Local-field effects in a dense ensemble of resonant atoms: Model of a generalized two-level system

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(Received 4 June 1998)

The local-field effects are considered which are feasible in a dense ensemble of resonant atoms modeled by multilevel quantum systems. Our approach is based on the generalized two-level system [V.S. Butylkin, A.E. Kaplan, and Yu.G. Khronopulo, Zh. Eksp. Teor. Fiz. 59, 921 (1970)]. Making use of this model, we take account of the nonresonant polarization and Stark shift of the absorption line due to the difference in linear polarizabilities of atoms in ground and excited states. With near dipole-dipole interaction and quadratic Stark effect included simultaneously, new features of the hysteresis behavior of the population difference as a function of the external-field intensity are predicted. [S1050-2947(99)01504-8]

PACS number(s): 42.65.Pc, 42.25.Gy

I. INTRODUCTION

In Ref. [1] it was shown that under conditions of resonant radiation absorption by a dense medium the near dipoledipole (NDD) interaction or local-field effect is a mechanism that leads to the spectral shift of the absorption line, which is equal to $n\omega_L$ where $n=n_1-n_2$ is the level population difference, $\omega_L = (4 \pi/3) N_0 \mu^2 / \hbar$ is the Lorentz frequency, μ is the dipole transition moment, and N_0 is the density of resonant atoms. This inference seemed to be an important implication for the prediction of the bistable dependence of population difference n on the radiation intensity I or intrinsic optical bistability (IOB) [2]. The local-field correction modifies the Bloch equations in a nontrivial way, and this is just what admits a two-valued solution n(I). The prediction of IOB [2] stimulated quite a number of investigations devoted to different aspects of this phenomenon (see, for example, Refs. [3-5]). The appearance of IOB is feasible at sufficiently high concentrations of resonant atoms. However, increasing the density of resonant atoms by itself does not necessarily provide the bistable behavior of the population difference as a function of the external-field intensity. In the relevant theory there is a criterion (necessary threshold condition) that has the form $b \equiv T_2 \omega_L \ge 4$ where T_2 is the lateral relaxation time [4]. Thus parameter b plays the role of the NDD interaction constant, that has to take on the "magic" value of not less than approximately 4. The above inequality imposes a very rigid restriction on the feasibility of IOB. For example, for a collision-broadened gas this condition is not satisfied in principle and the optical bistability is not attainable [4]. These considerations to a great extent motivated our study, the results of which are presented in this paper. Here we show that a dense ensemble of atoms modeled by multilevel quantum systems displays a more complicated (in comparison to ordinary two-level systems) dynamics of IOB due to the renormalization of the local-field amplitude and additional shift of the resonant absorption line caused by the quadratic Stark effect. It is remarkable that a dense ensemble of multilevel resonant atoms gives a chance of reducing b(up to b < 1) to attain conditions required for the emergence of IOB. Our examination presumes the adiabatic character of the change in the external-field intensity and is not concerned with the radiation propagation effects.

To describe the resonant interaction of the field ${\cal E}$ $= Ee^{-i\omega t}$ with the multilevel quantum systems under conditions of the one-photon resonance, use is made of the model of a generalized two-level system proposed and developed by Butylkin, Kaplan, and Khronopulo [6]. In the framework of this model one can correctly take into account the change in the resonant medium polarization influenced by the quadratic Stark effect related to the difference between the linear polarizabilities of atoms in ground $|1\rangle$ and excited $|2\rangle$ states.

It is worthwhile to divide our further treatment of the subject as follows. In Sec. II we address the modification of Bloch equations that are intended for the adequate description of multilevel systems. Section III is the principal one, where the basic idea is formulated into the proper equation for the population difference. Section IV illustrates a variety of feasible situations where the high density of multilevel atoms results in novel properties of IOB. In conclusion, in Sec. V some experimental points and possible applications are briefly discussed.

II. GENERALIZED TWO-LEVEL MODEL. MODIFIED BLOCH EQUATIONS

In the case of the one-photon resonance the Bloch equations for the generalized two-level system can be written as [6]

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$$\dot{n} = -\frac{4\mu}{\hbar} \operatorname{Im}(E_L^* P) - \frac{n-1}{T_1},$$
 (1)

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$$\dot{P} = i \frac{\mu}{\hbar} E_L n - \frac{P}{T_2} [1 - i(\delta - \Omega)], \qquad (2)$$

where *P* is the polarization amplitude, T_1 is the time of longitudinal relaxation, and $\delta = T_2(\omega - \omega_0)$ is the normalized detuning of the frequency ω of the acting field from the frequency ω_0 of resonant transition $|1\rangle \leftrightarrow |2\rangle$. Equations (1) and (2) are distinguished from the usual Bloch equations for the ordinary two-level system [7] by the parameter

$$\Omega = \gamma |E_L|^2 \equiv \frac{\kappa_1 - \kappa_2}{\hbar} T_2 |E_L|^2.$$
(3)

It determines the Stark shift of the absorption line due to the nonresonant interaction of the field with a multilevel quantum system. Here

$$\kappa_j = \frac{2}{\hbar} \sum_{n>2} \frac{|\mu_{nj}|^2 \omega_{nj}}{\omega_{nj}^2 - \omega^2} + \frac{(-1)^j |\mu|^2}{\hbar(\omega_0 + \omega)} \tag{4}$$

are the tensor components of the linear atomic polarizability in the *j*th state (*j*=1,2) at the frequency ω , μ_{nj} and ω_{nj} being the dipole moments and frequencies of nonresonant transitions $|n\rangle \leftrightarrow |j\rangle$. The local-field amplitude E_L involved in Eqs. (1) and (2) is defined taking into account the Lorentz-Lorenz correction:

$$E_L = E + \frac{4\pi}{3}\mathcal{P},\tag{5}$$

where \mathcal{P} is the macroscopic polarization amplitude for an ensemble of resonant atoms. As is shown in Ref. [6], in the generalized two-level model of quantum systems the polarization amplitude is

$$\mathcal{P} = \mathcal{P}_{\text{nonr}} + \mathcal{P}_{\text{res}} = \frac{N_0}{2} [\kappa_1 + \kappa_2 + (\kappa_1 - \kappa_2)n] E_L + \mu N_0 P.$$
(6)

In view of Eq. (6) the local-field amplitude is reduced to

$$E_L = \alpha \left(E + \frac{4\pi}{3} \mu N_0 P \right), \tag{7}$$

with

$$\alpha = \left(1 - \frac{2\pi}{3}N_0[\kappa_1 + \kappa_2 + (\kappa_1 - \kappa_2)n]\right)^{-1}, \qquad (8)$$

and the inequality $(2\pi/3)N_0[\kappa_1 + \kappa_2 + (\kappa_1 - \kappa_2)n] < 1$ takes place. Consequently, the influence of the nonresonant part \mathcal{P}_{nonr} results in the renormalization of the local-field amplitude. It is interesting that this renormalization resembles that of the local field in the case of an ensemble of resonant atoms embedded into a host (background) dielectric medium [8]. However, in our case because $\kappa_1 \neq \kappa_2$ the renormalization parameter α depends on the population difference *n* of resonant levels. Equations (1) and (2) can be recast using Eqs. (3) and (7), so that

$$\dot{n} = -\frac{4\mu d}{\hbar} \operatorname{Im}\left(E^*P\right) - \frac{n-1}{T_1},\tag{9}$$

$$\dot{P} = i \frac{\mu d}{\hbar} nE - \frac{P}{T_2} \{ 1 - i [\kappa + \eta |P|^2 + \sigma \operatorname{Re}(E^*P)] \},$$
(10)

where the new symbols introduced are $\kappa = \delta + \alpha b n$ $-\alpha^2 \gamma |E|^2$, $\eta = -\gamma [(4\pi\alpha/3)N_0\mu]^2$, $\sigma = -\gamma \alpha^2 (8\pi/3)N_0\mu$; $b = (4\pi/3\hbar)\mu^2 N_0 T_2$ is the above introduced constant of NDD interaction.

Thus in the model of a generalized two-level system the Stark shift of the absorption line is caused by two mechanisms: the NDD interaction and nonresonant medium-field interaction. Note that when $\gamma \rightarrow 0$ and $\alpha \rightarrow 1$ Eqs. (9) and (10) go over into the modified Bloch equations for the ordinary two-level system [1–5,8,9].

III. BASIC RELATIONSHIPS

Let us analyze Eqs. (9) and (10) in the stationary approximation $(\dot{n}=0, \dot{P}=0)$. As a matter of convenience we represent Eq. (10) as a system of two equations for two components U and V (P=U+iV):

$$\alpha \frac{\mu}{\hbar} n E_{\rm I} - \frac{V}{T_2} + \frac{U}{T_2} [\kappa + \eta (U^2 + V^2) + \sigma (E_{\rm I} U + E_{\rm II} V)] = 0,$$
(11)

$$\alpha \frac{\mu}{\hbar} n E_{\rm II} - \frac{U}{T_2} + \frac{V}{T_2} [\kappa + \eta (U^2 + V^2) + \sigma (E_{\rm I} U + E_{\rm II} V)] = 0.$$
(12)

Here $E = E_{I} + iE_{II}$. From stationary equation (9) it follows that

$$(E_{\rm I}V - E_{\rm II}U) = \frac{\hbar}{4\mu\alpha} \frac{1-n}{T_1},\tag{13}$$

and bearing it in mind, after simple algebraic manipulations from Eqs. (11) and (12) one can obtain the relationship

$$U^2 + V^2 = \frac{n(1-n)}{2\Delta},$$
 (14)

where $\Delta = 2T_1/T_2$. Equation (14) is essentially an analog of the probability conservation law that holds for time intervals substantially less than T_1 and T_2 [7], its form in our notation being $U^2 + V^2 + 4n^2 = \text{const}$. Relationship (14) enables Eqs. (11) and (12) to be transformed to the single quadratic equation for *P*

$$\left(\frac{\sigma}{2}E^*\right)^2 P^2 + i\left[1 - i\left(\kappa + \frac{\eta}{2\Delta}n(1-n)\right)\right]P + En\left(\frac{\sigma}{4\Delta}(1-n) + \frac{\alpha\mu T_2}{\hbar}\right) = 0, \quad (15)$$

the solution of which has the form

$$P = \frac{E}{i\sigma|E|^2} \left\{ \left[1 - i\left(\kappa + \frac{\eta}{2\Delta}n(1-n)\right) \right] - \sqrt{\left[1 - i\left(\kappa + \frac{\eta}{2\Delta}n(1-n)\right) \right]^2 + 2\sigma|E|^2n\left(\frac{\sigma}{4\Delta}(1-n) + \frac{\alpha\mu T_2}{\hbar}\right)} \right\}.$$
 (16)

It should be noted that the second root of Eq. (15) has no physical meaning, since at $\gamma \rightarrow 0$ and $\alpha \rightarrow 1$ it gives a divergent solution for the polarization. And solution (16) becomes the well-known expression for the polarization amplitude corresponding to the conventional two-level system [9]. To simplify further analysis, let us reduce the number of parameters by introducing corresponding physical quantities. In particular, the condition $\Omega = 1$ permits us to introduce a constant that determines the intensity of the Stark field,

$$|E|_{\mathrm{St}}^2 = \frac{\hbar}{|\kappa_1 - \kappa_2|T_2},\tag{17}$$

at which the absorption line shifts by a value of $1/T_2$. Another characteristic parameter of the problem is the intensity of the resonant transition saturation at $\delta = 0$:

$$|E_0|^2 = \left(\frac{\hbar}{2\mu}\right)^2 \frac{1}{T_1 T_2}.$$
(18)

As will be shown below, the quantity

$$k = \operatorname{sgn}\left(\gamma\right) \frac{|E_0|^2}{|E|_{\operatorname{St}}^2} \tag{19}$$

is, together with α , an additional dimensionless parameter that arises in changing over to the model of a generalized two-level system. Substitution of Eq. (16) into stationary Eq. (9) results in an equation of the sixth degree for the population difference of resonant levels:

$$\sum_{m=0}^{6} \beta_m n^m = 0, (20)$$

the coefficients being functions of parameters of our system b, δ , Δ , k, $\phi = 1 - (2\pi/3)N_0(\kappa_1 + \kappa_2)$, and the normalized intensity $I = |E|^2/|E_0|^2$:

$$\begin{split} \beta_{0} &= (-bk+\phi)[I^{2}k^{2}+2k\phi^{2}(2b^{2}k-I\delta)-4bk\phi^{3}+\phi^{4}(1+\delta^{2})], \\ \beta_{1} &= 8b^{4}k^{4}\Delta\phi+2b^{3}k^{2}[Ik+\phi^{2}\delta+4k\phi^{2}(2-3\Delta)]-4b^{2}k\phi\{Ik(1+k\delta\Delta)+\phi^{2}[\delta+6k-5k\Delta-k\Delta\delta^{2}]\} \\ &-\phi\{I^{2}k^{2}+I\phi^{2}(1-2\delta k)+\phi^{4}(1+\delta^{2})\}+b\{I^{2}k^{3}(2-\Delta)+Ik\phi^{2}[3+2k\delta(3\Delta-2)]\} \\ &+b\phi^{4}\{2\delta+2k(\delta^{2}+5)-5k\Delta(1+\delta^{2})\}, \\ \beta_{2} &= bI^{2}k^{3}(\Delta-1)-b^{5}(k^{3}+4\Delta^{2}k^{5})-2b^{2}\phi Ik^{2}(3\Delta-4)(1+k\Delta\delta)-b^{4}k^{2}\phi[-3+4k\Delta(\delta+8k-6k\Delta)] \\ &-Ikb\phi^{2}[3-3\Delta+2k\delta(3\Delta-1)]+b^{2}\phi^{3}\{1+2k[12k+4\delta-4\Delta(\delta+k\delta^{2}+5k)+5k\Delta^{2}(1+\delta^{2})]\} \\ &-b\phi^{4}\{2\delta+5k+k\delta^{2}-5k\Delta(1+\delta^{2})\}-b^{3}k(-2Ik^{2}[k\delta\Delta^{2}+2\Delta-3] \\ &+3\phi^{2}+6k\phi^{2}\{\delta-2\delta\Delta+4k+k\Delta[-12+\Delta(5+\delta^{2})]\}), \end{split}$$

$$\begin{split} \beta_{3} &= -b^{3}k^{3}\{-4b^{2}+b^{2}\Delta[3+2k\Delta(4k\Delta-\delta-8k)]\} - b^{3}k^{3}I[-6+\Delta(8+4k\delta\Delta-2k\delta\Delta^{2}-3\Delta)] \\ &+ b^{2}k^{2}\phi I[-4+6\Delta-3\Delta^{2}+2k\Delta\delta(3\Delta-2)] + k^{2}b^{4}\phi\{-9+6\Delta-12k\delta\Delta(\Delta-1)\} + 4k^{2}\Delta[12-18\Delta+\Delta^{2}(5+\delta^{2})]\} \\ &- b^{3}k\phi^{2}\{3\Delta-6-6k\delta-12k\delta\Delta(\Delta-2)+2k^{2}[-8+36\Delta-30\Delta^{2}-6\Delta^{2}\delta^{2}+5\Delta^{3}(1+\delta^{2})]\} \\ &- b^{2}\phi^{3}\{1+4k\delta-8k\delta\Delta+8k^{2}-4k^{2}\Delta(5+\delta^{2})+10k^{2}\Delta^{2}(1+\delta^{2})\}, \end{split}$$

$$\begin{split} \beta_4 &= -b^3 I k^3 (\Delta - 1) (2\Delta - 2 + 2k\Delta^2 \delta - \Delta^2) - b^5 k^3 \{ 6 - 9\Delta + \Delta^2 [3 + 6k\delta - 4k\delta\Delta + 24k^2 - 24k^2\Delta + k^2\Delta^2 (5 + \delta^2)] \} \\ &- b^4 k^2 \phi \{ -9 - 3\Delta (\Delta - 4) + 4k\delta\Delta [3 + 2\Delta (\Delta - 3)] + k^2\Delta [32 - 72\Delta + 8\Delta^2 (5 + \delta^2) - 5\Delta^3 (1 + \delta^2)] \} \\ &- b^3 k \phi^2 \{ 3 - 3\Delta + 2k [\delta + 6\delta\Delta (\Delta - 1) + 2k + k\Delta (-12 + 15\Delta + 3\Delta\delta^2 - 5\Delta^2 - 5\Delta^2\delta^2)] \}, \end{split}$$

$$\begin{split} \beta_5 &= -b^5 k^3 \{-4 + 9\Delta + \Delta^2 [-6 - 2k\,\delta(\Delta - 1)(\Delta - 3) + \Delta + k^2(\Delta - 2)(8 - 8\Delta + \Delta^2 + \Delta^2\delta^2)] \} \\ &- b^4 k^2 \phi \{3 + 3\Delta(\Delta - 2) - 4k\,\delta\Delta(\Delta - 1)(2\Delta - 1) + k^2\Delta [24\Delta - 8 - 4\Delta^2(5 + \delta^2) + 5\Delta^3(1 + \delta^2)] \}, \\ \beta_6 &= b^5 k^3(\Delta - 1) \{1 - 2\Delta + \Delta^2 [1 - 2k\,\delta(\Delta - 1) - 4k^2(\Delta - 1) + k^2\Delta^2(1 + \delta^2)] \}. \end{split}$$

In going over to the conventional two-level system (when $k \rightarrow 0$ and $\phi \rightarrow 1$) Eq. (20) gives the well-known cubic equation for n [1-5,8,9].

IV. DISCUSSION OF RESULTS

Equation (20) predicts qualitatively new features of the dependence of the population difference on the external-field intensity. They are due to the nonresonant polarization component and joint exhibition of the NDD interaction of atoms and Stark shift of the absorption line because of the distinction in the linear polarizabilities in the ground and excited states. That is, the more complicated nonlinear behavior of the population difference is provided by the renormalization of the field amplitude and different nature of the absorption line shift versus the external-field intensity because of NDD interaction and quadratic Stark effect.

As a result, our theory predicts less rigid conditions of the IOB appearance and substantial change in its properties, depending on parameters of a dense resonant medium and the external field. In Ref. [10] ensembles of ions O_2^- in KCl crystals and coupled I_2 excitons in CdS monocrystals are proposed as promising systems for the experimental observation of IOB. However, such dense disordered ensembles are similar, in their spectroscopic properties, to gaseous media where the same mechanism is responsible for both the local-field effects and absorption line broadening [11]. As is shown in Ref. [12], the local-field correction is important at the relatively high density N_0 of resonant atoms when the collision broadening is dominant and, respectively, we arrive at the relation

$$T_2 \approx \frac{\hbar}{\eta_C \pi N_0 \mu^2},\tag{21}$$

where, for instance, for transition $j=1 \rightarrow j=0$ the parameter $\eta_C = 2\pi/3$. It is easily seen that in this case the NDD interaction constant *b* does not exceed the value $2/\pi \approx 0.64$. Actually, this is related to the fact that the increase in the density N_0 of resonant atoms is compensated by the corresponding decrease in the lateral relaxation time T_2 . This result is valid in the framework of the binary-collision approximation when $N_0\rho_0^3 \ll 1$ [13], where ρ_0 is the Weisskopf radius. For gases at $\mu^2 \le 10^{-36}$ erg cm³ and the mean velocity of thermal motion $V \approx 10^5$ cm s⁻¹ this condition holds up to $N_0 \approx 10^{20}$ cm⁻³. Thus by virtue of the restriction on the NDD constant for gaslike media, the theory based on the ordinary two-level model fails to substantiate the IOB phenomenon, for the requirement $b \ge 4$ cannot be fulfilled. It is notable that in the case of the generalized two-level model the situation drastically changes [and this is corroborated by



FIG. 1. Population difference of resonant levels as a function of the external-field intensity for different values of the parameter $\chi_0 = \pi N_0 \bar{\kappa}$ ($k = \pi m \chi_0 / 3\Delta$, $\phi = 1 - 2\chi_0 / 3$) at b = 0.64, $\delta = -0.5$, $\Delta = 5$, and m = 0.8: (b) $\chi_0 = 0.41$; (c) $\chi_0 = 0.82$. Curve (a) corresponds to the ordinary two-level model with $\chi_0 = 0$ (k = 0 and $\phi = 1$). With the density of resonant atoms chosen, the maximum value $\chi_0 = 0.82$ corresponds to $\bar{\kappa} = 5 \times 10^{-21}$ cm³.



FIG. 2. Same as Fig. 1 but for different values of the coefficient Δ at b=6, $\delta=-2$, $\phi=0.4$, and k=0.025: (b) $\Delta=4$; (c) $\Delta=4.5$; (d) $\Delta=5$; (e) $\Delta=10$; (f) $\Delta=20$. As with Fig. 1, curve (a) is plotted in accordance with the ordinary two-level model (k=0 and $\phi=1$).

the numerical so-lution of Eq. (20)]; namely, the Stark shift of the absorption line and field amplitude renormalization related to the nonresonant interactions results in the IOB for b < 4.

Figure 1 illustrates the curve n = n(I) numerically obtained from Eq. (20) for different values of coefficients *k* and ϕ . The parameters chosen are typical for dense gaseous media: $N_0 = 5.2 \times 10^{19} \text{ cm}^{-3}$, $\mu^2 = 10^{-38} \text{ erg cm}^3$, $T_2 = 2.9 \times 10^{-10}$ s. The numerical values of the susceptibility tensor components for the κ_j ($\kappa_1 + \kappa_2 \equiv \bar{\kappa}$ and $|\kappa_1 - \kappa_2|/\bar{\kappa} \equiv m$ <1) are quoted from the monograph by Butylkin *et al.* [14]. As is seen in Fig. 1, with allowance for the nonresonant polarization component and quadratic Stark effect, the dependence n(I) undergoes substantial changes. In particular, while increasing $\bar{\kappa}$, the initially monotonic curve n(I) displays the hysteresis (IOB) in the region of comparably small values of the external-field intensity I ($I \leq 2$) at b < 1. Our analysis shows that the reason for the IOB appearance is an additional shift because of the quadratic Stark effect. And the local-field amplitude renormalization due to the nonresonant polarization provides a displacement of the IOB region in the direction of smaller values of I. It is important to emphasize that in the high-intensity region (I > 2) there occurs a monotonic increase in the population difference up to the limiting (unit) value because of the "system drift from resonance" due to the Stark shift of the absorption line.

For the IOB observation to be achievable in an ensemble of two-level atoms at $b \ge 4$ one might use ordered system-

where the NDD interaction is responsible for the emergence of exciton bands. These might be molecular crystals [11], in which case exciton lines must be observed in optical spectra at low temperatures [15]. The very existence of exciton states suggests that the exciton bandwidth $\epsilon \simeq \omega_L$ is greater than the reciprocal of lateral relaxation time T_2^{-1} . Accordingly, the necessary condition $\omega_L T_2 \equiv b \gg 1$ can be realized at low temperatures [11].

In Fig. 2 we show the hysteresis behavior of n(I) in ordered systems for different values of $\Delta = 2T_1/T_2$ at b=6, $\delta = -2$, $\phi = 0.4$, and k = 0.025, which corresponds to the following values of physical parameters: $N_0 = 1.5 \times 10^{20}$ cm⁻³, $\bar{\kappa} = 2 \times 10^{-21}$ cm³, m = 0.5, $\mu^2 = 10^{-38}$ erg cm³, $T_2 = 10^{-9}$ s. As is seen from Fig. 2, accounting for the nonresonant polarization component and quadratic Stark effect makes the dependence n(I) more complicated. In particular, at $\Delta = 4$ the function n(I) is a bistable curve that has two hysteresis loops in different regions of the field intensity. With a relatively small increase in Δ , the region of the second hysteresis loop is appreciably extended in the direction of smaller values of the external-field intensity, which leads then to the intersection of the hysteresis loop regions. For $\Delta > 10$ the dependence n(I) is again transformed into a oneloop curve, the area thereof being much greater than that of a bistable curve corresponding to the ordinary two-level model.

Note that in our calculations the time T_2 was assumed to be fixed. Then, in fact, a change in Δ implies a change in the time T_1 and, as a result, intensity of the resonant transition saturation [see Eq. (18)]. That is why, when the curves in Fig. 2 for different values of Δ are compared, the renormalization of the external-field intensity $I(\Delta)$ should be regarded.

Thus the behavior and properties of a dense ensemble of resonant multilevel systems is a direct consequence of the concerted influence of nonresonant and NDD interactions. This gives rise to qualitatively novel bistable effects: (i) appearance of bistability in the region of small values of the NDD constant (b < 1); (ii) emergence of two hysteresis loops; (iii) intersection of hysteresis loops.

We underscore that in our study of IOB the field intensity is assumed to be a given external parameter. Such a consideration is valid for thin optical samples where the attenuation of the field propagating therein may be ignored. For dense resonant media the absorption length $L_0 \sim \lambda/10$ (λ is the wavelength) [2] and, respectively, the external-field intensity may be supposed as a given parameter for samples of thickness $L \le L_0$. The analysis of propagation effects requires the solution of the self-consistent Maxwell-Bloch system. We have restricted ourselves only to the investigation of medium polarization that determines the medium permittivity. The bistable polarization behavior is sure to result, for example, in the hysteresis reflection from a dense resonant medium. Without solving the very boundary value problem here, one can set forth, nevertheless, some interesting peculiarities to be expected. Undoubtedly the hysteresis in reflection must correlate with the formation of near-surface phase transition layers (domains). In the case of the bistable curve with two hysteresis loops [see Fig. 2(b)] two light-induced interfaces and, respectively, two domains of different-level excitation must be formed. Then there will be a double light-induced sharp discontinuity in the nonlinear permittivity. Of course, this will have an effect on reflectivity. To be more exact, the intensity-dependent light reflection coefficient is expected to have two hysteresis loops too. If b < 1, i.e., a medium is virtually rarefied enough, in solving the problem one may take advantage of the slowly varying amplitude approximation, with applicability limits holding good; that is, the examination of propagation effects can be simplified there. This boundary value problem will be considered elsewhere.

V. CONCLUSION

At present we have no information concerning direct experimental evidence of IOB in dense resonant media. With respect to the recent highly interesting research reported in Refs. [17,18] where the authors presumably attribute the observed bistability to the local-field influence, in our opinion, the question of a veritable bistability mechanism here is, nevertheless, still open and requires an additional analysis. To our knowledge, to date there are only reliable experimental data on the exhibition of local-field effects in a dense atomic potassium vapor where their influence leads merely to a deformation of the profile of the resonant absorption line [19]. The plain fact is that the main difficulty in attaining IOB in dense resonant media described by the conventional two-level model lies in the relatively large required value of the NDD interaction constant b ($b \ge 4$). Our results show that preferable media might be ones described by the generalized two-level model and having a significantly smaller threshold value of b (b < 1).

Therefore the successful search for resonant media with the suitable constant of the Stark shift can be expected to lift the rather severe restriction on the parameter *b*. Our estimations look quite encouraging for finding desirable parameters. The feasible reduction of the constant *b*, which facilitates the emergence of IOB, is likely to offer a real prospect for the hysteresis phenomenon observation in relatively dilute resonant media (with b < 1) and possible applications in optical computing. In particular, the two-loop bistability can be used in producing multiplexer logic elements and different devices of integrated optics [16].

ACKNOWLEDGMENTS

The preliminary results obtained were reported at the 7th International Seminar on Quantum Optics, Minsk, Belarus. The research was partially supported by the Fundamental Research Foundation of the Republic Belarus (Grant No. Φ 97-062).

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