc and M refers to the absolute magnitude. Now for small z, i.e., $z \ll 1$,

$$d = 42.38 - 5\ln\left(\frac{H}{100}\right) + \frac{1.086}{2}z, \quad \text{taking} \quad q_0 = \frac{1}{2}. \tag{32}$$

After simplification we can write

$$z = \frac{d - C}{0.543}, \quad \text{where} \quad C = 42.38 - 5\ln\left(\frac{H}{100}\right).$$

Now substituting this value of z in Eq. (30) we can write

$$d = -\frac{1.086}{\ln E} \ln W + N, \tag{33}$$

where

$$\ln E = \ln(Ka^2) \quad \text{and} \quad N = C + 0.543.$$

The above relation between the distance modulus and the width has a striking similarity to the Tully-Fisher relation but without any angular dependence. The reason is obvious since we have considered the shift and width due to scatterings only without considering any rotational effects. It appears that in the case of photons which are emitted perpendicular to the plane of the galaxy, we will be observing those photons only without any rotational effects. It should be mentioned that Bottenelli *et al.* [4] considered linear turbulence correction for the profile width as

$$W_{20,i,c} = \frac{[W_{20,\mathrm{obsv}} - W_{t,20}]}{\sin i}, \tag{34}$$

where $W_{20,\mathrm{obsv}}$ is the observed (profile) width at 20% peak maximum, corrected for instrumental broadening, $W_{(t,20)}$ is the correction term for turbulence, and the factor $\sin i$ corrects for disk inclination.

It would be interesting to compare this contribution for galaxy spectra due to the nonrotational part with the width calculated from multiple-scattering theory. These will be considered in a subsequent publication.

V. POSSIBLE IMPLICATIONS

Unlike the Doppler effect where the width of a line is unrelated to the shift, the Wolf mechanism predicts a tight relationship between the width and the shift of a line. As such, it is evident from the above analysis that *dynamic multiple-scattering theory* within Wolf's framework might play a significant role for QSO's in the following ways.

(1) The width and shifts of various spectral lines in different regions, say for UV, optical, etc., for active galactic nuclei (AGNs) and quasars may be explained in a consistent manner if the information regarding the environments around these objects were available.

(2) Since the Wolf contribution becomes prominent for larger z as can be seen from Fig. 2, this mechanism may shed new light in explaining the redshifts of quasars which tend to be large and peaking at $z \sim 2-3$ as well as for high redshift galaxies $(z > 1)$.

(3) For blueshift (i.e., $\alpha' < |\beta|$), as the broadening is much larger than the shift, as is evident from Eq. (22), one would get a continuous spectrum with no discrete lines.

(4) For low redshifts we can derive a relation like the Tully-Fisher type as a function for the distance modulus and width given by a physical mechanism. We also derived a general relation for higher redshifts [Eq. (30)], which can be verified from observations. Further studies can be made regarding the theoretical basis of the Baldwin effect [15] for line and continuum correlations in AGN.

Lastly, we should mention that the critical source frequency relating to the screening effect which plays a crucial role in explaining both redhsift and spectral width can be tested in *laboratory experiments*.

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