# Quantum interference in phase conjugation by resonant, nearly degenerate four-wave mixing

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We present a theoretical investigation of optical phase conjugation by use of nearly degenerate four-wave mixing in a two-level atomic system. The description is based on the semiclassical dressed-state approach, which offers a very transparent picture of the physics involved in the nonlinear process. The theory provides an explanation in terms of a quantum interference between two channels for the strong reduction of the optical phase-conjugate yield when the pump fields are tuned to the atomic resonance. In addition, the model predicts an efficient generation of the optical phase conjugate if the system is prepared in a pure dressed state. [S1050-2947(99)08708-9]

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# I. INTRODUCTION

Initiated by the discovery of Liao, Bloom, and Economou [1] that resonant effects can be used to increase the nonlinearity in atomic media, degenerate four-wave mixing (DFWM) in resonant media has been extensively studied. Especially the alkali-metal vapors have undergone very systematic investigation. Generally, in the strong pump regime the DFWM spectrum, i.e., the DFWM yield as a function of the pump detuning from resonance, exhibits a two-peaked structure with a vanishing DFWM yield when the pump is tuned to the atomic resonance [1,2]. Nearly degenerate fourwave mixing (NDFWM), i.e., the situation where the probe field has a frequency different from the pump frequency, has been investigated experimentally by Steel and Lind [3], Nilsen, Gluck, and Yariv [4], and more recently by Lin, Rubiera, and Zhu [5]. They studied the effect of the pumpprobe detuning on NDFWM spectra in sodium vapor, respectively, rubidium vapor, and observed multiresonant behavior of the reflectivity as they scanned the frequency difference between the pump and the probe fields. In these papers the results are explained qualitatively in terms of the ac Stark splitting of the atomic lines.

Different theoretical approaches have been applied to the four-wave mixing problem. Third-order, time-dependent perturbation theory was used by Nilsen and Yariv [6]. Perturbation theory is adequate to describe the four-wave mixing interaction when the fields are weak. With pump intensities well in excess of the saturation intensity, which for a twolevel atom corresponds to Rabi frequencies large compared to the linewidth, splitting of the atomic transition can be observed [7,8]. This necessitates a theoretical approach that accounts for the effect of the strong pump beams on the energy levels. Harter and Boyd [9] made a theoretical study of NDFWM in a medium characterized by a single strong atomic resonance. Based on the density matrix formalism they solved the problem of a strong pump and a weak probe field interacting with a two-level atom and predicted very large responses for the probe frequency detuned by the pump frequency by approximately the generalized Rabi frequency. Also based on the density matrix approach, Lucht, Trebino, and Rahn [10] performed a numerical investigation on multiwave mixing in gas-phase sodium. Although correct, the bare state density matrix equations do not lead to an elucidating physical picture concerning the generation of the FWM signal.

An approach to NDFWM in terms of population oscillations has been described by Steel et al. [11]. Here the fourwave mixing process is considered as a scattering process: the forward pump and the probe interact to spatially modulate the population difference. The backward pump then scatters from this spatial modulation to produce the conjugate wave. In the same paper it was noted that, for closed or nearly closed quantum systems, coherence effects can result in interference effects that eliminate certain resonances in the spectra obtained when the pump-probe detuning is scanned. In the following we will derive a semiclassical approach to NDFWM in a two-level atom. We treat the atomic interaction with the pump field first and consider a probe field interacting with the resulting dressed atom. This technique has been used for some other applications of dressed states in quantum optics. It plays a key role in the occurrence of quantum jumps in the coherently excited three-level atom [12,13]. Further it is used in the fields of lasing without inversion [14,15] and in electromagnetically induced transparency [16]. The derivation is based on the dressed state formalism as described by Courtens and Szöke [17], an approach that was also successful in providing a physical interpretation of resonance fluorescence in the saturation regime [18,19]. The dressed atom technique has also been applied to DFWM [20] and NDFWM [21]. Dressed state based calculations have also been done on transient probe spectra in the presence of a strong pump field [22,23]. In these calculations it was shown that the transient spectra of probe absorption are sensitive functions of the initial conditions of the dressed atom.

In this paper we restrict the calculations to the four-wave mixing scheme in which two pump fields with the same frequency  $\omega$  interact with a probe field of frequency  $\omega_p$ , slightly detuned from the pump frequency, to create a response at the frequency  $\omega_c = 2\omega - \omega_p$ . The analysis presented here is specifically oriented towards studying the NDFWM yield as a function of the initial dressed-state amplitudes. It is shown that the generation of the phase-conjugate signal proceeds via two distinct channels, where each channel corresponds to the atom starting in a pure dressed state. These channels interfere destructively such that

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FIG. 1. (a) Two-level atom and relevant frequencies. (b) The situation shown in (a) but now transformed to the dressed atomic states picture.

the yield vanishes when the atom is initialized in any 50:50 linear combination of dressed states. This presents an explanation of the vanishing NDFWM yield when the pump is tuned to line center. As a consequence it is shown how the NDFWM yield can be increased dramatically if the atomic gas has a pure dressed state equilibrium, rather than a ground state equilibrium. The possibility of controlling dressed state populations by the use of a phase-controlled driving field has been demonstrated [24] and therefore offers the possibility of realizing very efficient phase-conjugate mirrors.

The effect of atomic motion will be included by considering the atom moving in the standing-wave intensity pattern of the counterpropagating pump fields and assuming that the atom adiabatically follows the spatially varying intensity pattern. Our analysis confirms that the atomic motion has no influence on the phase-conjugate yield at line center and thus shows the invariance under atomic motion of the phaseconjugate "hole" at resonance.

#### **II. DRESSED-STATE THEORY OF NDFWM**

### A. Basic theory

Consider the response of a two-level atom with ground state  $u_g(\mathbf{r})$  and excited state  $u_e(\mathbf{r})$ , to an oscillating electric field, the pump field, of the form

$$\widetilde{E}(t) = \frac{1}{2} [\mathcal{E} \exp(-i\omega t) + \text{c.c.}], \qquad (1)$$

where  $\mathcal{E}$  is a constant, see Fig. 1(a). With use of the rotatingwave approximation and electric-dipole approximations, and expressed in a frame rotating at the laser frequency  $\omega$ , the atom-field interaction Hamiltonian can be expressed as [24]

$$\hat{\mathcal{H}}_{R} = \begin{pmatrix} 0 & -\frac{1}{2}\hbar\Omega^{*} \\ -\frac{1}{2}\hbar\Omega & -\hbar\Delta \end{pmatrix}, \qquad (2)$$

where  $\Omega = \mu \mathcal{E}/\hbar$ , and  $\mu$  is the electric-dipole moment. The eigenstates associated with Eq. (2) are given by

$$\psi_{+}(\mathbf{r},t) \equiv u_{+}(\mathbf{r}) \exp\left(-i\frac{E_{+}}{\hbar}t\right)$$
$$= \left[\cos\Theta u_{g}(\mathbf{r}) - \sin\Theta u_{e}(\mathbf{r})\right] \exp\left(-i\frac{E_{+}}{\hbar}t\right),$$
(3a)
$$\psi_{-}(\mathbf{r},t) \equiv u_{-}(\mathbf{r}) \exp\left(-i\frac{E_{-}}{\hbar}t\right)$$

= 
$$\left[\sin \Theta u_g(\mathbf{r}) + \cos \Theta u_e(\mathbf{r})\right] \exp\left(-i\frac{E_-}{\hbar}t\right).$$
(3b)

Here we have introduced  $\cos \Theta$  and  $\sin \Theta$  through the relations

$$\cos \Theta = \left(\frac{\Omega' + \Delta}{2\Omega'}\right)^{1/2} \text{ and } \sin \Theta = \left(\frac{\Omega' - \Delta}{2\Omega'}\right)^{1/2}.$$
 (4)

 $\Omega' = \sqrt{|\Omega|^2 + |\Delta|^2}$  is the generalized Rabi frequency. The eigenstates  $\psi_{\pm}$  are the semiclassical dressed atomic states discussed elsewhere [17,25]. The corresponding eigenenergies  $E_{\pm}$  are given by

$$E_{\pm} = \hbar \left( -\frac{1}{2} \Delta \pm \frac{1}{2} \Omega' \right). \tag{5}$$

Note that the dressed state  $u_+(\mathbf{r})$  has a higher energy than  $u_-(\mathbf{r})$ .

We will now investigate the effect on the dressed atomic states when a weak probe field  $\tilde{E}_{p}(t)$  of the form

$$\widetilde{E}_{p}(t) = \frac{1}{2} \left[ \mathcal{E}_{p} \exp(-i\omega_{p}t) + \text{c.c.} \right]$$
(6)

is interacting with the dressed atom, see Fig. 1(b). The frequency of the probe is detuned from the pump frequency by  $\delta$ , see Fig. 1(a), where  $|\delta| \leq \omega$ . The Schrödinger equation including the probe interaction can be written as

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = (\hat{\mathcal{H}}_D + \hat{\mathcal{V}}_D) \psi(\mathbf{r}, t).$$
(7)

Here  $\hat{\mathcal{H}}_D$  and  $\hat{\mathcal{V}}_D$  are the interaction Hamiltonian and the probe interaction operator in the dressed atomic basis. The components of  $\hat{\mathcal{V}}_D$  can easily be shown to be

$$\mathcal{V}_{D,++} = -\mathcal{V}_{D,--}$$
$$= -\frac{\mu \sin 2\Theta}{4} [\mathcal{E}_p \exp(-i\,\delta t) + \mathcal{E}_p^{\star} \exp(i\,\delta t)],$$
(8a)

$$\mathcal{V}_{D,+-} = \mathcal{V}_{D,-+}^{\star}$$
  
=  $-\frac{1}{2}\mu\mathcal{E}_p\sin^2\Theta\exp(-i\,\delta t) + \frac{1}{2}\mu\mathcal{E}_p^{\star}\cos^2\Theta\exp(i\,\delta t).$  (8b)

We will assume that the probe field is weak in the sense that it does not participate directly in the dressing of the atom, but merely acts as a small perturbation to the dressed states given by Eqs. (3a) and (3b). The perturbation treatment follows the standard procedure of introducing a parameter  $\lambda$ characterizing the strength of the perturbation, where  $\lambda = 1$  is the real situation, and then rewriting Eq. (7) in the form

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = (\hat{\mathcal{H}}_D + \lambda \hat{\mathcal{V}}_D) \psi(\mathbf{r}, t).$$
(9)

We will attempt to find a solution to first order in the probe field, so that the trial solution  $\psi(\mathbf{r},t)$  can be written as

$$\psi(\mathbf{r},t) = \alpha_{+}^{(0)} \psi_{+}(\mathbf{r},t) + \alpha_{-}^{(0)} \psi_{-}(\mathbf{r},t) + \lambda [\alpha_{+}^{(1)}(t) \psi_{+}(\mathbf{r},t) + \alpha_{-}^{(1)}(t) \psi_{-}(\mathbf{r},t)].$$
(10)

The zero-order coefficients  $\alpha_{+}^{(0)}$  and  $\alpha_{-}^{(0)}$  are time independent and specify the initial conditions imposed on the system. Substituting Eq. (10) into Eq. (9) results in two expressions for the first-order coefficients  $\alpha_{+}^{(1)}$  and  $\alpha_{-}^{(1)}$ :

$$\alpha_{+}^{(1)}(t) = \frac{1}{i\hbar} \int_{0}^{t} \left[ \alpha_{+}^{(0)} \mathcal{V}_{D,++} + \alpha_{-}^{(0)} \mathcal{V}_{D,+-} \exp(i\Omega't') \right] dt',$$
(11)

$$\alpha_{-}^{(1)}(t) = \frac{1}{i\hbar} \int_{0}^{t} [\alpha_{+}^{(0)} \mathcal{V}_{D,-+} \exp(-i\Omega't') + \alpha_{-}^{(0)} \mathcal{V}_{D,--}] dt'.$$
(12)

The coefficients  $\alpha_{+}^{(1)}$  and  $\alpha_{-}^{(1)}$  give the respective timedependent probability amplitudes for the dressed atom to make a transition to the states  $\psi_{+}(\mathbf{r},t)$  and  $\psi_{-}(\mathbf{r},t)$  under the influence of the probe field. These coefficients generally have a resonant character [20]. Consider, for example, the situation where the system is initially in the dressed state  $\psi_{-}$ . Written out explicitly the first-order coefficients for this case have the form

$$\alpha_{+}^{(1)}(t) = -\frac{\mu}{2\hbar} \left( \mathcal{E}_{p} \sin^{2} \Theta \frac{\exp[-i(\delta - \Omega')t] - 1}{\delta - \Omega' - i\gamma} + \mathcal{E}_{p}^{\star} \cos^{2} \Theta \frac{\exp[i(\delta + \Omega')t] - 1}{\delta + \Omega' + i\gamma} \right),$$

$$\alpha_{-}^{(1)}(t) = \frac{\mu \sin 2\Theta}{4\hbar} \left( \mathcal{E}_{p} \frac{\exp[-i\delta t] - 1}{\delta - i\gamma} - \mathcal{E}_{p}^{\star} \frac{\exp[i\delta t] - 1}{\delta + i\gamma} \right).$$
(13)

Here  $\gamma$  was added in order to avoid the first-order coefficients diverging in an unphysical manner close to the resonances. The value of  $\gamma$  is equal to the decay rate of the excited state.

The first term in the expression for  $\alpha_{+}^{(1)}$ , resonant when  $\delta = \Omega'$ , is interpreted as the dressed atom absorbing a probe photon with the consequence that a simultaneous transition from  $\psi_{-}$  to  $\psi_{+}$  is induced, which produces the phase-conjugate photon. The second term, resonant when  $\delta = -\Omega'$ , corresponds to the probe field stimulating the dressed atom to make a transition from  $\psi_{-}$  to  $\psi_{+}$ . The terms appearing in the expression for  $\alpha_{-}^{(1)}$  can be interpreted in a similar fashion. Here it must be remembered that  $\alpha_{-}^{(1)}$  is the probability amplitude that the dressed atom stays in the  $\psi_{-}$  state upon the probe interaction. This explains why the terms



FIG. 2. Energy-level structure for a resonant NDFWM interaction. The nonlinear effects couple the two pump beams with frequency  $\omega$  and the probe beam with frequency  $\omega + \delta$  to generate an outgoing signal wave, the optical phase conjugate.

are resonant when  $\delta = 0$ . The present formalism elucidates the interrelationship between probe and conjugate beam in the pumped atomic vapor. This is in fact a general property of NDFWM materials, where in the presence of pump beams the probe and conjugate are intimately connected in the sense that the one cannot propagate without the other and vice versa [26].

With the determined first-order coefficients the expectation value of the induced polarization can be derived through the relation

$$\langle \tilde{\mathcal{P}}(t) \rangle = \int \mathbf{d}\mathbf{r} \, \psi^*(\mathbf{r}, t) \hat{\mu}_D \psi(\mathbf{r}, t).$$
 (14)

Among the various frequency components of  $\langle \tilde{\mathcal{P}}(t) \rangle$  we are interested in the component at the frequency  $\omega - \delta$ , for which we will denote the amplitude by  $\langle \tilde{\mathcal{P}}(\omega - \delta, t) \rangle$ . This is identified as the phase-conjugate signal. It is a result of the four-wave mixing process indicated in Fig. 2. The illustrated four-wave mixing scheme belongs to a broader class of elastic photon-scattering phenomena in which it is demanded that the atom is returned to its initial quantum state upon completion of the NDFWM process [27].

### **B.** Initial-state dependence of $\langle \tilde{\mathcal{P}}(t) \rangle$

The induced polarization (14) appears to be a sensitive function of the specific initial condition imposed on the system. A straightforward analysis, starting from Eq. (14), using Eqs. (10)–(12), and keeping only terms that are (i) first order in  $\lambda$  and (ii) proportional to  $\exp[\pm i(\omega - \delta)t]$  then yields the result

$$\langle \tilde{P}(\omega - \delta, t) \rangle = (|\alpha_{+}^{(0)}|^{2} - |\alpha_{-}^{(0)}|^{2}) P_{0}(\Omega, \Delta, \delta)$$
$$\times \exp[-i(\omega - \delta)t] + \text{c.c.}, \quad (15)$$

where

$$P_{0}(\Omega, \Delta, \delta) \equiv -\frac{\mu^{2} \mathcal{E}_{p}^{\star}}{8\hbar} \frac{\Omega^{2}}{\Omega^{2} + \Delta^{2}} \times \left(\frac{1}{\Omega' - \delta - i\gamma} + \frac{1}{\Omega' + \delta + i\gamma}\right). \quad (16)$$

From Eq. (15) we conclude that the phase-conjugate signal is composed out of contributions from two channels, one channel starting in the pure dressed state  $u_+(\mathbf{r})$  and the other similarly in  $u_-(\mathbf{r})$ . Starting from an arbitrary state  $u(\mathbf{r})$  $= \alpha_+^{(0)} u_+(\mathbf{r}) + \alpha_-^{(0)} u_-(\mathbf{r})$ , each channel provides a weighing factor  $|\alpha_{\pm}^{(0)}|^2$ , with opposite signs. Hence, the channel contributions tend to counteract each other, such that a maximum yield is obtained when starting in one of the pure dressed states, while zero yield results from any 50:50 linear superposition of dressed states.

For instance, if the initial state is the ground state, then using Eq. (4) we have that

$$|\alpha_{+}^{(0)}|^{2} - |\alpha_{-}^{(0)}|^{2} = \frac{\Delta}{(\Omega^{2} + \Delta^{2})^{1/2}}.$$
(17)

For  $\Delta = 0$ , i.e., the pump field tuned exactly to line center, the ground state is a 50:50 linear combination of the dressed states and hence, as expressed in Eq. (15), the phase-conjugate yield vanishes as a result of destructive interference.

In the case where the system is initially in a pure dressed state the yield exhibits a maximum when the pump beam is tuned to line center, see Fig. 3(a). This is in contrast to the situation described above where the system was initially in the ground state, shown in Fig. 3(b). The complete vanishing of the phase-conjugate yield when the pump field is tuned to line center has indeed been observed in experiments [1,28]. Note that as the pump field is made stronger, i.e., implying a larger Rabi frequency  $\Omega$ , a broadening occurs of the detuning region around line center for which the probabilities  $|\alpha_{-}^{(0)}|^2$  and  $|\alpha_{+}^{(0)}|^2$  are of comparable size. This gives rise to widening of the dip, see Fig. 3(b). If the system can be prepared in one of the dressed states there will be no destructive channel interference effect and hence the phase-conjugate yield will not vanish at line center.

The results of decreasing the pump field are shown in Fig. 4. In Fig. 4(a) the system is initially in a pure dressed state, and a three-peaked spectrum is observed. The side bands for  $\Delta = 0$  occur because the respective probe and phase conjugate become resonant with a dressed state transition. The situation where the atom is initially in the ground state is shown in Fig. 4(b). It is seen that the dip observed when  $\Delta = 0$  has become very narrow in accordance with the narrower region for which  $|\alpha_{-}^{(0)}|^2$  and  $|\alpha_{+}^{(0)}|^2$  become equal in magnitude.

#### **III. INCLUSION OF ATOMIC MOTION**

Here we will investigate the effects of atomic motion on the NDFWM signal. A rigorous treatment would be to derive new time-dependent dressed state wave functions and then do the perturbation calculation with the probe field. However, this would be complicated because the atom, in the general case, is exposed to two pump fields with different frequencies leading to a complicated dynamical behavior. A conventional dressed state approach to resonance fluorescence for a two-level atom in a strong bichromatic field has been demonstrated, however, only for the case where the pump fields are detuned symmetrically around the atomic resonance [29]. This approach leads to very involved calcu-



FIG. 3. Phase-conjugate yield as a function of the pump detuning from line center  $\Delta$  in two situations with different initial conditions. In (a) the system is initially in a pure dressed state, i.e.,  $u_{-}(\mathbf{r})$ or  $u_{+}(\mathbf{r})$ . As can be seen from the figure the phase-conjugate yield exhibits a maximum whenever the pump beam is tuned to exact resonance. (b) shows the situation where the system is initially in the ground state. In this case the phase-conjugate yield vanishes completely when the pump is tuned to resonance. In both (a) and (b) the pump-probe detuning  $\delta$  is equal to 50 MHz. The damping parameter has the value  $\gamma = 3.8 \times 10^7 \text{ s}^{-1}$ . Solid curve corresponds to the case where  $\Omega = 100 \text{ MHz}$ , dashed curve to  $\Omega = 200 \text{ MHz}$ .

lations outside the scope of this paper. We will propose an approximation to the problem by merely considering the adiabatic following effect of the moving atom experiencing an oscillating Rabi frequency. The two counterpropagating pump beams give rise to a spatial standing-wave pattern of the total electric field. Hence, an atom moving through this standing-wave pattern will experience an oscillating Rabi frequency. For simplicity we consider the situation where the two pump fields have equal amplitudes. In this case the total electric field can be written as

$$\widetilde{E}(t) = \frac{1}{2}E(x)[\exp(-i\omega t) + \text{c.c.}], \qquad (18)$$

where E(x) is given by

$$E(x) = 2\mathcal{E}\cos\left(\frac{2\pi}{\lambda}x\right).$$
 (19)



FIG. 4. Phase-conjugate yield as a function of the pump detuning from resonance  $\Delta$ . In this case the pump Rabi frequency is low so that  $\Omega \sim \gamma$ , and the pump-probe detuning  $\delta$  is set to 50 MHz. (a) shows the situation where the atom is initially in a pure dressed state. The features observed when  $\Delta = \pm \delta$  occur when the probe, respectively, phase conjugate is resonant with a dressed-state transition. (b) is the case where the dressed atom is initially in the ground state. In this case the dip at line center is very narrow. Again the resonant contributions at  $\Delta = \pm \delta$  are observed.

Transitions between the dressed states will be induced by the atomic motion, due to the spatial variation of the dressed levels, at a rate proportional to  $\dot{\Theta}$  [17]. Using Eqs. (4) and (19) it can be shown that

$$|\Theta| \simeq \frac{\pi}{\lambda} \frac{\Delta}{\Omega' + \Delta} \left(\frac{\Omega}{\Omega'}\right)^2 \dot{x}.$$
 (20)

Here  $\dot{x}$  is the speed of the atom. If the dressed atom has to adiabatically follow variations of the standing intensity pattern we require that  $|\dot{\Theta}|$  is small compared to the time scale set by the Hamiltonian (2). This leads to the condition  $|\dot{\Theta}| \ll \Omega'$ . This in turn leads to the following constraint on the velocity:

$$|\dot{x}| \ll v_{\rm cr} \equiv \frac{\lambda}{\pi} \frac{\Omega'(\Omega' + \Delta)}{\Delta} \left(\frac{\Omega'}{\Omega}\right)^2.$$
 (21)



FIG. 5. Critical velocity  $v_{\rm cr}$  and the average velocity for the case where  $\Omega = 200$  MHz, T = 373 K, and M = 87u.

Note that for a resonant pump field, i.e.,  $\Delta = 0$ , the adiabaticity condition (21) holds for any velocity as was also noted elsewhere [30].

Now consider an ensemble of Rb atoms at a temperature T=373 K. The critical velocity  $v_{cr}$  together with the average speed of the atoms is shown in Fig. 5. As can be seen, a fair amount of atoms fall into the group where the adiabatic approximation is valid. Further, it has been noted [31] that resonance enhancement is essential if atoms are to contribute to the FWM signal, but that the Doppler shift excludes this enhancement for atoms with  $|\mathbf{k} \cdot \mathbf{v}| > \Gamma$  where  $\Gamma$  is the transition linewidth and  $\mathbf{k}$  is the propagation vector for the pump beam.

In the present approach, the phase-conjugate yield from an atomic gas with a standing pump-wave pattern is obtained by taking the average over all the Rabi frequencies, which for a gas in equilibrium is given by

$$P = \frac{4}{\lambda} \int_0^{(1/4)\lambda} \langle \tilde{\mathcal{P}}(\omega - \delta, \Omega(x)) \rangle dx, \qquad (22)$$

where

$$\Omega(x) = 2\Omega \left| \cos \left( \frac{2\pi}{\lambda} x \right) \right|, \quad \text{where} \quad x \in \left[ 0, \frac{\lambda}{4} \right]$$
 (23)

and  $\Omega = \mu \mathcal{E}/\hbar$ .

The effect of the atomic motion is shown in Fig. 6 for the situation where the atom is initially in the ground state. The solid line represents the case where all atoms are standing still. The effect of the motion is seen in the dashed curve. In Fig. 6(a) the pump-probe detuning is  $\delta = 50$  MHz, and in Fig. 6(b) the pump-probe detuning is  $\delta = 100$  MHz. In both cases the pump Rabi frequency is  $\Omega = 200$  MHz. Note that the yield incorporating atomic motion is larger than the yield when the atoms are standing still. This is because, as a result of the atomic motion, the atoms are distributed equally over the standing-wave intensity pattern. Atoms situated in the nodes will not contribute to the phase-conjugate signal since there is no pump field, whereas atoms situated at antinodes contribute maximally. However, a certain fraction of atoms will be in regions where the pump field is low enough to



FIG. 6. Effect of atomic motion on the phase-conjugate yield. Here it is assumed that the atom is initially in the ground state. Solid line is the situation without atomic motion, dashed line with atomic motion. In both cases the pump Rabi frequency is chosen as  $\Omega = 200$  MHz. In (a) the pump-probe detuning is  $\delta = 50$  MHz and in (b)  $\delta = 100$  MHz.

fulfill the resonance condition  $\Omega' \equiv \sqrt{|\Omega|^2 + \Delta^2} = \delta$ . With the values of the parameters chosen here, this resonance condition is not satisfied for the atoms at rest. That this is indeed the case can be deduced from the fact that the yield is dramatically reduced when  $|\Delta| > |\delta|$ . Note that the dip at line center remains in the case where atomic motion is included.

The corresponding situation but for the case where the atom is initially in a dressed state is shown in Fig. 7. From this figure it is clear that the enhancement of the phaseconjugate yield at line center, observed when the dressed atoms are at rest, persists to the situation where atomic motion is included.

## **IV. CONCLUSION**

We have studied optical phase conjugation by means of nearly degenerate four-wave mixing in a two-level atomic system. The model used is based on the semiclassical dressed states in which first the interaction between pump field and atom is studied, and second the interaction of the probe field with the dressed atom is described by lowestorder perturbation theory. Our results are thus correct to all orders in the pump field and to first order in the probe field.



FIG. 7. Effect of atomic motion on the phase-conjugate yield. Here it is assumed that the atom is initially in a dressed state. Solid line is the situation without atomic motion, dashed line with atomic motion. In both cases the pump Rabi frequency is chosen as  $\Omega = 200$  MHz. In (a) the pump-probe detuning is  $\delta = 50$  MHz and in (b)  $\delta = 100$  MHz.

The generation of the phase-conjugate signal naturally appears as the consequence of the dressed atom interacting with the probe field: each absorption of a probe photon is intimately connected with the emission of a phase-conjugate photon.

It is shown that the generation of the phase-conjugate signal is a sensitive function of the specific initial condition of the dressed atom: in the case where the dressed atom is initially in a superposition state of the lower dressed state  $u_{-}(\mathbf{r})$  and the upper dressed state  $u_{+}(\mathbf{r})$  the phase-conjugate yield vanishes when the pump field is tuned to the atomic resonance. In contrast, starting the atom in a pure dressed state leads to maximum yield when the pump is tuned to atomic resonance. As a consequence the yield is an order of magnitude higher than in the situation where the atom is in the ground state. We therefore propose efficient phase conjugation if the system is prepared initially in a pure dressed state. It was demonstrated by Bai, Yodh, and Mossberg [24] using an amplitude- and phase-controlled field, that it is indeed possible to prepare atoms in pure dressed states.

The vanishing of the phase-conjugate yield at line center is interpreted as a quantum interference effect. The generation process in general proceeds via two distinct channels, where each channel corresponds to the atom being initially in one of the pure dressed states. The channel contributions are equal in magnitude but have opposite signs. For the atom in a given initial state  $\alpha \psi_+ + \beta \psi_-$  the channel contributions to the phase-conjugate yield are weighted by  $|\alpha|^2$  and  $|\beta|^2$ . In the atomic vapor, due to spontaneous emission, most of the atoms start from the ground state, which at line center is in fact a 50:50 combination of dressed states. Therefore at line center there must be a dip in the phase-conjugate yield. Note that the explanation of the dip in the phase-conjugate yield given here differs from the one given by Grynberg, Pinard, and Verkerk [20] for the case of DFWM.

The effect of atomic motion is included by considering an atom moving in the standing intensity pattern of two counterpropagating pump fields assuming that the atom adiabatically follows the spatially varying intensity. The vanishing of the phase-conjugate yield when the pump is tuned to the atomic resonance is shown to be invariant under atomic motion. Further, for the case where the atom is initially in a pure dressed state, the enhancement of the phase-conjugate yield for the pump tuned to resonance is shown to persist to the situation where atomic motion is included.

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