Bloch-like quantum multiple reflections of atoms

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We show that under certain circumstances an atom can follow an oscillatory motion in a periodic laser profile with a Gaussian envelope. These oscillations can be well explained by using a model of energetically forbidden spatial regions. The similarities and differences with Bloch oscillations are discussed. We demonstrate that the effect exists not only for repulsive but also for attractive potentials, i.e., quantum multiple reflections are also possible. [S1050-2947(99)09408-1]

I. INTRODUCTION

During the past few years the development of different and powerful laser cooling techniques [1] has led to the experimental observation of several striking matter-wave phenomena for cold atoms. These phenomena resemble the usual effects observed in standard optics, and therefore have been the subject of study of a fast-developing research field properly called *atom optics*. Some atom optics examples could be the diffraction of atoms [2], interferometers [3] and lenses [4] for atoms, and very recently even the atom optics equivalent of a laser source, or atom laser [5]. Atomic mirrors have also been constructed using evanescent laser fields [6] or, more recently, with magnetic fields [7]. Both mirrors, although based on different physical effects, share the same physical background, i.e., the atom ''feels'' a repulsive potential, and if its initial velocity is sufficiently small, it reaches a turning point, being reflected. Note that after the turning point the atomic density of probability follows an exponential decay, resembling the case of an electric field impinged onto a metallic surface. Therefore, these atomic mirrors can be considered the atom optics counterpart of metallic mirrors in light optics.

On the other hand, in the last few years, the behavior of a cold atom in periodic laser potentials (light lattices) has aroused great interest, in particular the resemblance of this physical situation and solid-state physics [8]. In this sense, Bragg scattering has been analyzed in the context of atomic waves [9], and so has the use of such an effect to construct atomic beam splitters and interferometers [10]. Bragg reflection is just a particular case of a more general situation, i.e., the so-called photonic-band-gap structures (or PBGS's) [11]. In dielectric periodic structures, some electromagnetic waves cannot propagate (basically because their corresponding energies are within an energetic gap produced by the periodic structure), and therefore are reflected. In previous papers [12] we proposed a laser arrangement that acts as an atomic-band-gap structure, in the sense that it resembles a PBGS but for atomic waves instead of electromagnetic ones. The atoms are reflected if their incoming kinetic energy lies within a gap produced by the laser periodicity. This effect produces a bandlike momentum spectrum of the reflected atoms. We have shown that the atoms can be reflected not only by repulsive laser potentials, but also by attractive ones, allowing

the atom optics equivalent of a multilayer dielectric mirror in light optics.

Among the different solid-state-like phenomena recently reported, is of special interest the analysis of the behavior of cold atoms accelerated in a periodic potential. In this sense, the well-known Bloch oscillations and their stationary counterpart, i.e. the Wannier-Stark ladders, have been experimentally observed in the context of atom optics [13].

The main new idea of the present paper is that an atomic beam dropped onto a laser potential like that of Ref. [12] can undergo under certain conditions multiple oscillations inside the laser region. We show that this effect can be physically explained using an image of energetically forbidden spatial regions, what we call spatial gaps. In particular the effect is produced due to a combination of partial Landau-Zener tunneling through sufficiently narrow spatial gaps, and a partial reflection on them. In this sense the laser arrangement behaves as an atomic-wave Fabry-Perot interferometer [14]. We have also analyzed the similarities and differences between these oscillations and Bloch oscillations. Due to these similarities and differences we call the effect Bloch-like oscillations. We show that the effect can be easily observed analyzing either the reflected momentum spectrum (where Wannier-Stark-like resonances appear) or the temporal evolution of an atomic wave packet. We prove that the multiple reflections also appear for attractive potentials, i.e., quantum multiple reflections are also possible.

The scheme of the paper is as follows. In Sec. II we briefly review the theoretical model already presented in Ref. [12]. Section III is devoted to the development of the spatial gaps image. Section IV uses the spatial gap image to explain the appearance of resonances in the momentum spectrum of the reflected atoms, and compare the effect with a Fabry-Perot interferometer. Section V discuss the similarities and differences between the presented oscillations and Bloch oscillations. In Sec. VI we present the temporal evolution of an atomic wave packet for the case of a repulsive potential, whereas in Sec. VII the case of quantum multiple reflections is considered. We finalize in Sec. VIII with some conclusions.

II. THEORETICAL MODEL

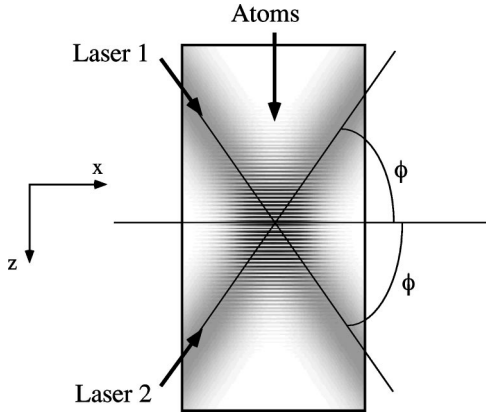In this section we briefly discuss the theoretical model we use in the paper. For a more detailed discussion, see Ref.

FIG. 1. Scheme of the considered laser arrangement. Two Gaussian lasers propagate, forming and angle ϕ and $-\phi$ with the x axis. The atoms are dropped from a trap onto the interference region of both lasers, where a periodic profile is formed.

[12]. Let us consider the same laser arrangement as that of Ref. [12], formed by the interference of two laser beams (Fig. 1) of the same intensity, polarization, and with the same Gaussian profile, but with respective wave vectors $\vec{k}_1 = k(\cos\phi\vec{u}_x + \sin\phi\vec{u}_z)$ and $\vec{k}_2 = k(\cos\phi\vec{u}_x - \sin\phi\vec{u}_z)$. In the zone very close to $x=0$ the electric field, which is assumed to be linearly polarized in the y-direction, can be written in the form

$$\vec{E} \simeq E_0\vec{e}_y \exp\left[\frac{-(z-z_c)^2\cos^2\phi}{d^2}\right]$$
$$\times \cos[k(z-z_c)\sin\phi]\cos(kx\cos\phi - \omega t), \quad (1)$$

where $E_0/2$ is the amplitude of each laser, z_c is the center of the laser region, and d is the halfwidth of the Gaussians. Let us assume that the laser frequency ω is quasiresonant with some atomic transition between the ground state and an excited state (of energies $\hbar\omega_g$ and $\hbar\omega_e$, respectively), and therefore we can treat the atom as a two-level system. In this paper, we assume a large internal detuning $[\Delta = \omega - (\omega_e - \omega_g)]$, in such a way that the adiabatic approximation will be valid [15]. In order to obtain a scalar Schrödinger equation to describe the atomic interaction with the laser, we follow the standard formalism developed in Ref. [15], except for the inclusion of the gravitational field. The scalar equation takes the form

$$-\frac{\hbar^2}{2M}\frac{d^2}{dz^2}\psi(z) = \left[\frac{q^2}{2M} - V(z)\right]\psi(z), \quad (2)$$

where

$$V(z) = -Mgz - \frac{1}{2}\hbar\Delta \mp \frac{1}{2}\hbar\sqrt{\Delta^2 + 4\Omega(z)^2}, \quad (3)$$

with $\Omega(z) = \Omega_0\exp[-\cos^2\phi(z-z_c)^2/d^2]\cos[k(z-z_c)\sin\phi]$. The sign in Eq. (3) depends on the detuning: the "$-$" sign corresponds to $\Delta<0$, and the "$+$" sign corresponds to $\Delta>0$. The coupling is given by $\Omega_0 = -\mu E_0/2\hbar$, which is the Rabi frequency associated with each laser, where $\mu = \langle\vec{e}_y\vec{\mu}\rangle$, with $\vec{\mu}$ the transition dipole. q is the z momentum

in $z=0$, sufficiently far away from z_c to consider that the field in $z=0$ is negligible. To make the model as realistic as possible, one could include the diffuse scattering resulting from spontaneous emission by adopting the lossy vector Schrödinger equation method of Ref. [16], i.e., $\Delta \rightarrow \Delta + i\gamma/2$, where γ is the spontaneous emission frequency. However, a large detuning is considered in this paper to avoid the effects of the spontaneous emission.

Using convenient units of length (k^{-1}), momentum $(\hbar k)$, and frequency $(\omega_\nu = \hbar k^2/2M)$, we can rewrite the Schrödinger equation in a dimensionless form:

$$\frac{d^2}{d\tilde{z}^2}\psi(\tilde{z}) = -\left\{\tilde{q}^2 + \beta\tilde{z} + \frac{1}{2}\tilde{\Delta} \pm \frac{1}{2}\sqrt{\tilde{\Delta}^2 + 4\tilde{\Omega}(\tilde{z})^2}\right\}\psi(\tilde{z}), \quad (4)$$

in which the tilde denotes dimensionless units. In Eq. (4), we find the function $\tilde{\Omega}(z) = \tilde{\Omega}_0\exp(-[(\tilde{z} - \tilde{z}_c)^2\cos^2\phi]/\tilde{d}^2)\cos[(\tilde{z}-\tilde{z}_c)\sin\phi]$. β is a gravitational parameter which introduces some differences depending on the mass of the atom. Since we shall assume that $\tilde{\Omega}_0 \ll |\tilde{\Delta}|^2$, we can define a parameter $\eta = \tilde{\Omega}_0^2/\tilde{\Delta}$ which determines the strength of the laser potential. This can be easily shown by introducing a Taylor expansion on the right hand side of Eq. (4),

$$\frac{d^2}{d\tilde{z}^2}\psi(\tilde{z}) = -\{\tilde{q}^2 + \beta\tilde{z} - \tilde{V}(z)\}\psi(\tilde{z}), \quad (5)$$

where

$$\tilde{V}(\tilde{z}) = \eta\exp[-2\cos^2\phi(\tilde{z}-\tilde{z}_c)^2/\tilde{d}^2]\cos^2[(\tilde{z}-\tilde{z}_c)\sin\phi]. \quad (6)$$

Note that the potential is formed by the product of a cosine squared function and a Gaussian envelope.

In all the figures throughout the paper, we analyze the case of the $2s$-$2p$ ^{7}Li transition, whose parameters are $\lambda = 670.8$ nm, $\omega_\nu = 3.96\times10^5$ s^{-1}, $\gamma = 3.72\times10^7$ s^{-1}, and $\beta = 2.93\times10^{-4}$. The position of the center of the laser region is at $z_c = 300\sqrt{2}\pi k^{-1} = 0.142$ mm, and the halfwidth of the laser Gaussians is given by $d = 100\pi k^{-1} = 33.5$ μm.

III. SPATIAL GAPS IMAGE

In this section we present a model which allows us to understand the physics behind the results we will observe in the following sections. From Eq. (6) we observe that the potential $\tilde{V}(\tilde{z})$ is quasiperiodic, except for the Gaussian envelope, its periodicity given by $\Delta\tilde{z} = \pi/\sin\phi$. In order to understand the effects of periodicity better, let us remove gravitation and spontaneous emission, and for the time being let us forget the smooth Gaussian dependence. With these assumptions, the laser potential is a simple cosine squared potential, whose amplitude (η) can be positive (if $\tilde{\Delta}>0$) or negative (if $\tilde{\Delta}<0$), depending on which dressed state is reached. It is well known that a periodic potential leads to an energy structure of allowed and forbidden bands [17]. Only the incoming momentum components q whose associated ki-
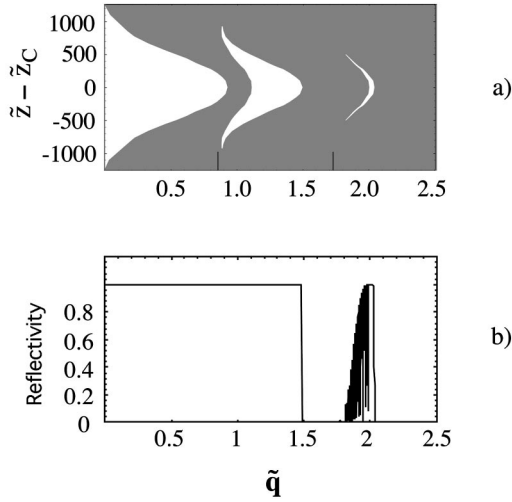
FIG. 2. Results as a function of the z momentum q at $z=0$ for $\eta=2.0$, $\tilde{z}_C=300\sqrt{2}\pi$, $\tilde{d}=100\pi$, and $\phi=\pi/3$. (a) Spatial band structure. White regions correspond to three different spatial gaps. Small vertical bars indicate the geometrical Bragg modes. The atoms are assumed to travel in the graph initially from down to up, beginning with momentum q at the bottom ($\tilde{z}=0$) of the figure. (b) Reflectivity without gravitation and spontaneous emission.

netic energy lies within an allowed band can propagate inside the potential. If q does not satisfy the band condition (i.e., if $q^2/2M$ lies in a gap), then the atoms with this incoming momentum cannot propagate inside the laser region and are therefore reflected, reflection bands being formed.

However, this image is excessively simple. In order to obtain physical insight into the numerical results, obtained by direct resolution of the Schrödinger equation (5), we must take into account the Gaussian laser envelope. The analysis is greatly simplified if, as we consider in this paper, the width of the Gaussian envelope is very large compared to the cosine squared periodicity. In particular, in the cases analyzed below the width of the Gaussian envelope of the potential at $1/e$ is 245 times the cosine squared period. If this condition is satisfied, we can consider that within small intervals of the envelope we have a large number of cosine squared oscillations of approximately constant amplitude. We can therefore define an energy band structure for each of these intervals considered as the band structure calculated for an infinitely extended periodic potential with this constant amplitude. We can extend this reasoning and define a local band structure for each position z_0 inside the laser region, or in other words for each value of the Gaussian envelope [$V_{\text{env}}(z_0)$]. This local band structure is calculated for a cosine squared potential of infinite number of periods with constant amplitude $V_{\text{env}}(z_0)$. Therefore, each position within the laser region is linked with an energetic spectrum of allowed and forbidden energies, a spatially dependent band structure being formed. Figure 2(a) shows such a spatial band structure for the case of a repulsive potential of $\eta=2.0$. In particular, for certain momenta some spatial regions are energetically forbidden (white regions). We will call these regions *spatial gaps*. Figure 2(b) shows the reflectivity for different incoming momentum components q, obtained by direct resolution of Eq. (5). The spatial gaps image allows an intuitive understanding of the physical processes behind
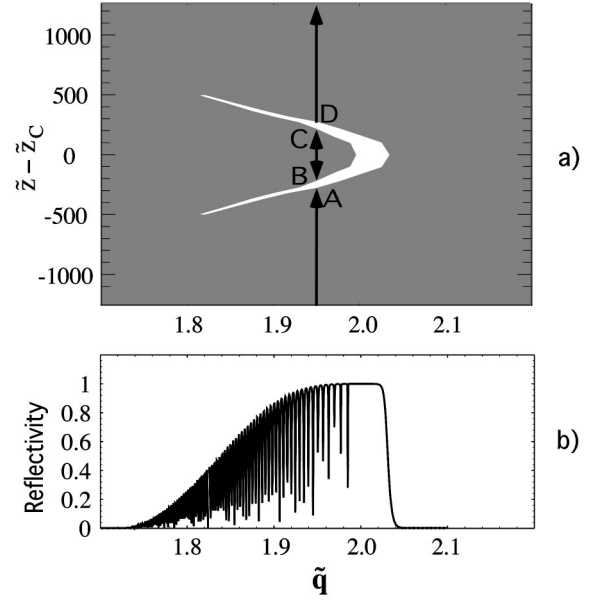

FIG. 3. (a) Detail of Fig. 2(a). (b) Detail of Fig. 2(b). Note the appearance of several resonances in the reflection spectrum.

these numerical results. If an atom with some momentum component finds during its travel inside the laser region a spatial gap, then the propagation through it is energetically forbidden, and consequently the atom is reflected. This point becomes clear by comparison between Figs. 2(a) and 2(b). In Ref. [12], we also show that this model explains very well the effects of the spontaneous emission and gravitation. In this paper we will assume a sufficiently large detuning to neglect the spontaneous emission effects. The gravitational effects will also be negligible for the momentum components of interest in this paper, and for the laser arrangement considered.

IV. FABRY-PEROT-LIKE BEHAVIOR

In Sec. III we have observed that an atom is reflected if it finds a spatial gap inside the laser region. Therefore the spatial gap acts as a potential barrier. Note the following.

(i) If the spatial gap is sufficiently narrow, then transmission via tunneling becomes possible. This tunneling produced from one allowed spatial region to another is no more than the well-known Landau-Zener tunneling between allowed energetic bands.

(ii) The form of the spatial gap depends on the specific incoming momentum one considers, and hence this effective potential barrier has the interesting property that it changes its form and width depending on the incoming atomic momentum.

For certain interval of momenta [around $\tilde{q}=2$ in Fig. 2(b)], several peaks in the momentum spectrum of reflection can be observed. These peaks are depicted in detail in Fig. 3(b). The physical process behind these peaks can be well understood by using the spatial gap image of Sec. III. In Fig. 3(a) we present in detail the region in Fig. 2(a) corresponding with the region depicted in Fig. 3(b). As we observe in this figure the spatial gap, i.e., the effective potential barrier, is very narrow in this region. This fact allows the possibility

of tunneling of the wave function through the spatial gap (from point A to point B), and therefore a partial reflection is produced. Since the envelope is symmetric, the transmitted part reaches again an spatial gap (point C), from where the atoms can be partially transmitted again by tunneling until point D, or can be reflected back to point B, from where they can be partially reflected and so on. This process leads to multiple oscillations between points B and C, i.e., the region from B to C acts as Fabry-Perot cavity [18], where B and C act as lossy mirrors. As we indicated previously, since the form of the effective barrier depends on the momentum, the length of the ''cavity'' depends also on the momentum. Following the optical analogy, this is the same case we would have if the length of a Fabry-Perot cavity was different depending on the wavelength of the light. The peaks we find in the momentum spectrum of the reflection can therefore be explained as resonances of this special Fabry-Perot cavity.

V. BLOCH OSCILLATIONS AND WANNIER-STARK LADDERS

Another way to understand these resonances in the reflection spectrum is provided by a well-known phenomenon of solid-state physics, namely the Wannier-Stark (WS) ladders [19]. This effect is simply a stationary counterpart (i.e., in the frequency domain) of another effect in the time domain called Bloch oscillations (BO's). Let us briefly review this concept. BO's appear when particles within a periodic potential are affected by a constant acceleration. We must point out that although this effect was initially observed in the solid-state context [20], several recent experiments [13] have reported the same phenomena in atom optics. BO's can be easily understood for weak potentials [21]; due to the acceleration, the momentum of the particles increases linearly according to Newton's law until it reaches a critical value satisfying the Bragg condition; then the atomic wave is reflected and its momentum is reversed. The atom travels again under Newton's law until it reaches other Bragg condition and then it is reflected again. Then the BO's can be understood as multiple reflections between two Bragg reflections. For larger potentials the BO's can be understood using the spatial gaps image (as pointed out previously, Bragg reflection is a particular case of the band-gap structures for weak potentials). Let us consider an infinitely extended periodic potential of constant amplitude affected by an external force. Let us suppose that this force is linear in the spatial coordinate. Hence the band structure varies in the space leading to tilted allowed and forbidden spatial regions [Fig. 4(a)]. Let us suppose a particle inside an allowed region. This particle moves until it reaches a forbidden region (point A), from where it is reflected. Then it travels again within the allowed region until it finds another forbidden region (point B), from where it is reflected, and so on, leading to multiple oscillations. Note that in absence of external force, no tilting is present [Fig. 4(b)], and hence there are not multiple oscillations.

We observe that the oscillations reported in Sec. IV are certainly similar to the BO's. In particular, both are due to the same reason, i.e., multiple reflections between forbidden regions. However several important differences can be pointed out.
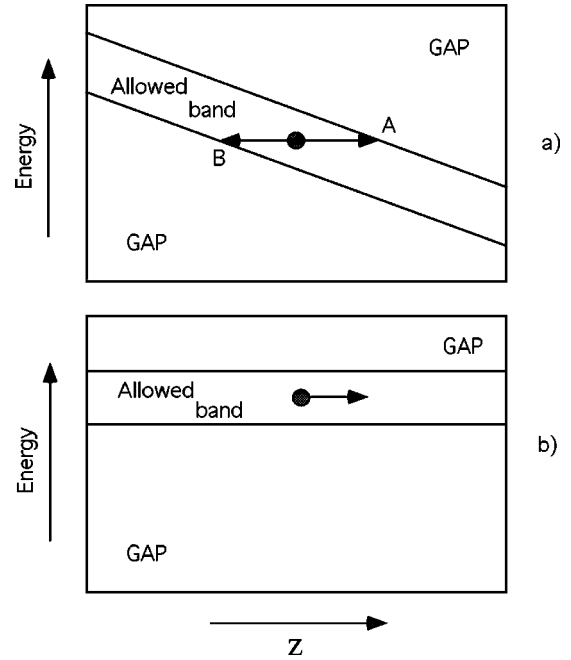

FIG. 4. (a) Scheme of an atom evolving inside an allowed region between two forbidden ones when an external constant force is applied. Therefore the atom is multiply reflected leading to the well-known Bloch oscillations. (b) Without external force there is no tilting of the spatial gaps, and the atom evolves freely inside the allowed band. Therefore, the oscillations are not produced. In both graphics the energy and z are represented in arbitrary units.

(i) The physical process which produces the ''cavity'' is basically different: in the BO's the tilting of the bands is due to an external force, while in our case is the symmetry of the Gaussian envelope.

(ii) Whereas in the BO's the reflection is produced between two different spatial gaps, in the oscillations reported here both sides of the ''cavity'' are produced by the same spatial gap, which is curved due to the Gaussian shape of the laser envelope.

(iii) In the BO's the atom is initially inside the allowed region between two forbidden ones. In our case the atom enters from outside, and therefore needs to tunnel a narrow spatial gap to enter into the internal allowed region where it oscillates.

Due to these similitudes and differences, we call the resonances in the reflection spectrum Wannier-Stark-like resonances.

VI. TIME DOMAIN. BLOCH-LIKE OSCILLATIONS

We have therefore interpreted the resonances appearing in Fig. 3(b) as Wannier-Stark-like resonances. As for standard WS ladders, the Wannier-Stark-like resonances are linked in the time domain with what we call Bloch-like oscillations. In order to observe this effect we have numerically calculated the evolution of an atomic wave packet through the laser arrangement. In order to achieve this we have solved the Schrödinger equation (5) evaluating the wave function inside and outside the laser region for different incoming momentum components q. Note that at this point turns very important the possibility to calculate the wave function inside the
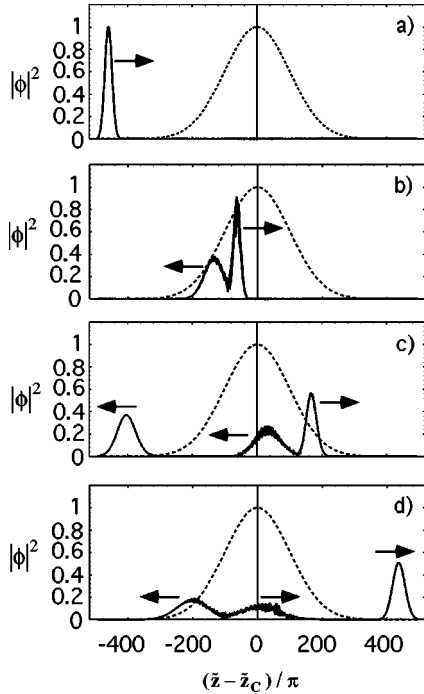
FIG. 5. Temporal evolution of an atomic wave packet through a laser region whose parameters are those of Fig. 2. The parameters of the wave packet are $\tilde{a} = 35\pi$ and $\tilde{q}_0 = 1.9$. (a) shows the density of probability $|\phi(z)|^2$ at time $t = 0$, while (b), (c), and (d), respectively, show successive snapshots at $t = 6T$, $t = 10T$, and $t = 14T$ where $T = 18\pi\omega_\nu^{-1}$. In all the figures the Gaussian shape of the laser envelope is shown in dashed lines for comparative purposes (an arbitrary scale is used to depict the Gaussian). The appearance of multiple oscillations inside the laser region is evident in this case.

laser region, which cannot be calculated by using a transfer-matrix method like that of Ref. [22]. We have employed a finite-differencing numerical method previously presented in Ref. [12], which allows us also to know the wave function inside the laser region. Once we have calculated the spatial behavior of the wave function for the different momentum components $\psi(q,z)$, we obtain the temporal evolution of an atomic wave packet by applying a one-dimensional Fourier transform:

$$\phi(z,t) = \int e^{-i(\hbar q^2 t/2M)} \psi(q,z) f_0(q) dq, \qquad (7)$$

where $f_0(q)$ is an initial Gaussian momentum distribution of the form

$$f_0(q) = e^{-a^2(q-q_0)^2/2} e^{-iqz_0}, \qquad (8)$$

where q_0 is the central momentum component of the atomic wave packet.

Figure 5 shows the evolution of an atomic wave packet with $\tilde{a} = 35\pi$ and $\tilde{q}_0 = 1.9$ through a laser arrangement as that of Fig. 3. The initial position of the center of the atomic wave packet is $\tilde{z}_0 = -35\pi$. Figure 5(a) shows the snapshot at $t = 0$, whereas Figs. 5(b), 5(c), and 5(d), respectively, show the snapshots at $t = 6T$, $10T$, and $14T$, with $T = 18\pi\omega_\nu^{-1}$. In all the figures the Gaussian form of the laser envelope is represented in dashed lines for comparative purposes. In Fig.
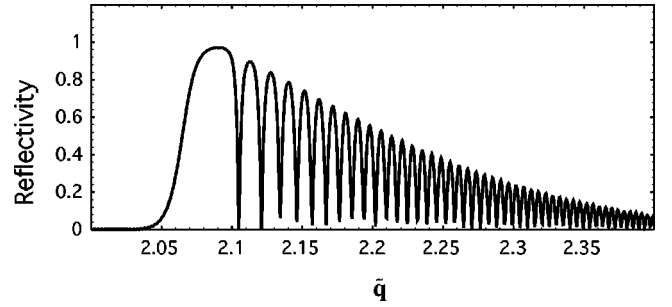

FIG. 6. Reflectivity as a function of the incoming z momentum q at $z = 0$ for $\eta = -5.0$, $\tilde{z}_C = 300\sqrt{2}\pi$, $\tilde{d} = 100\pi$, and $\phi = \pi/3$. Note the appearance of several resonances in the reflection spectrum.

5(b) a partial reflection and tunneling is clearly produced. The reflected wave packet travels to the left, disappearing, while the transmitted wave packet travels within the laser region. In Fig. 5(c) the previously transmitted wave packet is again split into two parts: a transmitted part which travels to the right, disappearing, and a reflected part which travels back in the laser region. Again in Fig. 5(d) a new splitting is produced. The observation of successive multiple reflections becomes more difficult due to the wave-packet spreading and due to the losses in the different partial transmissions.

VII. QUANTUM MULTIPLE REFLECTIONS

In this section we show that the previously reported multiple oscillations can also be observed for the case of attrac-
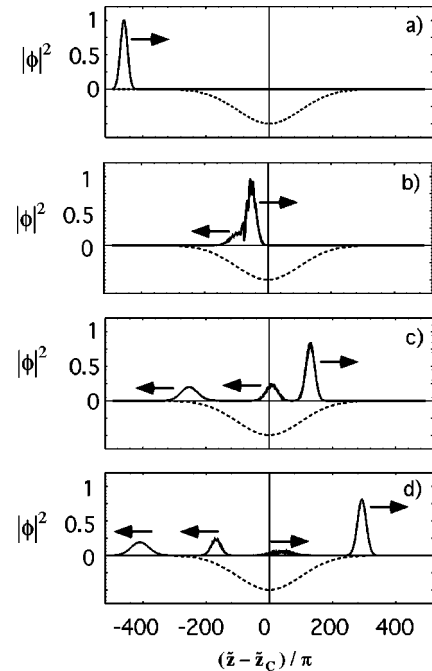

FIG. 7. Temporal evolution of an atomic wave packet through a laser region whose parameters are those of Fig. 6. The parameters of the wave packet are $\tilde{a} = 35\pi$ and $\tilde{q}_0 = 2.2$. (a) shows the density of probability $|\phi(z)|^2$ at time $t = 0$, while (b), (c), and (d), respectively, show successive snapshots at $t = 5T$, $t = 7T$, and $t = 9T$, where $T = 18\pi\omega_\nu^{-1}$. In all the figures the (negative) Gaussian shape of the laser envelope is shown in dashed lines for comparative purposes (an arbitrary scale is used to depict the Gaussian). The appearance of multiple oscillations inside the laser region is evident, even considering that in this case the potential is attractive.

tive laser potential, i.e., it is possible to achieve multiple reflections in a laser potential that classically allows none. Figure 6 shows a detail of the reflection spectrum for the case of an attractive potential with $\eta = -5.0$. We clearly observe the appearance of several resonances like those of Fig. 3. These distortions are no more than the previously analyzed WS-like resonances. As for the repulsive case, a narrow spatial gap appears, leading to the already analyzed effect of partial Landau-Zener tunneling, and therefore to multiple reflections, whose effect in the momentum domain is the appearance of the WS-like resonances. As in the previous case, Bloch-like oscillations can be observed if we monitor the evolution of an atomic wave packet through the laser region. Figure 7 shows this evolution for the case of the same atomic wave packet of Fig. 5, but now the laser is that of Fig. 6. The first snapshot [Fig. 7(a)] is at $t = 0$, whereas Figs. 7(b), 7(c), and 7(d), respectively, are obtained at $t = 5T$, $t = 7T$, and $t = 9T$, with $T = 18\pi\omega_\nu^{-1}$, where we observe different wave-packet splittings due to partial tunneling and reflection. As in the repulsive case multiple oscillations are possible, i.e., quantum multiple reflections also appear.

VIII. CONCLUSIONS

In this paper we have analyzed the reflection of an atomic beam dropped onto a laser with a periodic profile modulated by a Gaussian envelope, formed in the interference region of two Gaussian laser beams. We have numerically calculated the atomic reflection on such an arrangement by direct resolution of the corresponding Schrödinger equation, and explained the bandlike character of the reflection momentum spectrum using a band-theory model of energetically forbidden spatial regions, or spatial gaps. We have shown that due

to a partial Landau-Zener tunneling through sufficiently narrow spatial gaps, and also due to the Gaussian symmetry of the laser potential, an atom can undergo multiple reflections inside the laser region, in a process which resembles a Fabry-Perot interferometer but for atomic waves. These multiple oscillations have been compared with the well-known Bloch oscillations of solid-state physics, analyzing the similarities and differences between both oscillations. In particular, although both processes are due to multiple reflections between energetically forbidden spatial regions, in the effect presented in this paper the spatial gap is curved due to the Gaussian symmetry, and the atom oscillates between two barriers which are actually part of the same spatial gap, contrary to the Bloch oscillations in which the atom oscillates between two different spatial gaps. Also, the reported multiple oscillations are not due to an external force like Bloch oscillations, but due to the Gaussian form of the envelope. Due to these similarities and differences we have called these oscillations Bloch-like oscillations. We have analyzed the effect both in the frequency domain (where Wannier-Stark-like resonances appear) and in the time domain (observing the Bloch-like oscillations). We have proved that the effect appears not only for repulsive laser potentials, but also for attractive ones, i.e., quantum multiple reflections are also possible.

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