Quantum correlations violate Einstein-Podolsky-Rosen assumptions

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The inconsistency between quantum mechanics and Einstein-Podolsky-Rosen assumptions about locality and reality cannot be avoided by restricting attention to correlations. This is demonstrated by reworking the logical scheme that was developed to show inconsistency without inequalities for two particles, adapting it to correlations in two pairs of particles, so the Einstein-Podolsky-Rosen arguments can be made entirely in terms of correlations. A simple example is given. [S1050-2947(99)07010-9]

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I. INTRODUCTION

A better understanding of quantum mechanics can be gained from Mermin's attempt to provide a new interpretation [1]. The idea of Mermin's interpretation is that correlations are physically real but values of quantities being correlated are not. Even without regard to the philosophy (which does appeal to my intuition), there is something to learn by seeing if this idea can work, and if it cannot, why not.

There is a problem. Correlations, as well as values of quantities, can violate Einstein-Podolsky-Rosen assumptions about locality and reality. This was pointed out by Cabello [2]. It opens questions to be explored. How ubiquitous is this? How can we see it most clearly? Does it mean correlations offer no advantage as a handle for interpretation? In the spirit of Mermin's approach, let us see what quantum mechanics can tell us.

Mermin has shown that a quantum state is determined by correlations between subsystems [1]. It is also determined by particular quantities having certain values. If there is a difference in physical reality between correlations and values of quantities, the difference must be in the details.

A quantum correlation is a mean value of a product of two (or more) Hermitian operators. We can restrict our attention to products of projection operators; all correlations can be calculated from these. If the projection operators are for different subsystems, their product is a projection operator. Its eigenvalues are 0 and 1. The correlation is 1, maximum, if and only if the state gives eigenvalue 1 for the projection-operator product, which means the quantity represented by the product operator has the value 1. A correlation is maximum if and only if a quantity has a definite value.

I use that here to construct a particularly clear demonstration that quantum correlations can violate Einstein-Podolsky-Rosen assumptions. I rework the logical scheme that was developed [3] to describe Hardy's discovery [4] that the inconsistency between quantum mechanics and Einstein-Podolsky-Rosen assumptions can be seen without inequalities for two photons or particles with spin $\frac{1}{2}$. I adapt it to correlations in two pairs of particles, so the Einstein-Podolsky-Rosen arguments can be made entirely in terms of correlations.

In the last section I work out an example with a state vector and projection operators that are so simple that the calculations can be done almost by sight.

II. SETUP

Consider two pairs of particles produced in a state that entangles all four particles. Suppose the two pairs come out of the source in opposite directions, say one pair goes east and the other pair goes west. For the pair going west, I measure one of two correlations between the two particles, either

$$\langle D\rangle = \langle |w1\rangle\langle w1||x1\rangle\langle x1|\rangle \tag{1}$$

or

$$\langle F\rangle = \langle |w\rangle\langle w||x\rangle\langle x|\rangle, \tag{2}$$

where $|w1\rangle$ and $|w\rangle$ are states for one of the particles of the pair and $|x1\rangle$ and $|x\rangle$ are states for the other particle. For the pair going east, you measure one of the correlations

$$\langle E\rangle = \langle |y1\rangle\langle y1||z1\rangle\langle z1|\rangle \tag{3}$$

or

$$\langle G\rangle = \langle |y\rangle\langle y||z\rangle\langle z|\rangle, \tag{4}$$

using states $|y1\rangle$ and $|y\rangle$ for one particle and $|z1\rangle$ and $|z\rangle$ for the other particle.

The state of the four particles is chosen so that [3]

$$\begin{aligned}\langle FG\rangle &= 0,\\ \langle D(1-G)\rangle &= 0,\\ \langle (1-F)E\rangle &= 0,\\ \langle DE\rangle &> 0,\end{aligned} \tag{5}$$

where D, F, E, and G are the operators inside the mean values (1)–(4). The example given below shows that this is possible.

Consider the eigenvalues d, f, e, and g for the operators D, F, E, and G. Since each of these operators is a projection,

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each eigenvalue can be only 0 or 1. If any of the correlations $\langle D\rangle$, $\langle F\rangle$, $\langle E\rangle$, or $\langle G\rangle$ is 1, maximum, the corresponding eigenvalue must be 1. Conversely, if the state gives eigenvalue 1 for any of the projection operators D, F, E, or G, the corresponding correlation must be 1.

Since FG is a projection operator with eigenvalues fg, its zero mean value tells us there is no probability that f and g are both 1. If the correlations $\langle F\rangle$ and $\langle G\rangle$ are measured, they will not be both 1. The zero mean value for $D\,(1-G)$ tells us that if d is 1 then $1-g$ must be 0, so g must be 1. If the correlations $\langle D\rangle$ and $\langle G\rangle$ are measured and $\langle D\rangle$ is 1, then $\langle G\rangle$ must be 1. Similarly, from the next equation, we see that if the correlations $\langle F\rangle$ and $\langle E\rangle$ are measured and $\langle E\rangle$ is 1, then $\langle F\rangle$ must be 1. The positive mean value for $\langle DE\rangle$ tells us that if the correlations $\langle D\rangle$ and $\langle E\rangle$ are measured, the result can be that both are 1; the probability for that is $\langle DE\rangle$.

III. MEASURING CORRELATIONS

A mean value $\langle D\rangle$ is a number calculated from the operator D and a state. How is it measured? We consider cases where $\langle D\rangle$ is 1. If the quantity represented by D has been measured and found to have the value 1, we know we have a state where $\langle D\rangle$ is 1. Specifically, the measurement distinguishes the states $|w1\rangle$ and $|x1\rangle$ of the west particles from orthogonal states. We know $\langle D\rangle$ is 1 when the west particles are in the states $|w1\rangle$ and $|x1\rangle$.

What does it mean that $\langle D\rangle$ is a correlation? Is it supposed to be a statistical property of a series of events? We can view it that way here. In each event, the four particles are produced and the measurements are made. This is repeated many times. Suppose the measurement in the west is the one just described. Then $\langle D\rangle$ is 1 for the subset of events where the west particles are in the states $|w1\rangle$ and $|x1\rangle$. The measurement produces this subset with this correlation $\langle D\rangle$ for the two west particles. The correlation is a property of the state of these west particles after the measurement, not the initial state of the four particles [2].

Suppose the measurement in the east is whether the particles are in the states $|y\rangle$ and $|z\rangle$. When they are, $\langle G\rangle$ is 1. The second of Eqs. (5) tells us that when the measurement in the west yields the eigenvalue 1 for D, the measurement in the east must yield the eigenvalue 1 for G. In the subset of events that yield west particles in the states $|w1\rangle$ and $|x1\rangle$, the east particles are found to be in the states $|y\rangle$ and $|z\rangle$. When the correlation $\langle D\rangle$ is 1 for the west particles, the correlation $\langle G\rangle$ is 1 for the east particles.

The other Eqs. (5) can be viewed similarly. The first says that when the measurement in the east is the one just described and the measurement in the west distinguishes $|w\rangle$ and $|x\rangle$ from orthogonal states, there are no events where the measurements yield particles in the states $|w\rangle$ and $|x\rangle$ in the west and $|y\rangle$ and $|z\rangle$ in the east. There are no events where the correlation $\langle F\rangle$ is 1 for the west particles and the correlation $\langle G\rangle$ is 1 for the east particles. The last Eq. (5) says that if the measurements are whether the particles are in the states $|w1\rangle$ and $|x1\rangle$ in the west and $|y1\rangle$ and $|z1\rangle$ in the east, there are events where the particles are found to be in those states. There is a subset of events where the correlation $\langle D\rangle$ is 1 for the west particles and the correlation $\langle E\rangle$ is 1 for the east particles.

IV. EINSTEIN-PODOLSKY-ROSEN ARGUMENT

So far, everything said has been in terms of quantum mechanics, but it certainly invites application of the Einstein-Podolsky-Rosen argument. We shall see that following it, making assumptions only about correlations, leads to a contradiction with quantum mechanics.

The first assumption is that the measurement in the west has no effect on the outcome of the measurement in the east, and the measurement in the east has no effect on the outcome in the west. The arrangement could make this assumption difficult to refuse: the measurements could be widely separated or well shielded; there may be no time for information to go from one measurement to the other without a signal traveling faster than light.

Suppose the correlation $\langle D\rangle$ is measured in the west and the correlation $\langle G\rangle$ is measured in the east. If the result in the west is that $\langle D\rangle$ is 1, the result in the east has to be that $\langle G\rangle$ is 1. This could be confirmed by repeated testing in experiments. The value of the correlation $\langle G\rangle$ in the east can be obtained from the measurement in the west without doing anything to the particles in the east. The Einstein-Podolsky-Rosen argument asks us to assume, therefore, that if $\langle D\rangle$ is measured and found to be 1, then $\langle G\rangle$ is 1 whether we choose to measure $\langle G\rangle$ or not. Similarly, we are asked to assume that if $\langle E\rangle$ is measured and found to be 1, then $\langle F\rangle$ is 1 whether we choose to measure $\langle F\rangle$ or not.

Suppose the correlations $\langle D\rangle$ and $\langle E\rangle$ are measured and both are found to be 1. Then are $\langle F\rangle$ and $\langle G\rangle$ both 1? The assumptions we are asked to make, together with quantum mechanics, imply that $\langle F\rangle$ and $\langle G\rangle$ are both 1. But $\langle F\rangle$ and $\langle G\rangle$ cannot be both 1. We have a contradiction between quantum mechanics and Einstien-Podolsky-Rosen assumptions about correlations.

V. EXAMPLE

For one simple example of a state and operators D, F, E, and G that satisfy Eqs. (5), let D, F, E, and G be as in Eqs. (1)–(4) with

$$|w\rangle=\tfrac{3}{5}|w1\rangle+\tfrac{4}{5}|w2\rangle,$$

$$|x\rangle=|x1\rangle,$$

$$|y\rangle=\tfrac{3}{5}|y1\rangle+\tfrac{4}{5}|y2\rangle,$$

$$|z\rangle=|z1\rangle,$$

using orthonormal state vectors $|wj\rangle$ for one particle going west, $|yk\rangle$ for one going east, and just one normalized state vector for each of the other two particles. Let the state be represented by

$$|\Psi\rangle=c^{-1}(|1111\rangle+\tfrac{4}{3}|2111\rangle+\tfrac{4}{3}|1121\rangle-\tfrac{41}{16}|2121\rangle),$$

where

$$|j1k1\rangle=|wj\rangle|x1\rangle|yk\rangle|z1\rangle$$

for $j=1$ and 2 and $k=1$ and 2.

It is easy to see that Eqs. (5) are satisfied, because

$$\langle DE\rangle = |c|^{-2},$$

$$DG|\Psi\rangle = GD|\Psi\rangle = D|\Psi\rangle,$$

$$FE|\Psi\rangle = E|\Psi\rangle,$$

and

$$FG|\Psi\rangle = |w\rangle\langle w\|y\rangle\langle y\|\Psi\rangle = 0,$$

because

$$\langle w|\langle y\|\Psi\rangle = c^{-1}\left(\tfrac{3}{5}\times\tfrac{3}{5}+\tfrac{4}{5}\times\tfrac{3}{5}\times\tfrac{4}{3}+\tfrac{3}{5}\times\tfrac{4}{5}\times\tfrac{4}{3}-\tfrac{4}{5}\times\tfrac{4}{5}\times\tfrac{41}{16}\right) = 0.$$

Normalization of $|\Psi\rangle$ requires $|c|^2$ to be about 11.12 so $\langle DE\rangle$ is about 0.09. Different choices for the operators and state could make $\langle DE\rangle$ bigger, but from the calculations I did to work out this example, I would guess that $\langle DE\rangle$ cannot be much bigger.

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[1] N. D. Mermin, Am. J. Phys. **66**, 753 (1998); see also Los Alamos e-print archive quant-ph/9801057 (unpublished).
[2] A. Cabello, Phys. Rev. A **59**, 113 (1999); **60**, 877 (1999).
[3] T. F. Jordan, Phys. Rev. A **50**, 62 (1994).
[4] L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993).