Overlapping identical resonances and radiative interference effects in recombination of heavy multicharged ions

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The quantum electrodynamical effects of radiative interference in the recombination of electrons with heavy multicharged ions are discussed. Numerical calculations of the corresponding cross sections in the vicinities of $KL_{12}M_{12}$ and $KM_{12}M_{12}$ dielectronic recombination resonances of the heliumlike uranium have been performed. The results obtained may serve as a focus for near-future experiments with the Super-EBIT facilities. [S1050-2947(99)07008-0]

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I. INTRODUCTION

Overlapping resonances have been thoroughly investigated in nuclear and particle physics. The most interesting case is the overlap of identical resonances, that is, resonances with identical quantum numbers, when the interference terms survive not only in the differential but also in the total cross section after integration over angles. As a consequence, these terms lead to some special interference effects, for example quantum beats, which are well known in neutral *K*-mesons [1] and the ^{8}Be nucleus [2,3].

In atomic physics, a similar situation was investigated theoretically [4] and observed experimentally [5] in the decay of coherently excited $2s$ and $2p$ states of the hydrogen atom in an external electric field. The electric field mixes even- and odd-parity states so that the resulting combinations have identical quantum numbers. Though the excited levels do not overlap in this case due to the repulsion in the electric field, they are close enough to be excited coherently and to give the interference effect. Overlap of resonances may arise, in principle, if an external magnetic field is also added [6].

Unlike the case of neutral atoms, where the radiative overlap of identical resonances is very rare, it can easily take place in the spectra of highly charged heavy ions, in particular in heliumlike uranium. The magnitude of these effects can be qualitatively estimated by the magnitude of the radiative broadening compared to the energy interval between the levels of a multiplet with identical parity and total angular momentum quantum numbers. If there are no special exclusions, the interference effect turns out to be of order $(\alpha Z)^3$ [7], that is, about Z times as large as the effect of nonresonant levels on the line shape [8]. From this estimate, one can see that the overlap in the spectra of multicharged heavy ions arises because the radiative shifts and widths become comparable with the interelectron interaction corrections at very-high-Z values. It means also that this phenomenon can be only described within QED theory, where all the radiative corrections are treated in a consistent manner. We use here the same technique as in Refs. [7,9–15] based on the *S*-matrix or on the Green-function approaches.

The theory of radiative decay of overlapping identical levels in multicharged ions has been developed in Refs. [9–12]. The process of recombination of an electron with a hydrogenlike heavy ion provides one possible practical way for preparation of the situation under investigation [14–16]. The total cross section of the recombination process generally includes resonant dielectronic-recombination (DR) and nonresonant radiative-recombination (RR) cross sections, and terms which describe the interference between DR and RR channels. Interference terms due to radiative overlap of identical DR resonances will also be present but their corresponding effects usually turn out to be masked by DR-RR interference [14–16]. The latter has been investigated in the vicinity of the *KLL* resonances theoretically [14–20] and recently observed experimentally [21].

As usual, we label here DR resonances by the principal quantum numbers of the electrons that form the excited intermediate state, that is, using the notation for the inverse Auger process. For example, the *KLL* DR resonances are those in which a continuum electron is captured into the L shell, while a K-shell electron is excited to the L shell. In heavy multicharged ions, the L shell splits into two distinct subshells: the L_{12} shell, containing $2s_{1/2}$ and $2p_{1/2}$ levels, and the L_3 shell, that contains the $2p_{3/2}$ state. This splitting gives rise to three groupings of *KLL* DR resonances, such as the $KL_{12}L_{12}$, $KL_{12}L_3$, and KL_3L_3 resonances.

It should be noted that in contrast with direct cross-section measurements, the technique of recording the photon energy and electron beam energy for every observed event developed in Refs. [21,22] allows for separation of different x-ray transitions from the dielectronic capture resonances. This way of doing the experiment provides a good means for observation of the double radiative interference effects first

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discussed in Ref. [7]. By these effects we mean the radiative interference in the recombination process on groups of mutually overlapping identical, for example doubly excited, levels. Such situations can give the largest radiative interference effect, since not only initial but also final recombined states overlap in this case. Also it seems to be possible to observe the pure radiative interference effects because, in contrast with them, the DR-RR interference terms in the cross section are suppressed by at least a factor of $1/Z$. In lowest-order perturbation theory, the amplitudes of excitation of the group of double-excited levels in the process of direct radiative capture of an electron by a hydrogenlike ion in the ground state with emission of only one photon vanishes due to the orthogonality of the wave functions. These amplitudes contribute to the cross section if one only takes into account higher-order graphs, i.e., either in at least two-photon processes or after improvement of the operator for the emission of a photon by the interelectron interaction corrections of the order of $1/Z$. The RR process manifests itself only as a background in this case. The characteristic x rays emitted by RR can be used to calibrate the total cross sections [21,22]. In experiment [21] the combined spectra of a mixture of highly charged uranium ions have been observed. However, the resonances of He-like ion are shifted in energy compared to those from other recombined ions. The situations considered in this paper are quite general and should also apply to other few-electron multicharged heavy ions. The purpose of the present work is to find out the particular cases which are most favorable for experimental observation of the interference effects having the radiative origin.

Relativistic units $\hbar=c=1$ with the fine-structure constant $\alpha=e^2/4\pi$ and the electron mass m_e are used throughout the paper.

II. THEORY

We shall use here the Furry picture in tightly bound-state QED and approximation of noninteracting electrons. The latter is also known as $1/Z$ expansion. In zero approximation, the electrons of multicharged heavy ion are supposed to interact only with external field of the nucleus. To classify the states of the ion, it is natural to use the jj-coupling scheme. In the case of two-electron ion, the unperturbed wave function of the state $|d\rangle=|JMn_1l_1j_1n_2l_2j_2\rangle$ is written as

$$\Psi_d(\mathbf{x}_1,\mathbf{x}_2)=\mathcal{N}_d^{-1}\hat{\mathcal{A}} \times\sum_{m_1,m_2} C_{JM}^{j_1j_2}(m_1m_2)\psi_{d_1m_1}(\mathbf{x}_1)\psi_{d_2m_2}(\mathbf{x}_2). \tag{1}$$

Here $|d_km_k\rangle=|n_kl_kj_km_k\rangle$ $(k=1,2)$ denote one-electron solutions of the Dirac equation in the Coulomb field of nucleus, J,M are the total angular momentum and its projection, and $C_{JM}^{j_1j_2}(m_1m_2)$ are the Clebsch-Gordan coefficients. Antisymmetrization operator $\hat{\mathcal{A}}=\Sigma_P(-1)^{[P]}\hat{P}$, where $[P]$ is the parity of permutation of the electron coordinates, and normalization factor $\mathcal{N}_d$ is given by

$$\mathcal{N}_d=\begin{cases}\sqrt{2} & \text{if } d_1\neq d_2\\ 2 & \text{if } d_1=d_2.\end{cases} \tag{2}$$

The zero-approximation energy $E_d^{(0)}$ of the two-electron states d is defined by the sum of the corresponding one-electron energies ε_{d_k} $(k=1,2)$, taking into account the finite nuclear-size effects. The energy $E_d^{(0)}$ does not depend on the parity and total angular momentum J of the ion. This degeneration is removed by interelectron interaction and radiative corrections. The latter split the states with the same J but with different parity. For double excited states, a multiplet may contain a few levels with identical quantum numbers.

We shall consider the DR process of an electron with a hydrogenlike multicharged ion $A^{(Z-1)+}$ in its ground state, which may be schematically represented as

$$e^-(\varepsilon)+A^{(Z-1)+}(1s_{1/2}) \rightarrow A^{(Z-2)+}(d)^{**}\rightarrow A^{(Z-2)+}(s)^{**}+\gamma(\omega)\rightarrow\cdots, \tag{3}$$

where e^- denotes the incident electron with energy ε and γ is the emitted photon with frequency ω. The labels d and s merely serve to identify two-electron states and do not refer here to any particular value of orbital angular momentum. The d and s states are assumed to be groups of the doubly excited mutually overlapping levels with identical quantum numbers. Then the resonance condition is $\varepsilon+E_{1s_{1/2}}\simeq E_d$, where $E_{1s_{1/2}}$ and E_d are the energies of the hydrogenlike and heliumlike ions, respectively. In addition, only photons with frequency in the region of $\omega\simeq E_d-E_s$ are supposed to be measured in all directions of emission.

Consider, for example, the simplest DR process (3) with stabilizing radiative decay of the intermediate state s to the ground state f of the two-electron ion with the emission of two photons γ' and γ'' in frequency regions $\omega'\simeq E_s-E_r$ and $\omega''\simeq E_r-E_f$, respectively. The amplitude of the process has been obtained within the resonance approximation in Ref. [7] and looks like

$$S_{\mathrm{DR}}(\omega,\omega',\omega'',\varepsilon)=-2\pi i\,\delta(E_{1s_{1/2}}+\varepsilon-E_f-\omega-\omega'-\omega'') \times\sum_r\frac{\langle f|\hat{R}_{\gamma''}|r\rangle}{E_{1s_{1/2}}+\varepsilon-\omega-\omega'-\mathcal{E}_r} \times\sum_s\frac{\langle r|\hat{R}_{\gamma'}|s_R\rangle}{E_{1s_{1/2}}+\varepsilon-\omega-\mathcal{E}_s} \times\sum_d\frac{\langle s_L|\hat{R}_{\gamma}|d_R\rangle}{E_{1s_{1/2}}+\varepsilon-\mathcal{E}_d}\langle d_L|\hat{I}|i\rangle. \tag{4}$$

Here $|i\rangle$ is a wave function describing the initial state of the system (one-electron ion in its ground $1s_{1/2}$ state plus continuum electron). In coordinate representation, it looks like

$$\Psi_i(\mathbf{x}_1,\mathbf{x}_2)=\frac{1}{\sqrt{2}}\hat{\mathcal{A}}\psi_{\varepsilon ljm}(\mathbf{x}_1)\psi_{1s_{1/2}m'}(\mathbf{x}_2). \tag{5}$$

In Eq. (5), $\psi_{1s_{1/2}m'}(\mathbf{x})$ is the wave function of the ground state of the one-electron ion. The function $\psi_{\varepsilon ljm}(\mathbf{x})$ describes the state of the electron in the external Coulomb field of nucleus with the energy ε ($\varepsilon > m_e$), angular momentum j, and parity l. These functions are normalized on the energy scale as

$$\int \psi^{\dagger}_{\varepsilon ljm}(\mathbf{x})\psi_{\varepsilon' l' j' m'}(\mathbf{x})d^3\mathbf{x} = \delta(\varepsilon-\varepsilon')\delta_{ll'}\delta_{jj'}\delta_{mm'}.$$

By $\mathcal{E}_d = E_d - i\Gamma_d/2$ in Eq. (4) we denote the complex eigenvalues of the non-Hermitian operator $\hat{\mathcal{H}} = E_d^{(0)}\hat{1} + U(E_d^{(0)})$ acting in the corresponding subspace of the unperturbed d states [13–15]. The quasipotential U is defined here in the lowest approximation. Matrix elements of $\hat{\mathcal{H}}$ are given by

$$\begin{aligned}\hat{\mathcal{H}}_{dd'} = {} & E_d^{(0)}\delta_{dd'} + \left\langle d\left|\sum_{k=1}^{2}\left[\Sigma_R^{(\mathrm{SE})}(k)+\Sigma_R^{(\mathrm{VP})}(k)\right]\right|d'\right\rangle \\ & + \frac{2\alpha}{\mathcal{N}_d\mathcal{N}_{d'}}\sum_{m_1,m_2} C^{j_1 j_2}_{JM}(m_1 m_2)\sum_{m_1',m_2'} C^{j_1' j_2'}_{JM}(m_1' m_2') \\ & \times\left(\left\langle d_1 m_1 d_2 m_2\left|\frac{\alpha_{1\mu}\alpha_2^{\mu}}{r_{12}}e^{i|\varepsilon_{d_1}-\varepsilon_{d_1'}|r_{12}}\right|d_1' m_1' d_2' m_2'\right\rangle\right. \\ & \left. - \left\langle d_2 m_2 d_1 m_1\left|\frac{\alpha_{1\mu}\alpha_2^{\mu}}{r_{12}}e^{i|\varepsilon_{d_2}-\varepsilon_{d_1'}|r_{12}}\right|d_1' m_1' d_2' m_2'\right\rangle\right),\end{aligned} \tag{6}$$

where $r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|$, $\alpha_{1\mu}\alpha_2^{\mu} = 1 - \boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2$, $\Sigma_R^{(\mathrm{SE})}$ and $\Sigma_R^{(\mathrm{VP})}$ are regularized operators of the electron self-energy and vacuum polarization, and $\mathcal{N}_d$ and $\mathcal{N}_{d'}$ are normalization factors given by Eq. (2) for states d and d', respectively. The right $|d_R\rangle$ and left $\langle d_L|$ eigenvectors of $\hat{\mathcal{H}}$ [13–15] (see also Ref. [9], where only right vectors have been employed) are defined as

$$\hat{\mathcal{H}}|d_R\rangle = \mathcal{E}_d|d_R\rangle, \quad \langle d_L|\hat{\mathcal{H}} = \langle d_L|\mathcal{E}_d. \tag{7}$$

The vectors satisfy the orthogonality and completeness conditions:

$$\langle d_L|d_R'\rangle = \delta_{dd'}, \quad \sum_d |d_R\rangle\langle d_L| = \hat{1}.$$

In the problem of two identical levels, the explicit expressions for the right and left vectors are given in the Appendix.

In Eq. (4), matrix elements of the operator

$$\hat{I}(r_{12};E_d^{(0)}) = \alpha\frac{\alpha_{1\mu}\alpha_2^{\mu}}{2r_{12}}\sum_{k=1}^{2} e^{i|\varepsilon_{d_k}-\varepsilon_{1s_{1/2}}|r_{12}} \tag{8}$$

describe the excitation process of the states d by radiationless capture, taking into account the retardation effect. Note that the one-electron energy $\varepsilon_{1s_{1/2}}$ does not include the Lamb shift corrections while $E_{1s_{1/2}}$ does. The operator for the emission of a photon with polarization $\mathbf{e}$ and momentum $\mathbf{k}$ is defined by

$$\hat{R}_{\gamma} = e\sum_{n=1}^{2}\frac{(\boldsymbol{\alpha}\cdot\mathbf{e}^*)}{(2\pi)^{3/2}\sqrt{2\omega}}e^{-i\mathbf{k}\cdot\mathbf{x}_n}.$$

To obtain the expression for the cross section of DR process, one needs to integrate the square of modulus of the amplitude (4) over the final states and average it over the initial states of the system. The averaging procedure leads to the summation over j,l,m, and $m' = \pm 1/2$, quantum numbers of function (5), and division by $2N_0$, where N_0 is the number of degenerate electron states with fixed energy ε in the unit phase space. The latter is most easily calculated, if the states of the free electron are described by the momentum and polarization. Keeping in mind that the degree of degeneracy is the same for any set of quantum numbers, one can write

$$N_0 = 2\int\frac{d^3q}{(2\pi)^3}\delta(q^2-\varepsilon^2+m_e^2) = 2\frac{4\pi p^2}{(2\pi)^3},$$

where $p^2 = \varepsilon^2 - m_e^2$. The factor 2 here takes into account two different polarization states of the electron. Due to the unitarity condition for the Clebsch-Gordan coefficients, the summation over m,m' in cross section can be substituted by the sum over the total angular momentum J and its projection M. Then one can finally get [23]

$$\begin{aligned}\sigma_{\mathrm{DR}}(\varepsilon) & = \frac{\pi^2}{2p^2}\sum_{j,l}\sum_{J,M}\sum_{s}\sum_{d,d'}\frac{W_{dd',s}\langle d_L|\hat{I}|i_J\rangle\langle d_L'|\hat{I}|i_J\rangle^*}{(\varepsilon+E_{1s_{1/2}}-\mathcal{E}_d)(\varepsilon+E_{1s_{1/2}}-\mathcal{E}_{d'}^*)}.\end{aligned} \tag{9}$$

Here we employ functions

$$\Psi_{i_J}(\mathbf{x}_1,\mathbf{x}_2) = \sum_{m,m'} C^{j1/2}_{JM}(mm')\Psi_i(\mathbf{x}_1,\mathbf{x}_2),$$

where $\Psi_i(\mathbf{x}_1,\mathbf{x}_2)$ is given by Eq. (5). Matrix elements of the operator $\hat{I}$ [Eq. (8)] are diagonal with respect to quantum numbers J and do not depend on projection M. It means that summation over M in Eq. (9) can be actually replaced by multiplication on the number of all possible projections of J, that is, $2J+1$ factor.

The nondiagonal partial widths $W_{dd',s}$ for radiative transitions between d and s states are defined by multipolar expansion of the expression

$$W_{dd',s} = 2\pi\omega^2\sum_{\mathbf{e}}\int d\Omega\langle s_R|\hat{R}_{\gamma}|d_R\rangle\langle s_L|\hat{R}_{\gamma}|d_R'\rangle^*, \tag{10}$$

where $d\Omega$ means integration over the directions of the photon emission. Note that Eq. (10) differs from the definition given in Ref. [7] and generalizes the corresponding expressions in Refs. [14–16]. However, with the present choice of the matrix elements involved in $W_{dd',s}$, our expression for

TABLE I. Listed are $KM_{12}M_{12}$ DR resonances d of heliumlike uranium, their binding energies E_d, total radiative widths Γ_d, resonance energies ε of the incoming electron (without the rest energy), the Auger widths $\mathcal{A}_d$ of the states d, the final recombined states s, the frequencies $\omega = E_d - E_s$ of the emitted photons, the partial widths $W_{dd,s}$ for the radiative decays of the states d to the states s, and nonorthogonality integrals $\langle d_R | d_R \rangle$ and $\langle d_R | d'_R \rangle$ for every pair of identical levels, d and d'. All quantities were calculated in the biorthogonal basis.

d	E_d (keV)	Γ_d (eV)	ε (keV)	$\mathcal{A}_d$ (eV)	s	ω (keV)	$W_{dd,s}$ (eV)	$\langle d_R \vert d_R \rangle$	$\langle d_R \vert d'_R \rangle$
$[3p_{1/2}3p_{1/2}]_0$	29.063	7.007	102.760	0.022	$[2s_{1/2}3p_{1/2}]_1$	19.457	$1.397+0.035i$	1.004	$-0.089i$
					$[2p_{1/2}3s_{1/2}]_1$	19.531	$0.237-0.035i$		
$[3s_{1/2}3p_{1/2}]_1$	29.120	9.207	102.703	0.016	$[2p_{1/2}3p_{1/2}]_0$	19.350	$0.534-0.022i$	1.000	
					$[2p_{1/2}3p_{1/2}]_1$	19.427	$0.939-0.025i$		
					$[2s_{1/2}3s_{1/2}]_0$	19.438	$0.022+0.022i$		
					$[2s_{1/2}3s_{1/2}]_1$	19.500	$0.173+0.025i$		
$[3s_{1/2}3p_{1/2}]_0$	29.145	9.207	102.677	0.044	$[2p_{1/2}3p_{1/2}]_1$	19.402	$0.002+0.001i$	1.000	
					$[2s_{1/2}3s_{1/2}]_1$	19.475	$0.135-0.001i$		
$[3s_{1/2}3s_{1/2}]_0$	29.146	11.405	102.677	0.030	$[2s_{1/2}3p_{1/2}]_1$	19.374	$1.543-0.078i$	1.004	$0.089i$
					$[2p_{1/2}3s_{1/2}]_1$	19.448	$0.730+0.078i$		

σ_{DR} looks similar to the formula for the DR part of the cross section published in Refs. [14–16]. Moreover, definition (10) keeps the Bell-Steinberger equality in its conventional form,

$$\sum_s W_{dd',s} = i(\mathcal{E}_d - \mathcal{E}^*_{d'}) \langle d'_R | d_R \rangle, \tag{11}$$

but now the final states s may also have identical quantum numbers. Note that, in contrast with Eq. (9), in Eq. (11) the summation over s includes *all* possible low-lying states.

The particular case when $d = d'$ in the sum of Eq. (9) corresponds to the superposition of Lorentz shapes of the DR process. The terms σ_{DR} with $d \neq d'$ describe the radiative interference due to overlap between the upper as well as between the lower states, and lead to the asymmetry of the summarized shape. It should be noted that the definition of the ''pure'' Lorentz shapes is not unique in different basis sets. This means that even if terms $d \neq d'$ are not taken into account in Eq. (9) the radiative interference turns out to be partially involved in σ_{DR} in the biorthogonal basis through the complex mixing coefficients of the identical states. Nevertheless, it is usual to use the orthogonal basis set and in this case the Lorentz terms do not include the radiative interference. Asymmetry of the shapes can be numerically characterized, for example, by Low's parameter [8]. We use for this purpose the nonorthogonality integrals $\langle d_R | d'_R \rangle$. In orthogonal basis, such parameter can be chosen to be the ratio of nondiagonal widths to energy intervals between the overlapping levels.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical calculations of the DR cross sections have been performed using Eq. (9) in the vicinities of $KL_{12}M_{12}$ and $KM_{12}M_{12}$ resonances of heliumlike uranium. The energies and rates were calculated in the framework of the $1/Z$ expansion. The results obtained are given in Tables I–III. The electrical and magnetical dipole contributions as well as

TABLE II. The parameters of $KL_{12}M_{12}$ resonances of heliumlike uranium. Notations are the same as in Table I.

d	E_d (keV)	Γ_d (eV)	ε (keV)	$\mathcal{A}_d$ (eV)	s	ω (keV)	$W_{dd,s}$ (eV)	$\langle d_R \vert d_R \rangle$	$\langle d_R \vert d_R \rangle$
$[2p_{1/2}3p_{1/2}]_0$	48.470	10.176	83.353	0.227	$[2s_{1/2}2p_{3/2}]_1$	14.797	0.421	1.017	$-0.187i$
$[2s_{1/2}3p_{1/2}]_1$	48.520	9.587	83.303	0.102	$[2p_{1/2}2p_{3/2}]_2$	14.816	0.029	1.003	$-0.077i$
					$[2p_{1/2}2p_{3/2}]_1$	14.833	0.006		
$[2s_{1/2}3p_{1/2}]_0$	48.524	7.056	83.299	0.186	$[2s_{1/2}2p_{3/2}]_1$	14.743	0.001	1.000	$-0.004i$
$[2p_{1/2}3p_{1/2}]_1$	48.548	1.424	83.275	0.001	$[2s_{1/2}2p_{3/2}]_1$	14.720	0.090	1.002	$-0.061i$
					$[2s_{1/2}2p_{3/2}]_2$	14.843	0.449		
$[2s_{1/2}3s_{1/2}]_0$	48.558	30.225	83.265	0.195	$[2s_{1/2}2p_{3/2}]_1$	14.709	0.134	1.017	$0.187i$
$[2p_{1/2}3s_{1/2}]_0$	48.592	31.671	83.231	0.157	$[2p_{1/2}2p_{3/2}]_1$	14.761	0.545	1.000	$0.004i$
$[2p_{1/2}3s_{1/2}]_1$	48.595	30.246	83.228	0.026	$[2p_{1/2}2p_{3/2}]_2$	14.742	0.427	1.003	$0.077i$
					$[2p_{1/2}2p_{3/2}]_1$	14.759	0.085		
$[2s_{1/2}3s_{1/2}]_1$	48.620	38.978	83.203		$[2s_{1/2}2p_{3/2}]_1$	14.647	0.001	1.002	$0.061i$
					$[2s_{1/2}2p_{3/2}]_2$	14.770	0.007		

TABLE III. Listed are $KM_{12}M_{12}$ DR resonances d of heliumlike uranium, the final recombined states s, the frequencies ω of the emitted photons, and the partial widths $W_{dd,s}$ for the radiative decays of the states d to the states s. All quantities were calculated in the biorthogonal basis.

d	s	ω (keV)	$W_{dd,s}$ (eV)
$[3p_{1/2}3p_{1/2}]_0$	$[2p_{3/2}3s_{1/2}]_1$	14.966	0.703
	$[2p_{3/2}3p_{1/2}]_2$	14.972	0.006
$[3s_{1/2}3p_{1/2}]_1$	$[2p_{3/2}3s_{1/2}]_1$	14.909	0.004
	$[2p_{3/2}3p_{1/2}]_1$	14.915	0.091
	$[2p_{3/2}3p_{1/2}]_2$	14.915	0.455
	$[2p_{3/2}3s_{1/2}]_2$	14.922	0.005
$[3s_{1/2}3p_{1/2}]_0$	$[2p_{3/2}3s_{1/2}]_1$	14.884	0.001
	$[2p_{3/2}3p_{1/2}]_1$	14.890	0.546
	$[2p_{3/2}3s_{1/2}]_2$	14.897	0.009
$[3s_{1/2}3s_{1/2}]_0$	$[2p_{3/2}3s_{1/2}]_1$	14.883	0.392
	$[2p_{3/2}3p_{1/2}]_1$	14.889	0.001
	$[2p_{3/2}3p_{1/2}]_2$	14.889	0.011

the electrical quadrupole ones have been taken into account in the rates. We have also tabulated the calculated diagonal and nondiagonal values of the nonorthogonality integrals for every pair of identical DR resonances, and the energies of the emitted photons. For convenience the identical states are classified according to the *LS* limit. However, such classification can be somewhat artificial. For example, contributions of the pure $[2s_{1/2}3s_{1/2}]_J$ and $[2p_{1/2}3p_{1/2}]_J$ configurations are redistributed under increasing Z values. The energies of the levels have the radiative [24–28] (electron self-energy and vacuum polarization) and the exact one-photon interelectron interaction corrections included. The effect of finite size of the nuclear charge distribution was taken into account by numerical solution of the Dirac equation in the Coulomb field of a homogeneously charged sphere.

First we have investigated the situations most promising for the experimental observation of the double radiative interference effects. In the case of $KL_{12}M_{12}$ DR resonances,

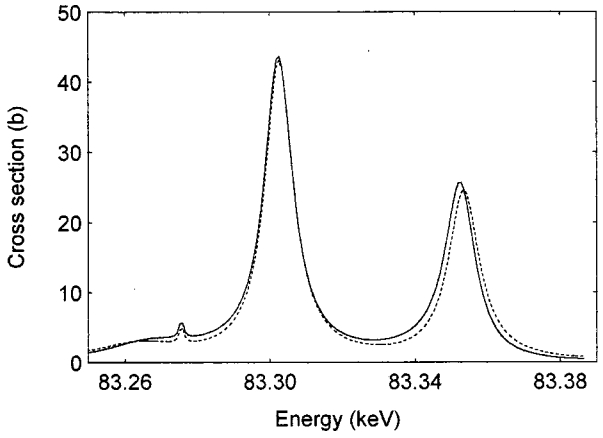

FIG. 1. Total DR cross section for U^{91+} in the vicinity of $KL_{12}M_{12}$ resonances resolved with regard to the $L_{12}L_{12}$ shell as a function of the incident electron energy (solid curve). The dashed curve corresponds to the Lorentz terms in orthogonal basis.

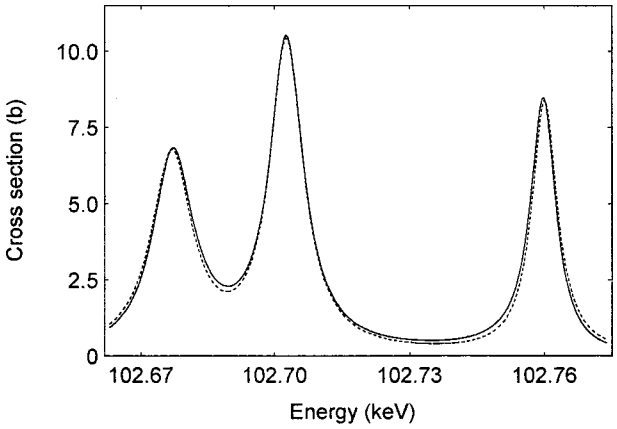

FIG. 2. Total DR cross section for U^{91+} in the vicinity of $KM_{12}M_{12}$ resonances resolved with regard to the $L_{12}M_{12}$ shell (solid curve); dashed curve, with no radiative interference included.

there are four pairs of identical levels. These are $[2s_{1/2}3s_{1/2}]_J$ and $[2p_{1/2}3p_{1/2}]_J$ and $[2s_{1/2}3p_{1/2}]_J$ and $[2p_{1/2}3s_{1/2}]_J$ levels with $J=0,1$. As final recombined levels in this case, the doubly excited $[2s^2_{1/2}]_0$, $[2p^2_{1/2}]_0$, and $[2s_{1/2}2p_{1/2}]_{0,1}$ states were chosen. The first pair here has the same parity and zero total angular momentum, but the second pair has a different total angular momentum and does not mix. The overlap parameters for these levels calculated in the biorthogonal basis are equal to [14]

$$\langle [2s^2_{1/2}]_{0_R} | [2s^2_{1/2}]_{0_R} \rangle = 1.016,$$

$$\langle [2s^2_{1/2}]_{0_R} | [2p^2_{1/2}]_{0_R} \rangle = 0.180i.$$

The results of the corresponding DR cross section calculations are shown in Fig. 1, and are in agreement with results of Ref. [7]. The second example is given by $KM_{12}M_{12}$ resonances. In this case, there are only two identical upper states, $[3s^2_{1/2}]_0$ and $[3p^2_{1/2}]_0$, and the cross section is resolved with regard to radiative decays to the $L_{12}M_{12}$ shell (see Table I and Fig. 2). As can be seen in both examples considered, the interference effects originate from the radiative overlap of the upper and lower groups of doubly excited states with identical quantum numbers. The situation shown in Fig. 1 looks more favorable for the experimental observation. The deviation from the Lorentz shape here is larger and reaches nearly 25% in the range of 83.34 keV for the energy of incoming electron.

We have also considered two other possible situations, where only upper groups of double excited levels overlap. The radiative interference terms are significant in cross section of $KL_{12}M_{12}$ resonances resolved with regard to the $L_{12}L_3$ shell. The latter splits into four $[2l_{1/2}2p_{3/2}]_J$ levels, different either by parity or by total angular momentum $J=1,2$ (see Table II). The results of the corresponding DR cross-section calculations are shown in Fig. 3. The interference terms here significantly defect the Lorentz shape in the 83.28 keV range. However, the absolute value of the DR cross section turns out to be rather small. In the case of $KM_{12}M_{12}$ resonances, we have calculated the cross section associated with stabilizing radiative transitions to the

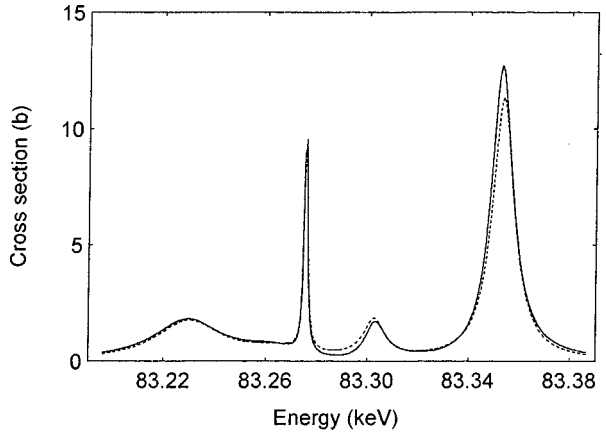

FIG. 3. Total DR cross section for U^{91+} in the vicinity of $KL_{12}M_{12}$ resonances resolved with regard to the $L_{12}L_3$ shell (solid curve); dashed curve, with no radiative interference included.

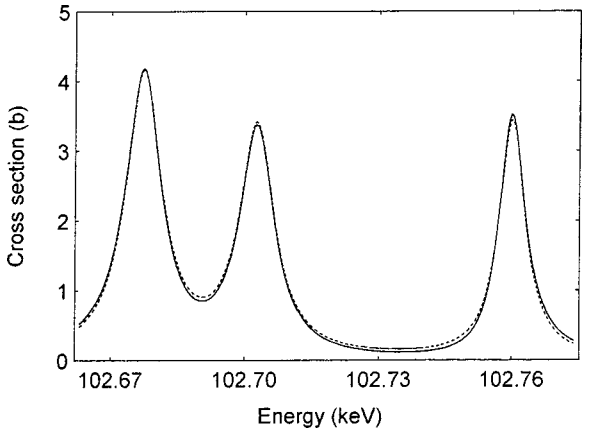

FIG. 4. Total DR cross section for U^{91+} in the vicinity of $KM_{12}M_{12}$ resonances resolved with regard to the L_3M_{12} shell (solid curve); dashed curve, with no radiative interference included.

$[2p_{3/2}3l_{1/2}]_J$ $(J=1,2)$ levels (see Table III and Fig. 4). The size of the interference effect is nearly 20%, however the DR contributions to the cross section are small again in the corresponding energy ranges.

Unforunately we could not find situations where the radiative interference terms originated only from the lower groups of double excited identical levels would give rise to an essential effect in the cross section. In this case, the resonance structure of the DR cross section is due to the upper group of nonoverlapping levels, which effectively reduces the relative value of the interference effect. One can expect some essential effects in the DR cross section, if the latter is integrated over the energy of incident electrons and is considered as a function of photon frequency in the region of $\omega \simeq E_d - E_s$ transitions.

The main approximation in the calculations is due to the omission of $1/Z$-order corrections in the evaluation of the radiative widths, which are as a consequence uncertain to about $2-3\,\%$. The magnitude of the effect looks large enough to be observable on the experiments. We should note that in work [21] the differential cross section was measured. To observe the effect we discuss, one should measure the total cross section, that is, to detect the photons emitted in all directions. The last problem is not principal restriction for Super-EBIT experiments.

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APPENDIX

Let us designate the matrix elements of the operator $\hat{\mathcal{H}}$ [Eq. (6)] in subspace of two degenerate states $|a\rangle$ and $|b\rangle$, which are constructed according to Eq. (1), as

$$\hat{\mathcal{H}} = \begin{pmatrix} \mathcal{H}_{aa} & \mathcal{H}_{ab} \\ \mathcal{H}_{ba} & \mathcal{H}_{bb} \end{pmatrix}.$$

The solution of the problem

$$\hat{\mathcal{H}}|d_R\rangle = \mathcal{E}_d|d_R\rangle$$

is given by

$$\mathcal{E}_d = \frac{1}{2}(\mathcal{H}_{aa}+\mathcal{H}_{bb}) \pm \left[\frac{1}{4}(\mathcal{H}_{aa}-\mathcal{H}_{bb})^2 + \mathcal{H}_{ab}^2\right]^{1/2}$$

and

$$|d_R\rangle = \eta_d|a\rangle + \xi_d|b\rangle.$$

The complex mixing coefficients η_d and ξ_d look like

$$\eta_d = (1+\zeta_d^2)^{-1/2}, \quad \xi_d = (1+\zeta_d^2)^{-1/2}\zeta_d,$$

where

$$\zeta_d = (\mathcal{E}_d - \mathcal{H}_{aa})/\mathcal{H}_{ab}.$$

We have taken into account here that, due to T invariance, $\mathcal{H}_{ab} = \mathcal{H}_{ba}$ [9]. As a result, the components of the vectors $\langle d_L|$ can be chosen to be equal to the corresponding components of the vector $|d_R\rangle$, that is,

$$\langle d_L| = |d_R\rangle^T \text{ and } \langle d_R| = |d_L\rangle^T,$$

but

$$\langle d_R| = |d_R\rangle^\dagger \text{ and } \langle d_L| = |d_L\rangle^\dagger.$$

In addition, for any pair of identical states d and d', if only $d \neq d'$, one can write

$$\langle d_R|d_R\rangle = \langle d'_R|d'_R\rangle, \quad \langle d_R|d'_R\rangle = \langle d'_R|d_R\rangle^*.$$

For states with different quantum numbers there is no difference between the left $|d_L\rangle$ and right $|d_R\rangle$ vectors.

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