Synchronous Sisyphus effect in diode lasers subject to optical feedback

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Diode lasers exposed to moderate optical feedback, when biased near threshold, exhibit low-frequency intensity fluctuations in the RF spectrum that materialize as dropout events in the intensity time series. We recorded high-quality intensity time traces and frequency spectra of these events that show a new phenomenon in which the dropout events synchronize and appear at regular time intervals. The fine structure of the individual dropouts is recorded with an ultrafast digitizer that reveals the presence of fast oscillations occurring in the picosecond range. Numerical work based on the single-mode single-delay Lang and Kobayashi rate equations shows a locked state consisting of sharp intensity pulses that are slowly modulated and locked to an extraordinarily regular pattern. [S1050-2947(99)01608-X]

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Over the past two decades, external cavity diode lasers have been successfully utilized for linewidth narrowing, suppression of unwanted secondary solitary modes, and reduction of modulation-induced frequency chirp [1]. Yet in spite of these technological achievements, diode lasers subject to optical feedback continue to pose challenging problems in nonlinear dynamics in optical systems. One subject under intense investigation is the sporadic dropout events, often referred to as low-frequency fluctuations (LFF's) [2]. LFF's manifest in the laser emission as a sudden reduction of laser intensity followed by a gradual return to full power. Tartwijk *et al.* [3], employing the metaphor of the mythical Greek character whose fate was to always roll a stone up a hill only to watch it roll back down, named this process the Sisyphus effect. This phenomenon was noticed as early as 1977 by Risch and Voumard [4] in the radio-frequency spectrum (RF) of their measurements. Over the years it has been investigated experimentally and analyzed theoretically in a number of stochastic and deterministic settings. However, the origin of the key mechanism inducing this behavior still remains unclear.

Recently, progress has been made towards elucidating the origin of the LFF's. Sano numerically investigated the Lang and Kobayashi (LK) rate equations near the threshold of the solitary laser [5]. He determined that the sudden intensity drops are caused by a crisis between local chaotic attractors and unstable saddle-type solutions of the steady-state equations, called antimodes [6]. The subsequent return to full power was shown to be a switching between compound cavity-mode oscillations. Tartwijk *et al.* [3] substantiated Sano's observations and presented numerical evidence concerning the existence of strong intensity pulses during the build-up of the average intensity. They suggested that these pulses are the manifestation of a form of imperfect mode locking that is frustrated by the drive towards the maximum gain mode [7]. Subsequently, Fisher *et al.* [8] reported streak camera observations of these irregular picosecond light pulses within the coherence collapse region, and attributed this pulsing behavior to chaotic itinerancy with drift. These investigations focused on the deterministic aspects of the events. In contrast, Hohl *et al.* [9] showed numerically that spontaneous-emission noise can significantly influence the statistics of the dropout events. By measuring the statistical distribution of the dropout events and the dependence of the mean time as a function of feedback strength, they found agreement with predictions of stochasticity made by Henry and Kazarinov [10]. Hohl *et al.* also concluded that for an accurate description of the dropout events, both deterministic and stochastic mechanisms are necessary.

Furthermore, in the earlier literature on LFF's, some very provocative experimental observations suggest that genuine locking phenomena may occur. Besnard *et al.* [11] found in a feedback experiment that when the external cavity laser operates in this unstable regime, a locking of the dropout events can occur, giving rise to a narrow resonance in the RF spectrum, which manifests as a self-modulation of the intensity. Attempts to explain this phenomenon by expanding the LK equations in external cavity modes produced the low-frequency features observed in the experiment. However, no experimental evidence was given for, or account taken of, the high-frequency component intrinsic to LFF as reported in Ref. [8] and numerically shown in Refs. [3,5]. Thus this earlier work cannot provide an adequate explanation of the locking phenomena. Understanding the physical mechanism of this locking phenomenon, which we call the *synchronous Sisyphus* effect, remains an open problem. The region of the pumping current where this effect occurs in unusually small and appropriate conditions of external cavity length and the degree of mirror tilting have to be established. The inherent noise in diode lasers tends to destabilize and destroy the locking mechanism. But even when these issues are re-

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solved, the ultrahigh-frequency oscillations are still at the limit of current data acquisition technology.

The objective of this paper is twofold. First, using high speed data acquisition, we investigate the ultrafast dynamical response of the compound cavity. Second, we present evidence of an observed synchronous Sisyphus effect and explain it in the context of the well established single delay LK equations. We emphasize that while care is needed in identifying the locked state, no adjustments or extensions are required to the LK equations in order to explain this locking phenomenon.

In the experiments, a 20 mW Toshiba TOLD9140 index guided InGaAlP laser diode, emitting at $\lambda = 693\,\mathrm{nm}$, was used. It was driven by a low-noise current supply and was temperature stabilized to $\pm 0.01\,°\mathrm{C}$. A high reflectivity silver mirror, located approximately 77 cm from the laser diode, created an external cavity with a fundamental frequency of $f_e = (c/2L) = 194\,\mathrm{MHz}$. An intracavity beam splitter directed a fraction of the light into a Hamamatsu C4258 photodetector with a 40 ps rise time. At these low current levels of operation, the diode lased in a few solitary longitudinal modes throughout the experiment, though the number of external cavity modes that the laser visited was typically very large.

The small photodetector signal was preamplified using a cascaded Hewlett-Packard 8447D dual amplifier. Each amplifier had a flat gain of 25 dB up to 1.5 GHz and a 3 dB bandwidth of 1.7 GHz. We measured the RF spectrum of the amplified signal with a Hewlett-Packard 8596E RF spectrum analyzer and recorded time series with a Tektronix RTD720A digitizer with a 500 MHz bandwidth. Additionally, we employed a Tektronix SCD5000 transient event digitizer with a 5 ps minimum sampling interval, 4.5 GHz bandwidth, and 1024 samples per trace, to record high-speed events. However, in order to generate a detectable signal for this digitizer, an additional amplifier with a gain of 12 dB and bandwidth of 1 MHz to 4 GHz was used.

The objective of our measurements was to faithfully capture the underlying fast dynamics of the locked state to reveal the nature of this phenomenon. Initially we measured the light-current characteristics for both the solitary laser case and the external cavity arrangement. The threshold of the solitary laser was at $I_{\mathrm{th}} = 38.3\,\mathrm{mA}$ and reduced to 33.6 mA upon coupling with the external mirror. The curve for the compound cavity laser showed a characteristic kink at the solitary threshold.

We have used the expedient way of adjusting the strength of the feedback by tilting the external mirror. The necessary adjustment of the feedback is very critical in order to stabilize the locked state completely. In our particular experimental arrangement, there was no other way of adjusting the strength of the external feedback. However, it was possible to grossly adjust the feedback strength by first inserting neutral density (ND) filters and then making small adjustments

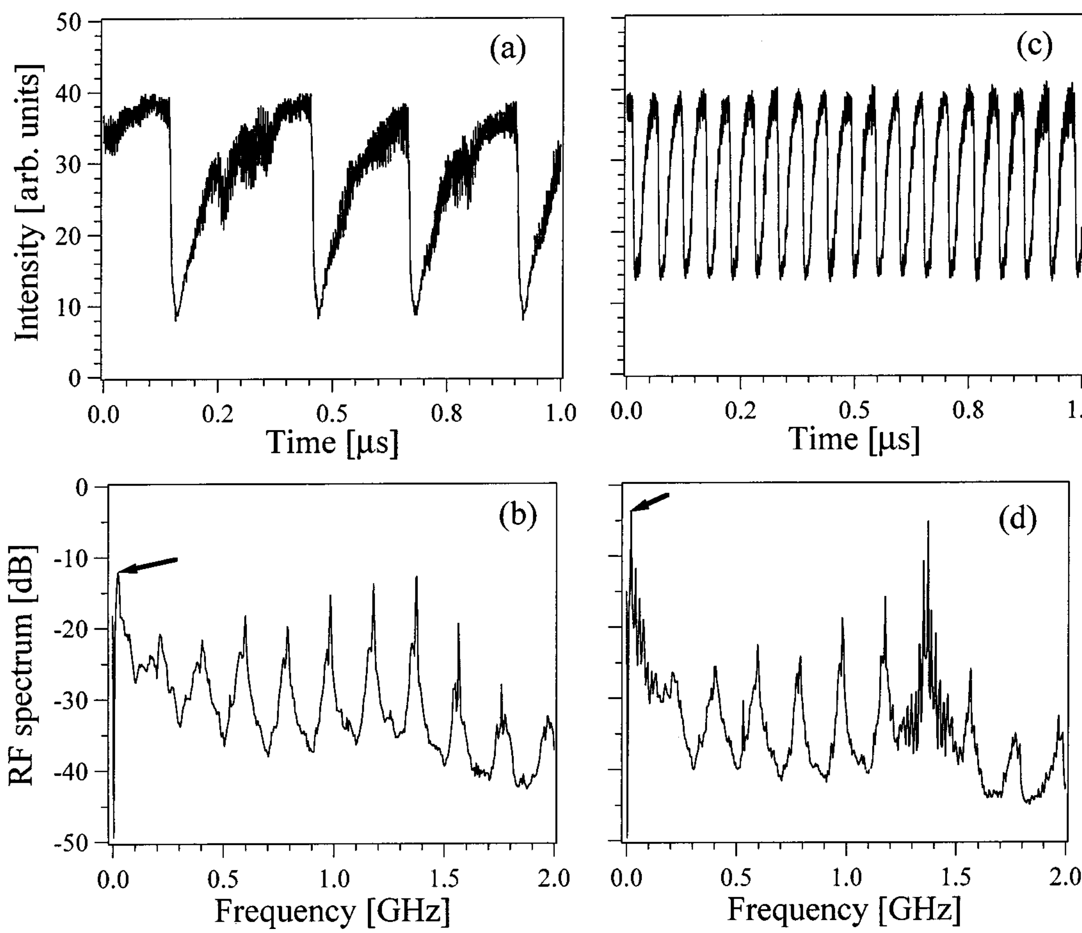

FIG. 1. (a) Experimentally obtained time series, $I = 38.0\,\mathrm{mA}$, of irregularly occurring dropout events, (b) corresponding RF spectrum, (c) experimentally obtained time series, $I = 39.0\,\mathrm{mA}$, showing periodically appearing events, and (d) corresponding RF spectrum. The arrows point to the LFF and the synchronous Sisyphus frequency of 19.4 MHz.

to stabilize the locked state. Indeed, using, for example, a filter to give an additional external feedback loss to the diode laser of 28%, the locked state was obtained even without tilting the mirror and remained stable and of identical form for a tilt adjustment up to 0.25 mrad about this position, beyond which it gradually lost stability. In both cases, with and without filter and appropriate tilting of the mirror, the power versus pumping current diagram was identical.

The experiment consisted of recording the intensity of the laser in the external cavity arrangement as the pumping current I was ramped in 1 mA increments from 35 to 50 mA. At each current step, RF spectra and time traces from both digitizers were recorded. In general, the time domain traces of the laser intensity are dominated by two characteristic features: sharp drops of the intensity and subsequent slow rises to full power. Furthermore, the RF spectra showed low-frequency peaks at a tenth of the external cavity mode spacing and clear evidence of external cavity modes.

At $I=38.0\,\mathrm{mA}$, a value of pumping slightly below the solitary laser threshold, the laser resides in a state between the solitary and external cavity laser thresholds. Figure 1(a) shows an intensity time trace measured with the Tektronix RTD720A digitizer. At this pumping level, dropout events are clearly observed. They occur at widely varying intervals, and due to the low level of pumping the return to full power is slow compared with the typical picosecond time scale of laser diodes. The corresponding Fourier spectra, using a 0–2-GHz sweep, shown in Fig. 1(b), clearly reveal high-frequency components not resolved in the time trace.

These features are in marked contrast to those obtained at a larger pumping level, $I=39.0\,\mathrm{mA}$, slightly above the solitary laser threshold. Here, regularly spaced dropout events appear and the rise time of the dropouts increases dramatically. Figure 1(c) shows such a time trace in which it is clear that even though the events seem to occur very periodically, nevertheless the shape of the oscillations show small period-to-period differences. For this low bandwidth measurement, the time trace looks almost sinusoidal and the intensity of the dropouts does not completely reach zero. Figure 1(d) shows that even though the frequency content remains very high, there is a characteristic frequency of 19.4 MHz (52 ns) that exhibits a very narrow bandwidth. This is the characteristic LFF frequency. As the pumping was varied around this value, we found the periodic state to exist for a certain amount of time before it destabilized into irregular dropout events. This happened with greater frequency as the pumping approached 39 mA, at which point stabilization of the state was complete. As was shown in Ref. [8] using streak camera observations, and also numerically outlined in Ref. [5], the underlying structure of the time series of dropout events exhibits a spiking behavior well into the picosecond regime. We find this behavior to persist for the periodic dropout state. A transient recording of this locked state is shown in Fig. 2. The actual drop in intensity occurs in only a few nanoseconds and is followed by a period in which the laser intensity appears to be weak. However, even during this time, there is considerable spiking such that the average intensity is not zero. In the recovery time we can resolve a dominant 1.36 GHz (0.735 ns) oscillation and the 194 MHz external cavity frequency. Notice also that the pulse bunches are separated by 5.1 ns, which matches the round trip time of the external cavity.

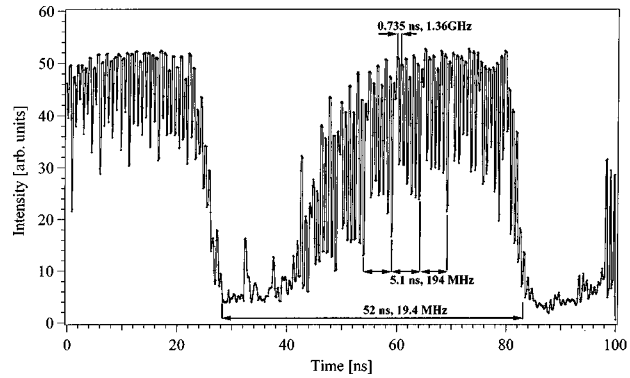

FIG. 2. Transient digitizer recording of the locked state at $I=39.0\,\mathrm{mA}$. Three time scales are evident 52 ns (19.4 MHz) corresponding to the period of the synchronous Sisyphus events, 5.1 ns (194 MHz) corresponding to the period of the external cavity modes and 0.735 (1.36 GHz) corresponding to the duration of the fast pulsations.

To elucidate the mechanism of this locking phenomenon, we have investigated numerically the Lang and Kobayashi delay differential equations [12]. The evolution of the carrier density N and the electric field $\mathcal{E}=A\exp[i\Phi(t)]$, where A is the field amplitude and Φ is the phase, is prescribed by the following set of dimensionless equations [13]:

$$\dot{\mathcal{E}}=(1+i\alpha)N\mathcal{E}+\eta e^{-i\omega\tau}\mathcal{E}(t-\tau), \tag{1}$$

$$T\dot{N}=P-N-(1+2N)|\mathcal{E}|^2. \tag{2}$$

In the equations, time t is measured in units of the photon lifetime τ_p ($\tau_p\sim 1$ ps), and $T=\tau_s/\tau_p$ is the ratio of the carrier lifetime ($\tau_s\sim 1$ ns) to the photon lifetime. The external cavity round trip time is $\tau=2L/c\tau_p$ and $\omega\tau$ is the round-trip phase mismatch, where ω is the solitary laser angular frequency. The excess pump current P is proportional to $(I/I_{\mathrm{th}})-1$, where I and I_{th} are the current and its value at the solitary laser threshold, respectively. The linewidth enhancement factor is denoted by α, and the amount of feedback is represented by η.

Naturally, a set of simplifying approximations is included in the derivation of the LK equations. For example, only one solitary mode is taken to lase, even though we determined that a large number of solitary laser modes were present in the experiment. However, we have determined that the nature of the locked state does not depend on the number of lasing modes. Calculations using a multimode model which includes gain suppression terms have shown that there is practically no difference in the nature of the locked state when the total intensity is detected. Some of the additional effects observed numerically and confirmed experimentally will appear in a future publication. The presence of a single delay term accounts for the empty cavity with a specific finesse. Including more delay terms in the model sharpens the finesse but does not alter the basic structure of the dynamics in phase space. In addition, these equations have successfully explained feedback effects for Fabry-Perot [14] and for

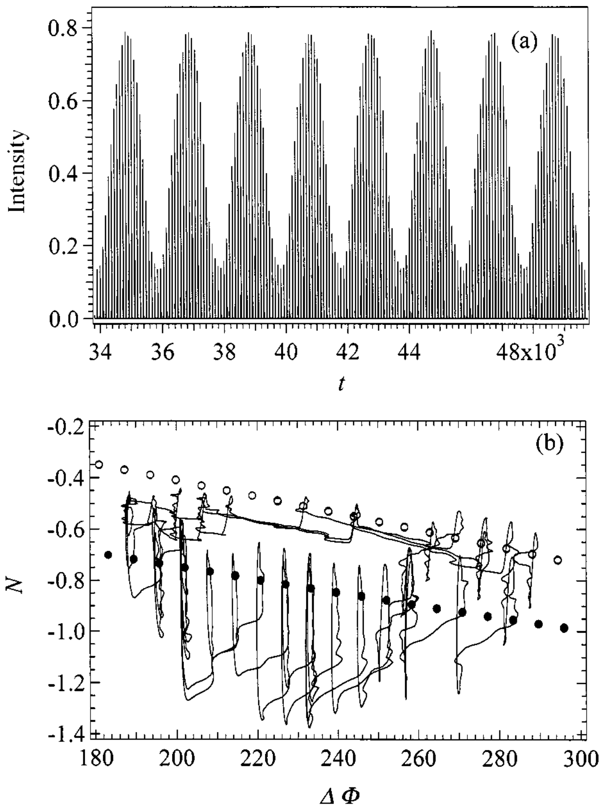

FIG. 3. (a) Numerical calculation of a representative locked state. We distinguish a sequence of equally spaced and slowly modulated sharp intensity pulses, and also recognize the high degree of regularity between the consecutive bunches of pulses. (b) Phase plot of two consecutive periods of the locked state. The diode laser visits approximately the same number of modes in each period though not necessarily the same ones. The circles (○) denote saddles and the bullets (•) designate nodes of the external cavity. The quantities in this figure are dimensionless.

distributed-feedback lasers [15]. The essence of the observed locking behavior, as shown below, originates from a large number of fixed points present in this operating regime.

We found a locked state by solving numerically the LK equations (1) and (2) utilizing a fourth-order fixed-step Runge-Kutta algorithm, with the following parameters: $P = -0.02$ (slightly below the solitary laser threshold), $\alpha=6$, $T=282$, $\tau=666.70$, and $\eta=0.1$. There are approximately 130 external cavity modes with these values, half of which are above threshold and therefore their chaotic ruins are regularly visited by the laser during the LFF process. The numerical calculation under these conditions shows that the structure of the LFF excursions locks to an almost regular pattern. The periodic state is shown in Fig. 3(a). It consists of a series of very short spikes which are modulated by a slow frequency of about 20 MHz, corresponding to one-tenth of the external cavity spacing. The phase space of N versus $\Delta\Phi=\Phi(t-\tau)-\Phi(t)$ of two successive periods of this solution exhibits the periodic visitation of the laser to a restricted set of external cavity modes in the range of 190–285, rather than 0–310, as would normally happen under LFF operation. As shown in Fig. 3(b), the laser visits approximately the same number of modes in each period though not necessarily the same modes. The dropout event happens when the laser, on its way towards the maximum gain mode, collides with approximately the same saddle, but the power does not drop to zero. Instead of the laser returning to the solitary laser mode, which is located at about $\Delta\Phi\sim 0$, the laser returns to a much larger intensity external cavity mode located at $\Delta\Phi\sim 190$. For comparison, we also show the fixed points in Fig. 3(b) (saddles are indicated by circles and the nodes by bullets). The parameters for this calculation were judiciously chosen in order to show vividly key features of a synchronous Sisyphus locked state: the sequence of sharp intensity pulses that are slowly modulated and locked to an extraordinarily regular pattern.

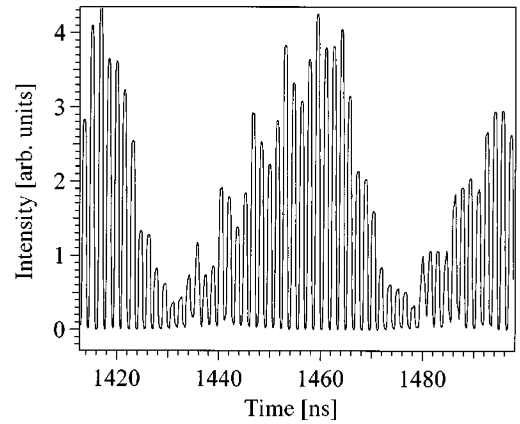

FIG. 4. Filtered intensity time trace of a computed locked state. Notice the qualitative correspondence with the experimentally recorded synchronous Sisyphus event of Fig. 2. The larger number of pulses in the experimental measurement, Fig. 2, may be attributed to the fact that the laser could be operating in a number of solitary longitudinal modes.

In Fig. 4 we show a temporally filtered time trace of a computed intensity time series that matches key features of the Sisyphus event obtained experimentally with the Tektronix SCD5000 transient digitizer and shown earlier in Fig. 2. These are (a) the duration of the basic cycles of about 50 ns corresponding to an envelope of 20 MHz close to the observed frequency of the 19.4 MHz in the RF spectrum, (b) sharp pulses of duration less than 1 ns, and (c) periodic bunches of pulses that are proportional in duration to the external cavity length, evident during the recovery of the laser intensity. Since the intensity was obtained from the middle of the external cavity laser through the use of a beam splitter, our observable consists of a coherent sum of multiple delayed terms [16]. This particular feature of our experimental setup was also incorporated in the filtering algorithm. However, we note that in Fig. 2 we have a larger number of pulses due to the fact that the diode laser may operate in a number of solitary longitudinal modes. Therefore, the agreement between our computations and the recorded data is at a qualitative level. For our computations we employed a typical set of parameters for diode lasers and in particular for the conditions at which the locked state was observed. These are $P=0.005$ (slightly above the solitary diode threshold), $\alpha=6$, $T=141$, $\tau=696.70$ (corresponding to an external cavity of 77 cm), $\eta=0.08$, $\omega\tau=0.0$, and photon decay rate of 1.357 ps. A cutoff frequency of 1.5 GHz was used for temporal filtering. The level of feedback was determined from an experimentally measured reduction of 8% in the threshold current for lasing compared to that for solitary laser.

Furthermore, extensive calculations with the addition of stochastic noise to mimic spontaneous-emission events reveal that there is a sensitive balance making the existence of the locked state possible. As the strength of the noise is gradually increased, the locked state loses its stability and the laser wanders into LFF only to return to the locked state for a certain amount of time. The RF spectrum, and in particular the shape of the LFF frequency, broadens and for large noise the signature and identity of the locked state are lost. A statistical analysis of the time intervals between dropout events has an almost Poissonian distribution for low pump levels and collapses to a δ function with the appearance of the locked state. In cases where the laser is noisier, the synchronous Sisyphus phenomenon is less readily observed. However, we have found clear evidence of bimodality in the distribution of the dropout occurrence times. This shows that the locked state can appear intermittently. Similar results regarding bimodality have been reported by Hohl [17].

In this paper we have studied experimentally the fast dynamics of the intensity dropout events as a function of pumping current. We have provided a detailed investigation of a locked state in which the dropout events occur periodically, including the underlying high-frequency components of its dynamics. In addition, we have numerically explored the LK equations and identified basic features of the locked state such as the sequence of sharp slowly modulated intensity pulses, locked to a markedly regular pattern. Also, through a filtering algorithm we made a comparison with our time-domain measurements of the intensity taken with the ultrafast digitizer. Even though we have clearly identified the presence of a locked state in the case of a single mode singe delay LK dynamical system, intriguing questions remain, for example, about the origin of the locked state. Does it originate from a global bifurcation? Or, is it some form of mode locking? These issues will be addressed in the future.

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