Prospects for *p*-wave paired Bardeen-Cooper-Schrieffer states of fermionic atoms

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We present theoretical prospects for creating *p*-wave paired BCS states of magnetic trapped fermionic atoms. Based on our earlier proposal of using dc electric fields to control both the strength and anisotropic characteristic of an atom-atom interaction and our recently completed multichannel atomic collision calculations we discover that *p*-wave pairing with ^{40}K and ^{82,84,86}Rb in the low field seeking maximum spin-polarized state represents excellent choices for achieving superfluid BCS states and may be realizable with current technology in laser cooling, magnetic trapping, and evaporative/sympathetic cooling, provided the required strong electric field can be applied. We also comment on the prospects of similar *p*-wave paired BCS states in ^{6}Li, and more generally on creating other types of exotic BCS states. Our study will open a new area in the vigorous pursuit to create a quantum degenerate fermionic atom vapor. [S1050-2947(99)02509-3]

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The success of atomic Bose-Einstein condensation (BEC) [1] has induced an exponential growth of interest in the properties of ultracold dilute quantum gases. Of particular interest now is the physics of the trapping and cooling of fermionic atoms [2]. Indeed the prospect of superfluidity with dilute atomic vapors has already been studied by several groups [3–6].

In this paper we present a theoretical study of the prospect of superfluidity in magnetic trapped fermionic atoms. We conduct our discussions for the three alkali-metal species currently being investigated at several labs: ^{6}Li ($I=1$), ^{40}K ($I=4$), and ^{82,84,86}Rb ($I=1,2,2$) [7]. This paper is organized as follows: We briefly review the existing proposals for BCS states of magnetically trapped atoms [3–5], after which we present our proposal based on the use of external dc-electric fields to induce anisotropic atom-atom interactions [8]. We then address its advantages and present results of our detailed multichannel atomic collision calculations. We conclude with a discussion of other possibilities of BCS states offered by our proposal, notably, the analogies of the Anderson-Brinkman-Morel [9] and Balian-Werthamer [10] states in ^{3}He [11,12].

In the standard BCS theory [13] of superconductivity the attractive interaction between electrons of opposite spin and momentum causes the (Cooper) instability of the filled Fermi sea ground state. Physically, this effect can be roughly understood as some kind of condensation of ''bosonic'' Cooper pairs [12,13]. In the weak-coupling limit when $k_B T_c \ll \hbar\omega_D$, ($\hbar\omega_D$ is a characteristic energy over which an attractive interaction persists around the Fermi surface), the transition temperature T_c is

$$k_B T_c \sim \hbar\omega_D \exp\left(-\frac{1}{N(0)|V|}\right). \tag{1}$$

$N(0)$ is the electronic density of states at the Fermi surface, and is $\propto k_F$, the Fermi momentum. V is the two-body scattering amplitude, assumed constant and negative around the Fermi surface. Although derived for the case of a homogeneous system, as argued in several recent studies [3,6], Eq. (1) should also provide good estimates for current magnetic traps where the local-density approximation is expected to be valid. We note that the computation of V is a complicated and difficult many-body problem, and reliable values can rarely be obtained. For an ultracold weakly interacting dilute atom gas, the situation is quite different; the scattering amplitude can be accurately computed, and is usually dominated by the lowest several partial waves (at sub-mini-Kelvin temperatures). The effective interaction V_l for the dominant lth partial wave term is $\sim\delta_l(k_F)/k_F$, where $\delta_l(k_F)$ is the phase shift for the lth partial wave.

In the pairing of two fermions the Pauli exclusion principle requires the total wave function to be antisymmetric. Therefore, the orbital angular momentum l has to be odd for spin symmetric pairs, and even for spin asymmetric ones. For spherically symmetric interatomic potentials, which vanish exponentially, $\delta_l(k)\sim k^{2l+1}$ at low energies when $k\to 0$. For a potential of the asymptotic form $-1/R^n$ ($n>4$), the phase shift scales as $\delta_l(k)\sim k^{2l+1}$ for $l<(n-3)/2$ and as $\delta_l(k)\sim k^{n-2}$ for $l>(n-3)/2$. At a large internuclear distance R, the typical ground-state interatomic potential is asymptotically dominated by the van der Waals term $-C_6/R^6$, where C_6 is the dispersion coefficient. Therefore, the phase shift scales as $\delta_l(k)\sim k^{2l+1}$ for $l=0,1$ and as $\delta_l(k)\sim k^4$ for $l\geqslant 2$. The only significant term is then from the s wave, described by the scattering length $s_{\rm sc} \equiv -\lim_{k\to 0}\delta_0(k)/k$.

Based on the observation above, one realizes that atoms in spin-symmetric states interact only through a vanishingly small p wave ($\sim k^3$). For ^{6}Li in the triplet (electronic spin-symmetric) state, the p-wave scattering length defined as $a_{\rm sc\text{-}p}^3 \equiv \lim_{k\to 0} -\delta_1(k)/k^3$ [14], is about -35 a.u., and corresponds to a very weak attraction, while the s-wave triplet scattering length $a_{\rm sc}$ is enormous at about ~ -2160 a.u. due to a shallow bound state [14]. In a recent paper Stoof and co-workers studied the prospects for superfluidity of magnetically trapped ^{6}Li [3]. In the maximum polarized state $|6\rangle$, a p-wave BCS pairing occurs at a transition temperature of

$$T_c \sim \frac{\epsilon_F}{k_B}\exp\left[-\frac{\pi}{2(k_F|a_{\rm sc\text{-}p}|)^3}\right], \tag{2}$$

too slow to be practical. For instance, with a density of $10^{12}/\mathrm{cm}^3$, we have $\epsilon_F/k_B \sim 600$ nK and $k_F|a_{\text{sc-p}}| \sim 7\times 10^{-3}$ [3].

Kagan and co-workers suggested the trapping of atoms in multiple hyperfine states [5]. They discovered that two atoms in the same state ($|6\rangle$ of ^{6}Li) can attract through phonon-induced interactions (caused by density fluctuations in the other state, $|5\rangle$ of ^{6}Li), and pairing occurs at

$$T_c \sim \frac{\epsilon_F}{k_B}\exp\left[-13\left(\frac{\pi}{2k_F|a_{\text{sc}}|}\right)^2\right], \tag{3}$$

where the large *s*-wave scattering length (for ^{6}Li) enters. Despite a dramatic enhancement (from the k_F dependence), this scheme is still difficult to realize. For a density of about $10^{12}/\mathrm{cm}^3$ per state (a total density of $2\times 10^{12}/\mathrm{cm}^3$), one gets $k_F|a_{\text{sc}}| \sim 0.4$. The prefactor 13 in the exponent dictates an excessively low temperature.

Recently Stoof and co-workers [3] proposed an *s*-wave pairing of atoms in different hyperfine states ($|5\rangle$ and $|6\rangle$ of ^{6}Li), which occurs at

$$T_c \sim \frac{\epsilon_F}{k_B}\exp\left(-\frac{\pi}{2k_F|a_{\text{sc}}|}\right). \tag{4}$$

For a density of $\sim 10^{12}/\mathrm{cm}^3$, T_c is about a few tens of a nanokelvin, potentially within experimental reach if the atomic decay channels are suppressed [16]. There are, however, major obstacles towards trapping more than one hyperfine state simultaneously: first, the two-body exchange loss channel is now open and the dipolar rates generally also increase; second, for stable BCS states, the pairing energies for atoms in different hyperfine states have to overcome the Zeeman energy deficit. An interesting proposal was also recently suggested by Modawi and Leggett [4] utilizing the *s*-wave pairing of atoms in spin-antisymmetric states with a result similar to Eq. (4).

In a recent letter [8] we proposed to control atom-atom interactions at ultralow temperatures with a dc electric field. In the presence of a dc electric field the spherical symmetry of the interacting atoms is distorted; consequently, the long-range interatomic potential is modified by the addition of an anisotropic dipole interaction term

$$V_E(R) = -\frac{C_E}{R^3}P_2(\cos\theta), \tag{5}$$

where $C_E = 2\mathcal{E}^2\alpha_1^A(0)\alpha_1^B(0)$, $\mathcal{E}$ is the strength of the dc electric field, and $\alpha_1^{A,B}(0)$ are the static atomic dipole polarizabilities for the two atoms denoted by *A* and *B*. P_2 is the Legendre polynomial of order 2 and θ is the angle between the directions of the electric field and the internuclear axis. A complete low-energy scattering treatment reveals that the total two-body (unsymmetrized) scattering amplitude takes the form [8]

$$f(\vec{k},\hat{k}') = 4\pi\sum_{lm,l'm'} t_{lm}^{l'm'}(k)Y_{lm}^*(\hat{k})Y_{l'm'}(\hat{k}'). \tag{6}$$

The reduced *T*-matrix elements $t_{lm}^{l'm'} \equiv \lim_{k\to 0} T_{lm}^{l'm'}/k$ (which has a physical meaning similar to that of the *s*-wave scattering length a_{sc}), are all nonvanishing asymptotically at $k \to 0$. One can intuitively understand this property ($T_{lm}^{l'm'} \sim k$ in the low-energy limit) based on the discussion of the k dependence of the phase shifts of an asymptotic potential $-1/R^n$. We found that the couplings between different channels are essentially given by $V_2 \sim 1/R^3$. Thus, for each channel, the effective potential generated by the couplings behaves in the limit of large R as $1/R^6$. Therefore, the character of the asymptotic R dependence of the total effective potential for each partial-wave equation is decided by the diagonal part of the total potential. For $l=0$ the effective potential behaves as $1/R^6$ and so $\delta_0 \sim k$. For $l\neq 0$ the effective potentials behave as $1/R^3$ and so $\delta_l \sim k$. The anisotropic nature required new numerical techniques to ensure the stability of the low-energy scattering calculations. Detailed discussions of the complete *T*-matrix elements for all alkali-metal atoms will be reported elsewhere [17].

Both the magnitude and sign of the $t_{lm}^{l'm'}$ are tunable by changing $\mathcal{E}$ [8]. Quite generally, we found that within the same l and l' manifold the $m=m'=0$ is always the largest $t_{lm}^{l'm'}$ term, and the larger m and m', the smaller $t_{lm}^{l'm'}$. Away from resonances, smaller l and l' correspond to larger $t_{lm}^{l'm'}$. Such a situation allows for the general pairing schemes as in ^{3}He [9,10,12]. One obtains the $T=0$ gap equation (in the weak-coupling limit),

$$\mathcal{C}(\vec{k}) = \frac{4\pi\hbar^2}{M}\sum_{\vec{k}'}\sum_{lm}\sum_{l'm'}(4\pi)\,t_{lm}^{l'm'}Y_{lm}^*(\hat{k})Y_{l'm'}(\hat{k}')\frac{\mathcal{C}(\vec{k}')}{2E_{\vec{k}'}}, \tag{7}$$

where the general quasiparticle excitation spectra is

$$E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + |\mathcal{C}(\vec{k})|^2} \tag{8}$$

for the homogeneous case. $\epsilon_{\vec{k}} = \hbar^2k^2/2M - \epsilon_F$ is the bare particle energy (measured from the Fermi surface at ϵ_F). Such a general pairing scheme results in an anisotropic gap $\mathcal{C}(\vec{k})$ and the excitation spectra $E_{\vec{k}}$ [9,10,12]. In particular, when one of the t_{lm}^{lm} is made dominant and attractive (positive), we can simplify the analysis by assuming

$$\mathcal{C}_{lm}(\vec{k}) \approx \Delta_{lm}Y_{lm}^*(\hat{k}). \tag{9}$$

One then obtains the ($T\neq 0$) gap equation

$$1 = \frac{4\pi\hbar^2}{M}\sum_{\vec{k}'}(4\pi)\,t_{lm}^{lm}|Y_{lm}(\hat{k}')|^2\frac{1}{2E_{\vec{k}}}\tanh\left(\frac{1}{2}\beta E_{\vec{k}}\right), \tag{10}$$

with $\beta = 1/k_BT$. At T_c, $\Delta_{lm}(T_c)=0$. Equation (10) can be approximately solved to give a result similar to Eq. (1),

$$T_c \sim \frac{\epsilon_F}{k_B}\exp\left(-\frac{\pi}{2k_F|t_{10}^{10}|}\right), \tag{11}$$

except for the substitution of $V \to -t_{lm}^{lm}$. The case of a *p*-wave pairing with t_{10}^{10} being the dominant (and positive) term is indeed true for ^{6}Li, ^{40}K, and ^{82,84,86}Rb in the maximum polarized state. These paired superfluid states are the analogues of the ^{3}He in the A_1 phase.

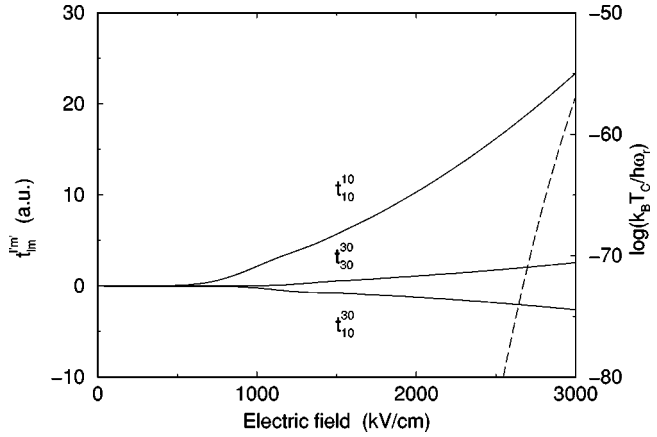

FIG. 1. For ^{6}Li with $\lambda=1$, $N=10^6$, and $\omega_r=(2\pi)400$ Hz. We then have $a_r=1.45$ μm, $\epsilon_F\approx 181.7\hbar\omega_r$, $k_F\approx 13.5/a_r$, and $n(\vec{r}=\vec{0})=1.36\times 10^{13}$ cm^3. The solid lines denote t_{lm}^{lm} in a.u. and refer to the left vertical scale, while the dashed line denotes k_BT_c (for pairing with t_{10}^{10}) in units of $\hbar\omega_r$ and refers to the (10 based) logarithmic right vertical scale.

Such a scheme is interesting since pairing involves only atoms in the same hyperfine state, yet the strength of the pairing can be as strong as that for a *s*-wave pairing in different hyperfine states [3,4,18]. This greatly increases the critical temperature for the BCS pairing to occur. The actual values of $t_{lm}^{l'm'}$ are illustrated by our calculations in Figs. 1, 2, and 3 for ^{6}Li, ^{40}K, and ^{82,84,86}Rb, respectively, although hyperfine structures are not included in these calculations presented. We expect they give the correct order of magnitude and the $\mathcal{E}$ dependence since collisions among atoms in the maximum polarized state mainly proceed along the triplet potential curves. We have also estimated the critical temperatures, Eq. (11), for typical trap parameters. They are plotted in dashed lines with respect to the logarithmic vertical axis to the right. The number density at the trap center is higher than the 10^{12} cm^3 used for previous estimates, justified since all inelastic processes would be slower in our scheme, the same reason why the BEC was originally realized in similar maximum polarized states. In the maximum polarized states, the spin-exchange collisional loss is eliminated and the two-body dipolar rates are suppressed. The three-body loss inside an electric field is a much more complicated issue currently under investigation. This rate seems highly reasonable since the centrifugal barrier in the *p*-wave collision channel prevents close encounters of two atoms at short distances.

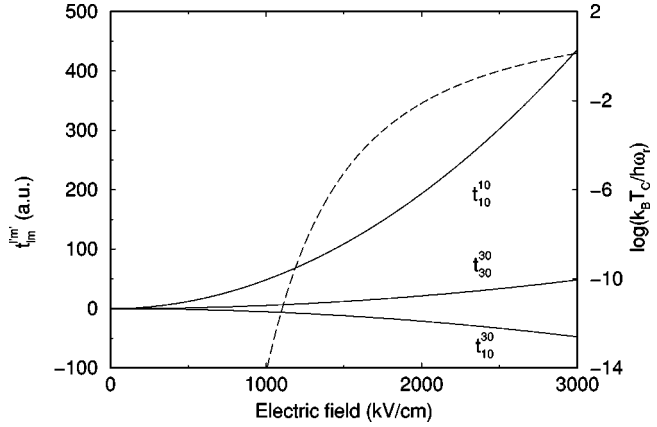

FIG. 2. The same as for Fig. 1, but now for ^{40}K with $\lambda=0.1$, $N=10^6$, and $\omega_r=(2\pi)400$ Hz. We then have $a_r=0.56$ μm, $\epsilon_F\approx 84.3\hbar\omega_r$, $k_F\approx 9.2/a_r$, and $n(\vec{r}=\vec{0})=7.45\times 10^{13}$ cm^3.

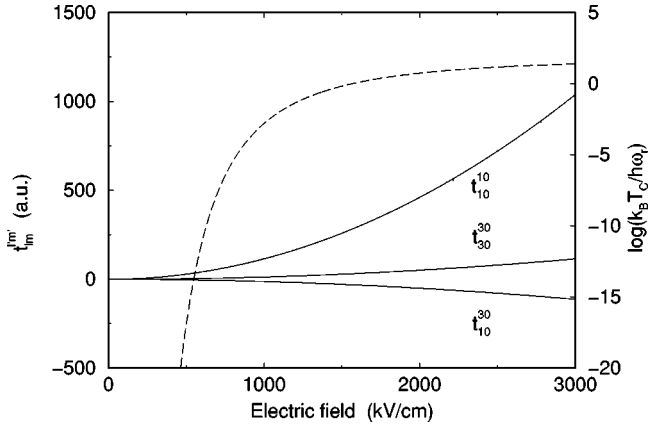

FIG. 3. The same as for Fig. 2, but now for ^{82,84,86}Rb with $a_r\sim 0.39$ μm and $n(\vec{r}=\vec{0})\sim 2.1\times 10^{14}$ cm^3. All three isotopes give similar results since hyperfine structures are not included here. The half lifetimes of the three isotopes are 1.25 min, 32.9 and 18.8 days, respectively.

More generally, one may consider possibilities of pairing with $t_{1\pm1}^{1\pm1}$ terms if resonance structures can be induced. Such resonances may also provide larger $t_{lm}^{l'm'}$ values at smaller dc electric fields. We are currently pursuing detailed atomic collision calculations including hyperfine structures inside a dc electric field to answer these questions. Inside an optical trap [19], different Zeeman states of the same hyperfine spin state are degenerate, but the effect of the dc electric field can still be induced. In such a case one would expect that similar to the analysis of the Anderson-Brinkman-Morel states, Balian-Werthamer states, and Leggett states in ^{3}He [9,10,12], one can consider all kinds of spin symmetries of the paired atoms [18]. This would represent a more interesting scenario than in the case of ^{3}He because the total spins can be much larger, e.g., for ^{40}K, $f=9/2$ and 7/2, respectively, in the two ground-state hyperfine manifolds [18].

As illustrated in Fig. 1 for ^{6}Li, within the range of the dc electric field considered, the induced *p*-wave interaction is too weak for the BCS pairing to occur at reasonable temperatures. This is a result of the small electric polarizability of ^{6}Li. However, for ^{40}K (Fig. 2) and ^{82,84,86}Rb (Fig. 3), pairing can be induced to occur at temperatures $\sim\hbar\omega_r/k_B$ for a dc electric field of about 3000 kV/cm and 1000 kV/cm, respectively. This should represent a reasonable prospect to aim for within current experimental capabilities. The estimate of T_c is based on the numbers for currently available cylindrical magnetic traps (with $\omega_z=\lambda\omega_r$, where ω_z and ω_r are, respectively, the axial and radial frequencies), i.e., with the width of the trap ground state $a_r=\sqrt{\hbar/2M\omega_r}\gg|t_{lm}^{lm}|$ [15,16]. The Fermi energy and the momentum are, respectively, $\epsilon_F=\hbar\omega_r(6\lambda N)^{1/3}$ and $k_F=(6\lambda N)^{1/6}/a_r$ for a noninteracting Fermi gas in these traps. The effective spatial density at the trap center is $n(\vec{r}=\vec{0})\approx(\lambda N/6)^{1/3}/(\pi^2a_r^3)$. Although the required electric field is high for a dc electric field configura-

tion, such field strengths can be easily achieved with lasers (e.g., CO_2 laser) in a quasistatic situation [20].

In conclusion, we have proposed a BCS pairing scheme for atoms inside an external dc electric field. We have performed detailed atomic calculations demonstrating the strong anisotropic pairing interactions for polarized atoms. By turning the *p*-wave scattering amplitude to larger values, we can effectively tune (higher) the critical temperature T_c. Compared with the existing proposals, our scheme seems to have several distinct advantages: First, the pairing only involves atoms in the maximum polarized state. This allows the same phase-space density to be reached at a lower spatial density, thus lowering three-body decays. Second, trapping inside the maximum polarized triplet hyperfine states completely eliminates the large (usually dominant) two-body decay mechanism—those due to spin exchange collisions. Additionally, it suppresses the two-body dipolar decay processes since the final decay channels are more restricted. Third, the applied dc electric field provides a tuning knob for optimal experimental control. All indications seem to be consistent with our proposal at the moment. We, therefore, encourage experimentalists to attempt the implementation of the strong electric fields required. It may also become possible to utilize the low-energy scattering resonances to induce dominant two-body attractions in *d*, *e*, and maybe even *f* waves. If the fermionic gases can be confined in an optical dipole trap similar to those recently realized for BEC at MIT [19], one may be able to create more exotic BCS states such as those of the *A* and *B* phases in ^{3}He, but with richer spin structures [4,18].

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APPENDIX: SCATTERING FROM AN ANISOTROPIC POTENTIAL

In this appendix we provide the essential formulation we developed for the scattering from the anisotropic potential containing the electric-induced dipole-dipole interaction. As was shown in [8], the atom-atom interaction potential inside a dc electric field of strength $\mathcal{E}$ is

$$V(\vec{R}) = V_0(R) + V_E(R), \tag{A1}$$

where V_0 is the symmetric part (for the usual collisional studies), and the induced anisotropic part is

$$V_E(R) = -\frac{C_E}{R^3} P_2(\cos\theta), \tag{A2}$$

where $C_E = 2\mathcal{E}^2 \alpha_1^A(0)\alpha_1^B(0)$ is the electric-induced dipole interaction coefficient and $\alpha_1^{A(B)}(0)$ are the static atomic dipole polarizabilities of atoms *A* and *B*, respectively. $P_2(.)$ is the Legendre polynomial of order 2 and θ is the angle between the directions of the electric field and the internuclear axis.

With such a potential (A1), the usual partial-wave expansion has to be modified. Consistent with the standard definition for the scattering amplitude, we denote the incident momentum (relative motion of the two atoms) by $\vec{k}$; then, the scattering wave function to be computed is

$$\psi(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}} + \frac{f(\vec{k},\hat{r})}{r} e^{ikr}. \tag{A3}$$

Note here $\vec{r}$ are the relative coordinates between the two nuclei. The scattering is described by $f(\vec{k},\hat{r})$, which (for elastic scattering) is always on the energy shell, i.e., the scattering momentum $\vec{k}' = k\hat{r}$. We also expand the incident wave according to

$$e^{i\vec{k}\cdot\vec{r}} = 4\pi \sum_{lm} i^l j_l(kr) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{r}), \tag{A4}$$

with the asymptotic expansion

$$j_l(kr) \sim \frac{1}{kr} \sin\left(kr - l\frac{\pi}{2}\right), \quad r \to \infty. \tag{A5}$$

Therefore,

$$e^{i\vec{k}\cdot\vec{r}} \sim \frac{4\pi}{kr} \sum_{lm} i^l \sin\left(kr - l\frac{\pi}{2}\right) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{r}). \tag{A6}$$

The scattering amplitude $f(\vec{k},\hat{r})$ can also be expanded onto the (complete) basis $Y_{lm}(\hat{r})$ to yield

$$f(\vec{k},\hat{r}) = \frac{4\pi}{k} \sum_{lm} T_{lm}(\vec{k}) Y_{lm}(\hat{r}). \tag{A7}$$

We then have for the scattering solution,

$$r\psi_{\vec{k}}(\vec{r}) = \phi_{\vec{k}}(\vec{r}) = \frac{4\pi}{k} \sum_{lm} i^l \left[Y_{lm}^*(\hat{k}) \sin\left(kr - l\frac{\pi}{2}\right) + T_{lm}(\vec{k}) e^{ikr - il\pi/2} \right] Y_{lm}(\hat{r}). \tag{A8}$$

Therefore, the solution $\phi_{\vec{k}}(\vec{r})$ can also be solved in the multichannel $Y_{lm}(\hat{r})$ basis. We have the coupled equation

$$H_l \phi_{lm}(\vec{r}) = \sum_{l'm'} i^{l'-l} \langle lm | V(\vec{r}) | l'm' \rangle \phi_{l'm'}, \tag{A9}$$

where here H_l is the effective Hamiltonian for the *l*th partial wave,

$$h_l = -\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}. \tag{A10}$$

The boundary conditions for the scattering are as from Eq. (A8),

$$\phi_{lm} \sim Y^*_{lm}(\hat{k}) \sin\left(kr - l\frac{\pi}{2}\right) + T_{lm}(\vec{k}) e^{ikr - il\pi/2}. \quad \text{(A11)}$$

Rewrite the total potential as

$$V(\vec{r}) = V_0 + V_2 \sqrt{4\pi} Y_{20}(\hat{r}), \quad \text{(A12)}$$

with

$$V_2(\vec{r}) = -\frac{C_3}{r^3}\frac{1}{\sqrt{5}}. \quad \text{(A13)}$$

We obtain

$$\langle lm|V(\vec{r})|l'm'\rangle = V_0 \delta_{ll'}\delta_{mm'} + \delta_{mm'} V_2 \langle lm|20|l'm\rangle, \quad \text{(A14)}$$

and the final multichannel form for the scattering equations,

$$h_l \phi_{lm}(r) = V_0 \phi_{lm}(r) + \sum_{l'} i^{l'-l} \langle lm|V(\vec{r})|l'm'\rangle \phi_{l'm}(r). \quad \text{(A15)}$$

Due to the symmetry of the matrix element $\langle lm|20|l'm'\rangle = 0$, if $l + l' + 2 = \text{odd}$. We immediately see that the even and odd parity (l) channels decouple. Specifically, if we keep open only the lowest two channels, we obtain

$$\begin{aligned} h_0 \phi_{00}(r) &= V_0 \phi_{00}(r) - V_2 \phi_{20}(r), \\ h_2 \phi_{20}(r) &= \left(V_0 + \frac{2}{7}\sqrt{5} V_2\right) \phi_{20}(r) - V_2 \phi_{00}(r), \end{aligned} \quad \text{(16)}$$

and

$$\begin{aligned} h_1 \phi_{10}(r) &= \left(V_0 + \frac{2}{\sqrt{5}} V_2\right) \phi_{10}(r) - 3\sqrt{\frac{3}{35}} V_2 \phi_{30}(r), \\ h_3 \phi_{30}(r) &= \left(V_0 + \frac{4}{3\sqrt{5}} V_2\right) \phi_{30}(r) - 3\sqrt{\frac{3}{35}} V_2 \phi_{10}(r), \end{aligned} \quad \text{(A17)}$$

$$\begin{aligned} h_1 \phi_{1\pm1}(r) &= \left(V_0 - \frac{1}{\sqrt{5}} V_2\right) \phi_{1\pm1}(r) - 3\sqrt{\frac{2}{35}} V_2 \phi_{3\pm1}(r), \\ h_3 \phi_{3\pm1}(r) &= \left(V_0 - \frac{1}{\sqrt{5}} V_2\right) \phi_{3\pm1}(r) - 3\sqrt{\frac{2}{35}} V_2 \phi_{1\pm1}(r). \end{aligned} \quad \text{(A18)}$$

To match the boundary conditions, we further expand the matrix scattering element according to

$$T_{lm}(\vec{k})/k = \sum_{l'm'} t_{lm}^{l'm'}(k) Y_{l'm'}(\hat{k}). \quad \text{(A19)}$$

As discussed in detail in [8], the nature of the asymptotic dipole-dipole interaction potential is such that all $t_{lm}^{l'm'}(k)$ become independent of k at low energies. Upon matching with the numerically integrated solutions, we obtain all $t_{lm}^{l'm'}(k)$.

The total scattering amplitude is, therefore,

$$f(\vec{k},\hat{r}) = f_{\text{even } ll'}(\vec{k},\hat{r}) + f_{\text{odd } ll'}(\vec{k},\hat{r}). \quad \text{(A20)}$$

For identical particle scattering one has to worry about the symmetrized (antisymmetrized) amplitude f_S (f_A) for bosons (fermions). In the spin-triplet electronic state of two atoms, one has

$$f_S(\vec{k},\hat{r}) = \frac{1}{\sqrt{2}}[f(\vec{k},\hat{r}) + f(\vec{k},-\hat{r})] = \sqrt{2} f_{\text{even } ll'}(\vec{k},\hat{r}), \quad \text{(A21)}$$

and

$$f_A(\vec{k},\hat{r}) = \frac{1}{\sqrt{2}}[f(\vec{k},\hat{r}) - f(\vec{k},-\hat{r})] = \sqrt{2} f_{\text{odd } ll'}(\vec{k},\hat{r}). \quad \text{(A22)}$$

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