Spatial instability of counterpropagating waves in nonlinear distributed feedback structures

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We investigate spatial instabilities (induced by small angular perturbations) of counterpropagating waves in nonlinear distributed feedback (DFB) structures. We determined the DFB-structure threshold length at which an absolute instability occurs and a nonhomogeneous spatial intensity distribution is generated. The evolution of the transverse intensity distribution is studied for counterpropagating waves as a function of the control parameters. [S1050-2947(99)04808-8]

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I. INTRODUCTION

A rich spatiotemporal dynamics of nonlinear distributed feedback (DFB) structures and prospects of their utilization in practical devices attract investigators to develop the theory of optical radiation propagation in a periodically modulated nonlinear media.

The propagation of plane waves in DFB structures was recently studied theoretically [1–12]. The model used predicts a series of effects such as optical multistability [1,8], temporal filtration of pulses with quadratic phase modulation [12], advent of solitons [6,7], self-oscillation, and chaos [3,10].

On the other hand, the various features of nonlinear DFB structures open interesting perspectives for applications in quantum electronic and integrated optics. In particular, a multistable regime of a laser with Bragg reflector based on a nonlinear DFB structure was predicted [13]. Futhermore, Refs. [14,15] claim experimental observation of multiple gap-soliton formation and an all-optical logic AND gate using a fiber Bragg grating.

The effects of a spatial modulation of the light beam reflected from a nonlinear DFB structure in the weak coupling approximation $kL \ll 1$ (where k is the coupling coefficient of counterpropagating waves and L is the DFB-structure length) were investigated using parabolic wave equations [16,17]. The analysis of those equations allows us [17] to conclude that generation of optical patterns in the transvere distribution of counterpropagating waves intensities may occur in a nonlinear DFB structure if the coupling is strong enough $kL \geqslant 1$ [18].

In this paper we investigate the effects of spatial instability of counterpropagating waves interacting in a nonlinear DFB structure for arbitrary coupling coefficients. We consider a transparent medium with nonrelaxing Kerr nonlinearity and we perform a stability analysis of the homogeneous stationary solutions (under the influence of small angular perturbations). We show that an absolute instability may arise leading to the generation of regular spatial structures (patterns). Examples of such structures were found through numerical integration of the fully nonlinear equations.

II. BASIC EQUATIONS

Let us consider a layer of a Kerr medium with periodically modulated refractive index

$$n = n_0 + n_1 \cos(qz) + n_2|E|^2, \tag{1}$$

where n_0 is the linear refractive index, $n_1 \ll n_0$ is the modulation amplitude with spatial frequency q, and n_2 is the magnitude characterizing the Kerr nonlinearity. Bragg scattering of radiation by the refractive index grating arised during the propagation in the medium. The field within the medium is given by the superposition of two counterpropagating waves

$$E(z,x,y,t) = [E_+(z,x,y)e^{iKz} + E_-(z,x,y)e^{-iKz}]e^{-i\omega t} + \text{c.c.}, \tag{2}$$

where $E_\pm(z,x,y)$ are the slowly varying amplitudes and the $K = (\omega/c)n_0$ is the wave number. Substituting Eqs. (1) and (2) into the wave equation we get [16,17]

$$\pm\frac{\partial E_\pm}{\partial z} - \frac{i}{2K}\Delta_\perp E_\pm = ikE_\mp e^{\pm i\Delta_0 z} + i\gamma(|E_\pm|^2 + 2|E_\mp|^2)E_\pm\,, \tag{3}$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, $\Delta_0 = q - 2K$ is the linear Bragg detuning ($|\Delta_0| \ll K$), $\gamma = Kn_2/n_0$, and $k = Kn_1/2n_0$. Boundary conditions for the system of equations (3) are

$$E_+(z=0,x,y) = E_{+0}(x,y), \quad E_-(z=L,x,y) = E_{-0}(x,y), \tag{4}$$

where E_{+0} and E_{-0} are arbitrary functions.

III. ANALYSIS OF LINEARIZED EQUATIONS

To get insight about possible physical situations we consider homogeneous solutions $\mathcal{E}_\pm(z)$ of Eq. (3). Of course, E_{+0} and E_{-0} are also independent of (x,y). Then we perform a linear stability analysis around such solutions for weak perturbations in transverse directions. The complex

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amplitudes of counterpropagating waves (2) can be written in the form

$$E_+ = \mathcal{E}_+(z) + [a_+(z)e^{i(K_x x + K_z z)} + a_-(z)e^{-i(K_x x - K_z z)}]e^{-iKz},$$

$$E_- = \mathcal{E}_-(z) + [b_+(z)e^{i(K_x x - K_z z)} + b_-(z)e^{-i(K_x x + K_z z)}]e^{iKz}, \tag{5}$$

where $a_\pm(z)$ and $b_\pm(z)$ are weak angular perturbation amplitudes $(|a_\pm|,|b_\pm| \ll |\mathcal{E}_\pm|)$ and $K_x \simeq K\theta$ and $K_z \simeq K(1-\theta^2/2)$, $\theta \ll \pi/2$ is the angle between the z axis and the direction of the perturbation propagation. The following equations can be obtained from Eq. (3) for the weak wave amplitudes in the linear approximation:

$$\frac{da_\pm}{dz} = ikb_\pm e^{i(\Delta+\Delta_0)z} + i\gamma[2(|\mathcal{E}_+|^2 + |\mathcal{E}_-|^2)a_\pm + 2\mathcal{E}_+\mathcal{E}_-^* b_\pm e^{i\Delta z} + \mathcal{E}_+^2 a_\mp^* e^{i\Delta z} + 2\mathcal{E}_+\mathcal{E}_- b_\mp^*],$$

$$-\frac{db_\pm}{dz} = ika_\pm e^{-i(\Delta+\Delta_0)z} + i\gamma[2(|\mathcal{E}_+|^2 + |\mathcal{E}_-|^2)b_\pm + 2\mathcal{E}_+^*\mathcal{E}_- a_\pm e^{-i\Delta z} + \mathcal{E}_-^2 b_\mp^* e^{-i\Delta z} + 2\mathcal{E}_+\mathcal{E}_- a_\mp^*], \tag{6}$$

where $\Delta = K\theta^2$. At $k=0$ Eq. (6) coincides with the corresponding equations from Ref. [19], where the analysis of counterpropagating wave spatial instabilities in a homogeneous nonlinear medium was carried out.

We look for the solution of Eq. (6) assuming that the intensities $I_\pm = |\mathcal{E}_\pm|^2 = I_0$ are constant and equal. Taking into account nonlinear phase raid appearing during the wave propagation in the medium, we have

$$\mathcal{E}_+(z) = \mathcal{E}_0 e^{i3\sigma z}, \quad \mathcal{E}_-(z) = \mathcal{E}_0 e^{i3\sigma(L-z)}, \tag{7}$$

where $\sigma = \gamma|\mathcal{E}_0|^2, \mathcal{E}_0 = \mathcal{E}_+(0) = \mathcal{E}_-(L) = \text{const}$. In the linear approximation we can take $\Delta_0 = 6\sigma$ without losing generality. In this case, it can be shown that the wave intensities remain practically constant and equal along the DFB structure if its length consists of an integer number of grating periods $(qL/2\pi = m,\ m = 0,1,2,\ldots)$. We introduce the quantities $A_\pm$ and $B_\pm$ as

$$a_\pm = A_\pm e^{i(\Delta/2+3\sigma)z}, \quad b_\pm = B_\pm e^{-i(\Delta/2+3\sigma)z}. \tag{8}$$

Equation (7) with account of Eq. (6) can then be written in matrix form

$$\frac{dY}{dz} = YX, \tag{9}$$

where $Y = (A_+ A_-^* - B_+ - B_-^*)^\top$,

$$X = \begin{pmatrix} x_1 & x_3 & -x_2 & -x_4 \\ -x_3 & -x_1 & -x_4^* & -x_2^* \\ -x_2^* & x_4 & -x_1 & -x_5 \\ x_4^* & -x_2 & -x_5^* & x_1 \end{pmatrix},$$

$x_1 = -i(\Delta/2 - \sigma)$, $x_2 = ik + i2\sigma e^{-i3\sigma L}$, $x_3 = i\sigma$, $x_4 = i2\sigma e^{i3\sigma L}$, $x_5 = i\sigma e^{i6\sigma L}$.

The boundary conditions become

$$A_+(0) = A_0, \quad A_-^*(0) = B_+(L) = B_-^*(L) = 0. \tag{10}$$

After simple calculations, the solution of the system of equations (9) can be represented in the form

$$Y(z) = \sum_{i=1}^{4} \begin{pmatrix} 1 \\ \beta_i \\ \alpha_i \\ \gamma_i \end{pmatrix} C_i e^{\lambda_i z}, \tag{11}$$

where C_i are integration constants,

$$\lambda_1 = -\lambda_2 = \frac{1}{2}\sqrt{24\sigma k\cos(3\sigma L) - \Delta^2 + 12\Delta\sigma + 4k^2},$$

$$\lambda_3 = -\lambda_4 = \frac{1}{2}\sqrt{8\sigma k\cos(3\sigma L) - \Delta^2 - 4\Delta\sigma + 4k^2}$$

are the roots of the characteristic equation of system (6), and

$$\alpha_{1,2} = \frac{-i2\lambda_{1,2} + \Delta - 6\sigma}{2(k + 3\sigma e^{-i3\sigma L})}, \quad \alpha_{3,4} = \frac{-i2\lambda_{3,4} + \Delta + 2\sigma}{2(k + \sigma e^{-i3\sigma L})},$$

$$\gamma_1 = \gamma_2 = -\gamma_3 = -\gamma_4 = e^{-i3\sigma L}, \quad \beta_i = \alpha_i\gamma_i.$$

We can write the matrix equation to determine the vector $C = (C_1\ C_2\ C_3\ C_4)^\top$ using boundary conditions (10):

$$DC = R, \tag{12}$$

where $R = (A_0\ 0\ 0\ 0)^\top$ is the free term column,

$$D = \begin{pmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 e^{\lambda_1 L} & \beta_2 e^{\lambda_2 L} & \beta_3 e^{\lambda_3 L} & \beta_4 e^{\lambda_4 L} \\ \gamma_1 e^{\lambda_1 L} & \gamma_2 e^{\lambda_2 L} & \gamma_3 e^{\lambda_3 L} & \gamma_4 e^{\lambda_4 L} \end{pmatrix}.$$

Solution of Eq. (12) is given by expression

$$C_i = D_i / D_0, \tag{13}$$

where D_0 is the determinant of matrix D, D_i is the determinant receiving from D by exchange the i column to the free term column. We can see from the general form of Eq. (13) that the weak perturbation amplitudes diverge at the parameter values making $D_0 = 0$. This means that an absolute instability in the transverse direction of the counterpropagating waves $\mathcal{E}_\pm$ may appear in a nonlinear DFB structure.

The following expression for the determinant D_0 can be obtained after simple mathematic calculations:

$$D_0 = P_4\Big\{2P_0 + 2[P_1 e^{-i6\sigma L} + P_2 e^{-i3\sigma L} + P_3] \times [\cosh[(\lambda_1 - \lambda_3)L] - \cosh[(\lambda_1 + \lambda_3)L]] + \frac{1}{4}\lambda_1\lambda_3[3\sigma^2 e^{-i6\sigma L} + 4k\sigma e^{-i3\sigma L} + k^2] \times [\cosh[(\lambda_1 - \lambda_3)L] + \cosh[(\lambda_1 + \lambda_3)L]\Big\}, \tag{14}$$

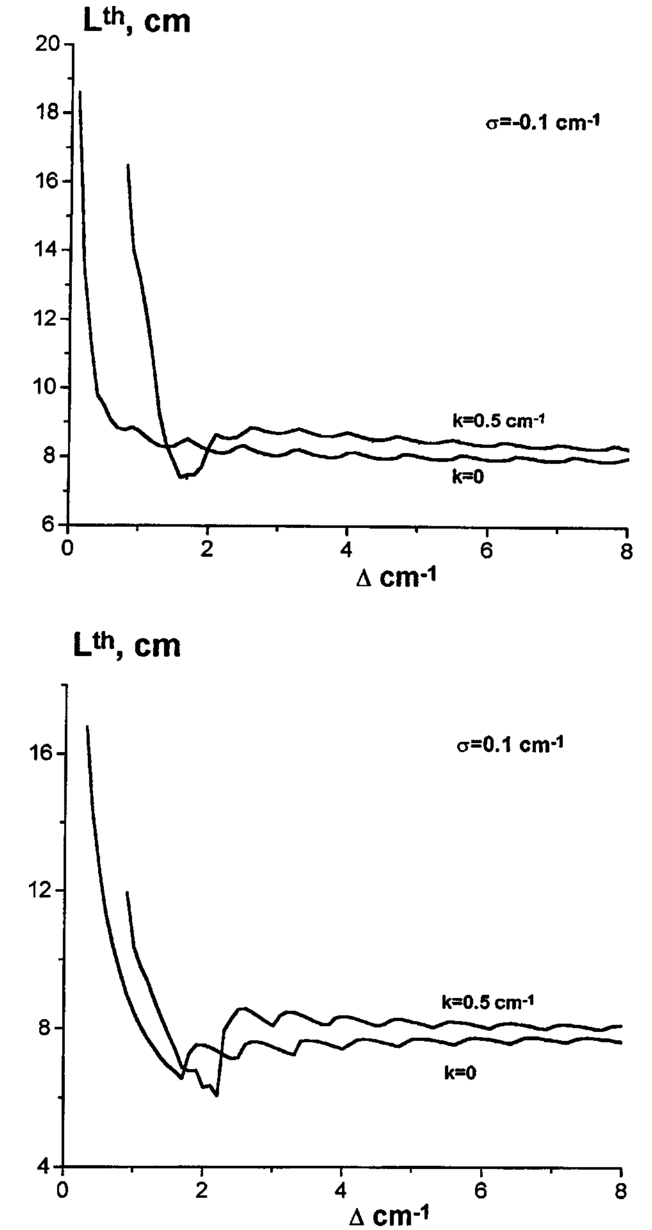

FIG. 1. DFB-structure threshold length L^{th} depends on Δ for different values of k.

where

$$P_0=\frac{1}{4}\lambda_1\lambda_3[k+3\sigma e^{-i3\sigma L}][k+\sigma e^{-i3\sigma L}],$$

$$P_1=-\frac{3}{2}\Delta^2\sigma^2+6\sigma^3\Delta+10\sigma^2k^2+36\sigma^4+24\sigma^3k\cos(3\sigma L),$$

$$P_2=8\Delta\sigma^2k+48\sigma^3k+8\sigma k^3-2\sigma k\Delta^2+24\sigma^2k^2\cos(3\sigma L),$$

$$P_3=-\frac{1}{2}\Delta^2k^2+2\sigma\Delta k^2+2k^4+16\sigma^2k^2+8\sigma k^3\cos(3\sigma L),$$

$$P_4=e^{-i6\sigma L}[k+3\sigma e^{-i3\sigma L}]^{-2}[k+\sigma e^{-i3\sigma L}]^{-2}.$$

At $k=0$ expression (14) coincides with the expression obtained in Ref. [19] for frequency-degenerated six-wave mix-

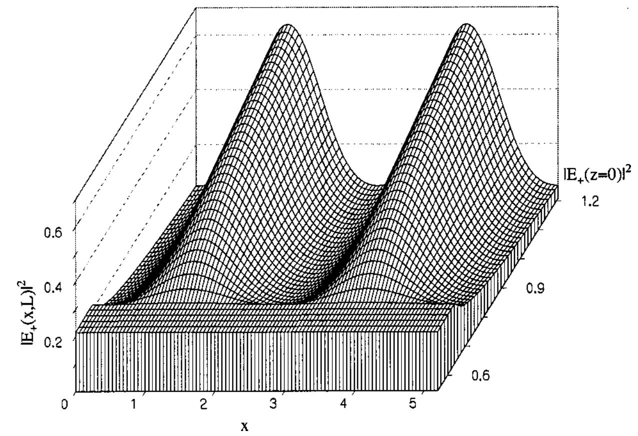

FIG. 2. Transverse distribution of transmitted radiation intensity depends on input one for $L=2$ cm, $k=1$ cm^{-1}, $\Delta_0=-4$ cm^{-1}.

ing. At $\Delta\to\infty$ we can neglect the processes of colight- and counterlight-induced diffraction. Then Eqs. (6) describe four-wave mixing [19] with the following condition for parametric generation:

$$2\sigma L=\pi/2. \tag{15}$$

In conclusion, the linear stability analysis of the homogeneous solution shows that at a critical length L^{th} of the DFB structure, D_0 vanishes and a transverse structure develops with a well defined wave number. The threshold length of the nonlinear DFB structure L^{th} as a function of Δ is shown in Fig. 1 for different values of the coupling coefficient and for both defocusing ($\sigma<0$) and focusing ($\sigma>0$) media. At $L=L^{\mathrm{th}}$ spatial instability of counterpropagating waves occurs for a definite value of Δ. As we can see from Fig. 1, at $k\neq0$ the value of L^{th} has a minimum and then increases in an oscillatory way with increasing Δ. For example, at $k=0.5$ cm^{-1}: for defocusing medium $L^{\mathrm{th}}_{\mathrm{min}}=7.41$ cm at $\Delta=1.6$ cm^{-1} and $L^{\mathrm{th}}=8.3$ cm at $\Delta=8$ cm^{-1}; for focusing medium $L^{\mathrm{th}}_{\mathrm{min}}=6.08$ cm at $\Delta=2.2$ cm^{-1} and $L^{\mathrm{th}}=8.12$ cm at $\Delta=8$ cm^{-1}.

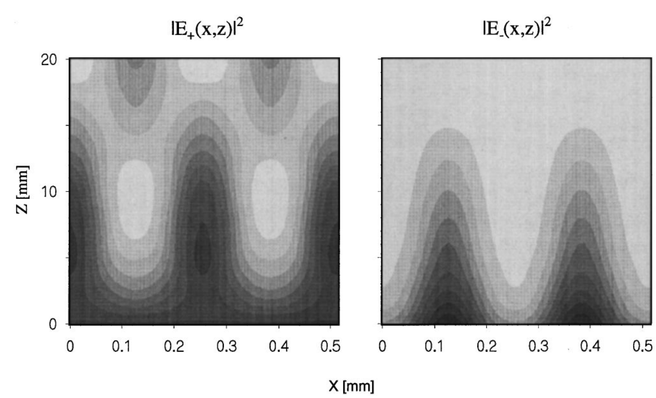

FIG. 3. Spatial distribution of forward and backward wave intensities within the nonlinear DFB structure for $k=1$ cm^{-1}, $J_{+0}=1.21$ cm^{-1}, $\Delta_0=-4$ cm^{-1}. More dark regions correspond to higher intensity.

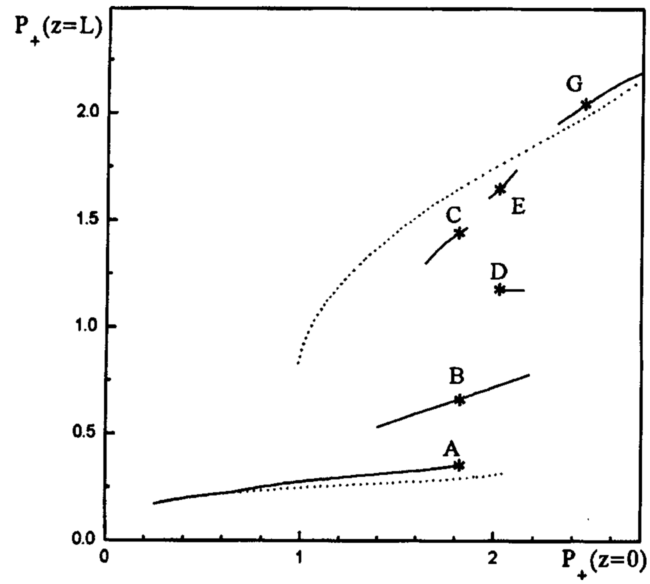

FIG. 4. Transmitted radiation average power depends on input radiation one for $k=1\ \mathrm{cm}^{-1}$, $\Delta_0=-4\ \mathrm{cm}^{-1}$, $x_0=5.2$ cm. Dotted lines correspond to the case of the plane wave approximation and solid lines are plotted with account of one-dimensional transverse diffraction.

IV. RESULTS OF NUMERICAL SIMULATIONS

We perform numerical simulations of Eqs. (3) describing the dynamics of the spatial evolution of counterpropagating wave intensities in the nonlinear DFB structure. For the sake of simplicity we consider the case of a two-dimensional pulse (the pulse size along the x axis is much smaller than the size along the y axis) incident on a defocusing medium. We solved the problem for the nonlinear DFB structure limited in transverse direction and assumed full reflection on sides boundary: $\partial E_\pm/\partial x|_{x=0}=\partial E_\pm/\partial x|_{x=x_0}=0$. All calculations were done for normalized value $\hat{E}_\pm(z,x)=\sqrt{|\gamma|}E_\pm(z,x)$. Boundary conditions (5) were chosen as

$$\hat{E}_+(0,x)=E_0+10^{-4}f(x),\quad \hat{E}_-(L,x)=0, \tag{16}$$

where $|f(x)|\leq 1$.

The dependence of the transverse distribution of the transmitted radiation intensity on the input one is shown on Fig. 2. As can be seen from Fig. 2, the spatial distribution of the transmitted intensity stays homogeneous if the input one $J_{+0}=|\hat{E}_+(z=0)|^2<0.6$. Above this threshold we can observe the fluent loss of stability of homogeneous field structure and the appearence of a periodic modulation of the intensity in the transverse direction. The modulation amplitude of the transmitted radiation increases when the input field intensity increases. It can be shown that the threshold intensity, at which counterpropagating wave instabilities appear in the nonlinear DFB structure, corresponds to the value $J_{+0}=0.6$, calculated from formula (14) at $D_0=0$ for the parameters of Fig. 2.

Spatial distributions of forward and backward wave intensities within the nonlinear DFB structure are shown in Fig. 3 for an input radiation intensity above thresold $J_{+0}>0.6$. As can be seen from Fig. 3, the spatial evolution of counterpropagating wave intensities is such that the forward wave regions with lower intensity correspond to the backward wave regions with higher ones and the opposite.

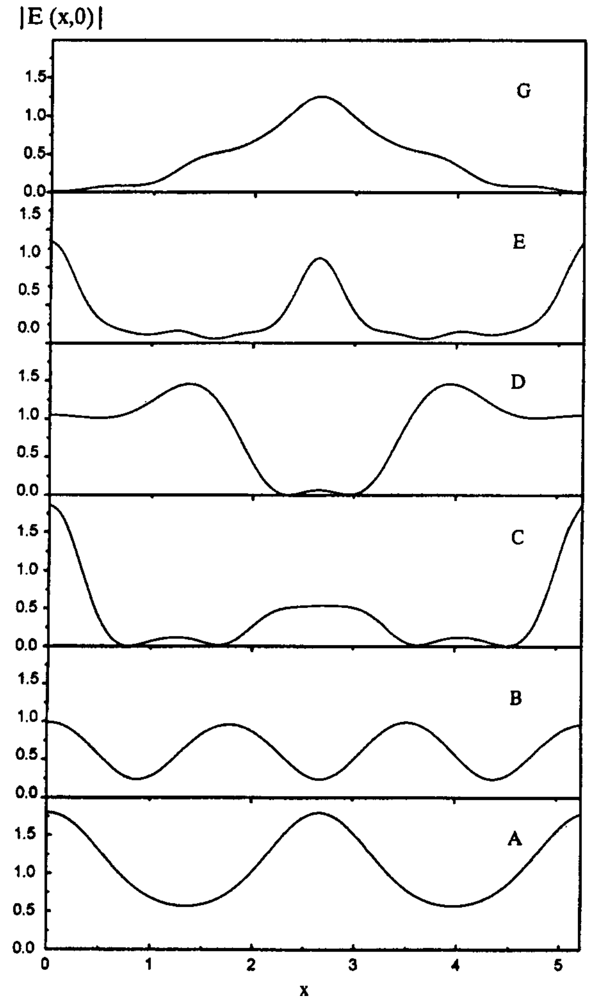

FIG. 5. Transverse structure of reflected radiation modulus amplitude for a different point of multistable curve than represented in Fig. 4.

It is known [1] that the reflection (transmission) coefficient of the nonlinear DFB structure is bistable with respect to the input wave intensity in the plane wave approximation. If we take into account diffractive effects, this result is valid only below the instability threshold of the homogeneous transverse field structure.

The appearence of wave periodic modulations due to the spatial instability changes the energy characteristics of this medium, for instance, its reflectivity. It is important to note that transverse effects induce multistability in the system which would otherwise be just bistable. This effect can be demonstrated by calculating the reflection (transmission) nonlinear DFB-structure index as a function of the input radiation intensity.

In the case of two-dimensional pulses it is convenient to introduce the counterpropagating waves average power $P_\pm=x_0^{-1}\int_0^{x_0}|\hat{E}_\pm(x)|^2dx$. Figure 4 shows how the average power of the transmitted radiation average power depends on the input radiation in the cases of the plane wave approximation (dotted lines) and taken into account one-dimensional transverse diffraction (solid lines). The aperture limitation leads to the discretization of transverse mode spectra. Due to this fact we can observe for one value of input radiation intensity at least three stable conditions of nonlinear Bragg reflector with different values of reflection coefficient and various transverse field structures (for example, points *A*, *B*, *C*, *D*, and *E* in Fig. 4). The transverse structures of the modulus of the reflected radiation are shown in Fig. 5 for different branches of the multistable curve.

In our calculations we use iterative solution methods of stationary counterpropagating wave interaction in nonlinear media based on a multicomponent scheme [20]. Note that the numerical method allows us to investigate the steady state of light fields in the nonlinear DFB structures in the presence of multistability [8].

V. CONCLUSION

In this paper we investigated the effects of spatial instability of counterpropagating waves interacting in a nonlinear DFB structure based on a transparent medium with nonrelaxing Kerr nonlinearity. We shown that homogeneous solutions destabilize under small angular perturbations at a critical value of the medium length. We performed a general linear stability analysis and we determined the threshold length of the DFB structure at which pattern formation occurs. We also studied the dynamics of the spatial distribution of the transmitted and reflected intensities by numerically solving the parabolic equations. It was shown that the spatial instability is responsible for the generation of optical patterns in the transverse distribution of the counterpropagating wave intensities in the nonlinear DFB structure. In particular, we predict that the appearance of wave periodic modulations changes the energy characteristics of the nonlinear DFB structure, for instance, its reflectivity. For example, transverse effects essentially influence the spectrum of the system generating multistable states.

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