Pump-induced correlation between two lasers

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We have studied the intensity correlation between two lasers induced by the pumping mechanism. This correlation is studied through the noise spectrum of the intensity difference between the two lasers. We have shown, in the context of atomic lasers, that this spectrum may be smaller than the corresponding shot noise if the pumping noises of the two lasers are correlated. 85% noise reduction with respect to shot noise is predicted under reasonable operating conditions. [S1050-2947(99)04608-9]

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In the last decade, great attention and a large literature have been dedicated to the control of quantum noise in optical systems [1]. Many theoretical and experimental developments have been achieved. In the experimental domain it is worthwhile to mention the demonstration of sub-Poissonian behavior in the micromaser [2], subshot-noise operation with semiconductor lasers [3], the production of squeezed light in bistable systems [5,6], and twin beams with optical parametric oscillators (OPO) [7]. It is also interesting to mention the recent demonstration of squeezed light generation with a vertical cavity surface emitting laser (VCSEL) [8]. These achievements are of great relevance from the fundamental point of view as well as for many interesting applications regarding noise control in telecommunications with optical systems or high-sensitivity measurements. Many experiments have been performed on high-sensitivity spectroscopy of trace elements with squeezed light where both OPOs [9] and diode lasers [10,11] were employed. Recently twin beams generated by an OPO were also used [12,13].

Quantum noise reduction in semiconductor lasers is based on the regular pumping principle originally proposed by Golubev and Sokolov [14]. They showed that squeezed light could be generated by lasers provided that pumping noise was suppressed. This idea was successfully applied by Machida and Yamamoto [3]. They obtained squeezed light from a diode laser by controlling the noise in the pump current through the high impedance principle. Another interesting achievement concerning laser noise control is the pump-induced correlation between two diode lasers. In Ref. [4] it was experimentally demonstrated both for diode lasers and light emitting diodes (LEDs) that intensity correlation below the standard quantum limit (SQL) may be obtained by correlating their pumping currents.

In the present work we study the intensity correlation between two lasers induced by the pumping mechanism. Our results are derived in the context of atomic lasers, in contrast with those of Ref. [4] which were obtained for semiconductor lasers. The difference between the photodetection signals obtained from the lasers may present fluctuations below the standard quantum limit (shot noise). The situation is analogous to the one found in twin beams produced by OPOs. In this case, however, the noise reduction is due to quantum correlation between the twin beams as a consequence of the two-photon nature of the radiation emission. In principle, the same kind of noise reduction may be obtained with two lasers, being in this case induced by pump correlation. In analogy to OPOs we will call the correlated lasers *twin lasers*. It may have many applications including high-sensitivity spectroscopy using differential measurement as in Ref. [12].

In order to calculate the quantum noise reduction in twin lasers taking into account the pumping noise properties, we have adopted a well known approach to laser noise theory [15,16]. This approach is suitable for atomic lasers and has been successfully used by several authors, showing good agreement with experimental data [17,18]. In Ref. [16] a general theory for quantum noise reduction in lasers was developed where arbitrary pumping statistics was taken into account and no adiabatic elimination of atomic variables was made. Thus, this theory is suitable for different kinds of atomic lasers having different relative magnitudes of the atomic and cavity decay rates. We will follow the basic steps of Ref. [16] where a set of quantum Langevin equations was derived for macroscopic atomic variables and electromagnetic field.

The atoms in the gain medium are modeled by a set of *open two-level systems* with upper $|e\rangle$ and lower $|g\rangle$ lasing levels as described in Fig. 1. They may decay from these levels to other levels whose dynamics will not be considered. The decay rates from levels $|e\rangle$ and $|g\rangle$ are, respectively, γ_e and γ_g, and the atomic polarization decay rate is $\gamma_{eg} \geqslant (\gamma_e + \gamma_g)/2$. The atoms are pumped from the underlying levels to the upper lasing level $|e\rangle$ with rate R. Cavity losses are described by the intensity decay rate κ. The atom-field interaction is treated under the usual electric-dipole and rotating-wave approximations, with g the coupling constant. We will restrict ourselves to the on-resonance case where the cavity is tuned to the atomic transition frequency. Under these assumptions one obtains the following quantum Langevin equations for the laser:

$$\dot{N}_e(t) = R - \gamma_e N_e(t) - g[a^\dagger(t)M(t) + M^\dagger(t)a(t)] + F_e(t),$$

$$\dot{N}_g(t) = -\gamma_g N_g(t) + g[a^\dagger(t)M(t) + M^\dagger(t)a(t)] + F_g(t), \tag{1}$$

$$\dot{M}(t) = -\gamma_{eg} M(t) + g[N_e(t) - N_g(t)]a(t) + F_M(t),$$

$$\dot{a}(t) = -\frac{\kappa}{2}a(t) + gM(t) + F_a(t),$$

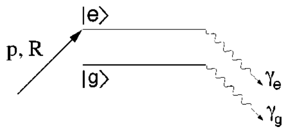

FIG. 1. Relevant atomic levels for the laser model.

where a and $a^\dagger$ are, respectively, the photon annihilation and creation operators, N_e and N_g are the macroscopic populations of levels $|e\rangle$ and $|g\rangle$, M is the macroscopic electric polarization, and F_μ, with $\mu = e,g,M,M^\dagger,a,a^\dagger$, are the corresponding fluctuation forces such that $\langle F_\mu \rangle = 0$. In the Markov approximation, the fluctuation forces are δ correlated:

$$\langle F_\mu(t) F_\nu(t') \rangle = 2 D_{\mu\nu} \delta(t-t'), \tag{2}$$

where $D_{\mu\nu}$ is the diffusion coefficient corresponding to variables μ and ν. The macroscopic atomic operators are defined in terms of the individual atomic operators by

$$N_e(t) = \sum_j \Theta(t-t_j) \sigma_e^j(t),$$

$$N_g(t) = \sum_j \Theta(t-t_j) \sigma_g^j(t), \tag{3}$$

$$M(t) = \sum_j \Theta(t-t_j) \sigma_{eg}^j(t),$$

where t_j is the time when the jth atom was pumped to level $|e\rangle$, $\sigma_{e(g)} \equiv |e(g)\rangle\langle e(g)|$, and $\sigma_{eg} \equiv |g\rangle\langle e|$. Θ is the Heaviside function which takes into account the fact that the jth atom starts to contribute to the macroscopic variables at time t_j. From the above definitions of the macroscopic variables one may obtain the macroscopic atomic fluctuation forces with contributions coming from the microscopic fluctuation forces and from pump noise. Since the atoms are pumped to level $|e\rangle$, the fluctuation force $F_e(t)$ will carry the properties of the pumping statistics. This fact will be particularly relevant to our future discussion. We shall omit the details of the calculations (see Refs. [15,16]) and just present the expression for the fluctuation force corresponding to the macroscopic population of the upper level:

$$F_e(t) = \sum_j \Theta(t-t_j) f_e^j(t) + \sum_j \sigma_e^j(t_j) \delta(t-t_j) - R, \tag{4}$$

where $f_e^j(t)$ is the upper level fluctuation force corresponding to atom j. We will be interested in the autocorrelation function

$$\langle F_e(t) F_e(t') \rangle = [\gamma_e \langle N_e(t) \rangle + R(1-p)] \delta(t-t'). \tag{5}$$

The pumping statistics is described by the parameter p ranging from 0 (Poissonian pumping) to 1 (regular pumping). The term proportional to $(1-p)$ in Eq. (5) comes from the last two terms in the right-hand side of Eq. (4). Since we will be interested in intensity fluctuations we shall restrict our discussion to the amplitude quadrature fluctuations $\delta X(t)$. For a stationary process we have

$$\langle \delta X(t) \delta X(t') \rangle = C_X(t-t'), \tag{6}$$

where $C_X(t-t')$ is the amplitude quadrature correlation function. In the strong field regime, where intensity fluctuations are much smaller than the average intensity $\langle I \rangle$, the intensity noise spectrum is given by

$$S_I(\Omega) = \langle I \rangle + 4 \langle I \rangle S_X^{\mathrm{NO}}(\Omega). \tag{7}$$

$S_X^{\mathrm{NO}}(\Omega)$ is the Fourier transform of the amplitude quadrature correlation function in normal ordering. The first term in the right-hand side of Eq. (7) is the shot-noise contribution. As in Ref. [16] the noise spectrum for the phase and amplitude quadratures of the electromagnetic field may be obtained by linearizing the laser equations [Eqs. (1)] around the steady state and applying the Fourier transform. A set of algebraic equations is then obtained for the Fourier amplitudes of the atomic and field variables. These algebraic equations may be solved for the amplitude quadrature fluctuation in terms of the Fourier transform of the fluctuation forces.

Let us now consider two identical lasers having the same values for γ_e, γ_g, γ_{eg}, κ, and g. We suppose that the lasers are pump correlated so that their pumping rate R as well as the pumping statistics parameter p are the same. We will calculate the noise spectrum of the intensity difference between the two lasers, $I_1 - I_2$. In the strong field regime this noise spectrum is given by

$$S_-(\Omega) = \langle I_1 \rangle + \langle I_2 \rangle + 4 \langle I_1 \rangle S_{X_1}^{\mathrm{NO}}(\Omega) + 4 \langle I_2 \rangle S_{X_2}^{\mathrm{NO}}(\Omega) - 4 \sqrt{\langle I_1 \rangle \langle I_2 \rangle} S_{X_{12}}(\Omega). \tag{8}$$

The first two terms on the right-hand side of Eq. (8) correspond to the shot noise. The other two terms are the individual noise contributions in normal ordering and the last term is the correlation term. This last term is the central point in the present work. If the two lasers are completely independent, there will be no correlation between them so that $S_{X_{12}}(\Omega) = 0$. However, if we suppose that the pumping mechanisms of the two lasers are correlated then we may have intensity correlation. We will show that in this case $S_-(\Omega)$ may present subshot-noise values as in twin beams generated by OPOs.

The correlation term may be written in terms of the correlation function for the amplitude quadrature fluctuations of the two lasers:

$$S_{X_{12}}(\Omega) = \int_{-\infty}^{\infty} \langle \delta X_1(t) \delta X_2(0) + \delta X_2(t) \delta X_1(0) \rangle e^{i\Omega t} dt. \tag{9}$$

We shall consider that all noise sources of the two lasers, apart from pump noise, are uncorrelated. Therefore, we expect that the correlation functions $\langle F_\mu^1(t) F_\nu^2(t') \rangle = 0$ except for $\mu = \nu = e$. In this case, one can easily show from the linearized laser equations that

$$S_{X_{12}}(\Omega) \propto \int_{-\infty}^{\infty} \langle F_e^1(t) F_e^2(0) + F_e^2(t) F_e^1(0) \rangle e^{i\Omega t} dt. \tag{10}$$

In order to calculate $\langle F_e^1(t) F_e^2(t') \rangle$ we will assume that atomic reservoirs from different lasers are uncorrelated so that all correlation will come from terms regarding pump noise [last two terms in Eq. (4)]. This correlation will involve the following averages:

$$\left\langle \sum_j \delta(t-t_j) \right\rangle \text{and} \left\langle \sum_{j,k} \delta(t-t_j^{(1)})\,\delta(t'-t_k^{(2)}) \right\rangle,$$

where $t_j^{(1)}$ is the time when the jth atom of laser 1 was pumped and $t_k^{(2)}$ is the time when the kth atom of laser 2 was pumped. The first average is just the pumping rate R [15,16] which we assume to be the same for both lasers. The second average may be easily calculated in the following limiting cases: (i) No correlation, so that $\langle F_e^1(t)F_e^2(t')\rangle=0$, and (ii) maximum correlation with time delay τ_0. In the last case, for every atom pumped to the upper level in laser 1, there will be another atom pumped in laser 2 after a time delay τ_0 so that $t_j^{(2)}=t_j^{(1)}+\tau_0$. In this case, the desired average may be calculated with the method described in Ref. [15], giving

$$\langle F_e^1(t)F_e^2(t')\rangle = R(1-p)\,\delta(t+\tau_0-t'). \tag{11}$$

In order to separate the effects due to pumping regularization from those stemming from pumping correlation we shall set $p=0$ (random pumping) from now on. However, our results may be easily extended to a general pumping statistics.

Partial correlation is taken into account in a straightforward manner. The two limiting cases considered above may be interpolated by averaging Eq. (11) over a probability distribution $P(\tau)$ for the time interval τ between two consecutive pumps, one in laser 1 and the other in laser 2. $P(\tau)$ will be characterized by the width α and the average time delay τ_0. These parameters, as well as the specific line shape for $P(\tau)$, may be arbitrarily chosen in order to fit future experimental data. No correlation is obtained when $\alpha\rightarrow\infty$ and maximum correlation corresponds to $\alpha\rightarrow 0$ when $P(\tau)$ tends to a δ distribution.

For two identical lasers we have $\langle I_1\rangle=\langle I_2\rangle$ and $S_{X_1}(\Omega)=S_{X_2}(\Omega)\equiv S_X(\Omega)$. In this case the intensity difference noise spectrum normalized to the shot noise is given by

$$\frac{S_-(\Omega)}{S_{\text{shot}}} = 1 + 4\left[S_X^{\text{NO}}(\Omega) - \frac{S_{X_{12}}(\Omega)}{2}\right]. \tag{12}$$

The expression for $S_X(\Omega)$ will not be presented here. We refer the reader to Ref. [16]. From the linearized laser equations we obtain our main result:

$$S_{X_{12}}(\Omega) = \frac{a^2bc^2r(r-1)(b^2+\tilde{\Omega}^2)}{8(a+b)D(\tilde{\Omega})}\,\frac{\tilde{P}(\Omega)+\tilde{P}(-\Omega)}{2}, \tag{13}$$

where we have used the normalized parameters

$$a\equiv\gamma_e/\kappa, b\equiv\gamma_g/\kappa, c\equiv\gamma_{eg}/\kappa, \tilde{\Omega}\equiv\Omega/\kappa, r\equiv R/R_{\text{th}}$$

and

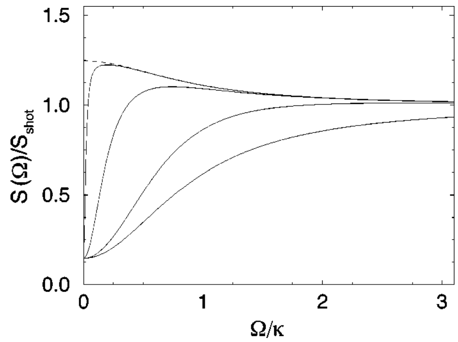

FIG. 2. Intensity difference noise spectrum $S_-(\Omega)$ normalized to shot noise. Dashed line corresponds to the individual laser noise spectrum $S_I(\Omega)/S_{\text{shot}}$. Solid lines from bottom to top correspond, respectively, to $\kappa\alpha=0,1,5,50$. For all curves $\tau_0=0$, $a=10^2$, $b=10^4$, $c=10^2$, $r=10$, $p=0$.

$$D(\tilde{\Omega}) \equiv \left| -i\tilde{\Omega}\left(\frac{1}{2}+c-i\tilde{\Omega}\right)(b-i\tilde{\Omega})(a-i\tilde{\Omega}) + \frac{abc(r-1)}{a+b}(a+b-2i\tilde{\Omega})(1-i\tilde{\Omega}) \right|^2.$$

R_{th} is the threshold pumping rate and $\tilde{P}(\Omega)$ is the Fourier transform of $P(\tau)$.

Different operating conditions may be considered with the theory developed here, which is valid for any relative magnitude of the atomic and cavity decay rates. However, we will show only the results corresponding to the so-called good-cavity limit for which a, b, $c\gg 1$. Besides, it is expected that the best squeezing is obtained for $b\gg a$ [14]. In Figs. 2 and 3 we plotted the intensity difference noise spectrum normalized to shot noise for different values of α and τ_0. For total correlation $(\alpha=0)$ about 85% noise reduction is expected at zero frequency under the operation conditions considered. The width of the squeezed region is of the order of κ. From Fig. 2 it can be seen that the correlation is washed out when $\kappa\alpha\gg 1$. In this case $S_-(\Omega)/S_{\text{shot}}$ tends to the individual noise spectrum of the lasers and the squeezed region of the spectrum becomes narrow. As an example, we have taken an exponential distribution:

$$P(\tau) = \frac{e^{-|\tau-\tau_0|/\alpha}}{2\alpha}. \tag{14}$$

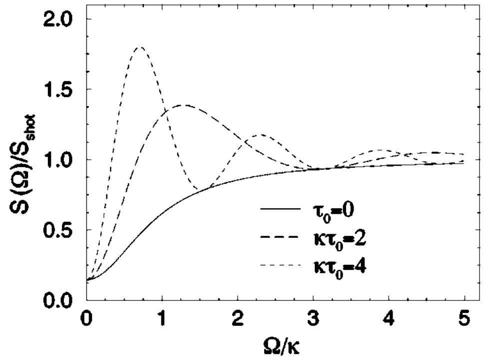

FIG. 3. $S_-(\Omega)/S_{\text{shot}}$ for different values of the time delay τ_0. For all curves $\alpha=0$, $a=10^2$, $b=10^4$, $c=10^2$, $r=10$, $p=0$.

The average time delay τ_0 produces oscillations in the noise spectrum also reducing the squeezed region as shown in Fig. 3. These results indicate that the conditions $\kappa\alpha$ and $\kappa\tau_0 \ll 1$ must be satisfied in order to achieve optimum noise reduction.

In conclusion, we have studied, in the context of atomic lasers, the intensity correlation between two lasers induced by the pumping mechanism. The intensity difference noise spectrum was calculated and subshot-noise behavior was predicted even for Poissonian pumping. The same kind of pump-induced correlation has already been demonstrated with two semiconductor devices [4]. This kind of intensity correlation is analogous to the one observed in twin beams generated by OPOs, which has been recently used for high-sensitivity spectroscopy [12].

A possible experimental approach for implementation of atomic twin lasers would employ two lasers optically pumped by twin beams generated either by an OPO or by two diode lasers as in Ref. [4]. The main experimental difficulty is related to the quantum efficiency of the pumping mechanism. Twin beams in a variety of wavelengths may be obtained if satisfactory quantum efficiencies are attained. Recently, special attention has been given to the development of low noise Nd:YVO_4 microchip lasers [19,18]. These lasers operate at 1064 nm and are optically pumped by diode lasers at 810 nm. An interesting possibility would be to pump two microchip lasers with two correlated diode lasers and check the correlation transfer. The theory developed here shows that this transfer is possible provided that enough quantum efficiency is obtained in the pumping process. The pumping noise model used has been shown to be suitable for Nd:YVO_4 microchip lasers [18].

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