Atomic soliton reservoir: How to increase the critical atom number in negative-scattering-length Bose-Einstein gases

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We study the stabilization of a large negative scattering length Bose-Einstein condensate by using a multi-soliton fringe system. Each fringe behaves like a quasiparticle and the whole cloud forms a stable system when the trapping in one or two directions is switched off. The resulting system can be used as a coherent source of atomic solitons. A discussion on how to generate the soliton fringe system experimentally is made. [S1050-2947(99)10208-7]

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I. INTRODUCTION

The recent development of the so-called atom laser [1] has triggered the study of the multiple exciting possibilities of optics of coherent matter waves. The experimental observation of interference fringes in the overlapping of two Bose-Einstein condensates [2] is a good example of one of the applications that can be expected in this active area of physics. These remarkable results have been possible due to previous important experiments on Bose-Einstein condensation (BEC) in ultracold atomic gases [3,4]. Since then, the investigation of the properties of this new state of matter has become a hot topic. Specifically, an important goal is to obtain condensates with higher number of particles in order to improve experimental results and to get a better understanding of the behavior of these coherent gases. Recently reported experiments [5,6], show a significant progress in this way.

The interaction between the constitutive bosons inside the condensate is defined in terms of the ground state scattering length a. When $a>0$ the interaction between the particles in the condensate is repulsive, whereas for $a<0$, the interaction is attractive. Most BEC experiments use gases with positive scattering length. However, recent experimental results [7], show that it is possible to use Feshbach resonances to continuously detune the value of a from positive to negative values, by means of external magnetic fields. This provides new interest to the analysis of attractively interacting condensates. However, in this case, the practical realization of the condensate is limited by a critical number of particles [4] above which the condensate is unstable and destroyed by the collapse phenomenon [8–12]. In spite of this serious difficulty, negative scattering length condensates have some peculiarities which make them interesting. One of them is that if the trap is removed in one direction, the attractive interaction yields (for a given number of particles) to a self-confined stationary state as it has been shown in a previous work [13]. In this case, the cloud as a whole behaves as a particle and can be controlled by acting on it with external fields. These solutions can be properly called solitons since they appear as a consequence of self-interaction and not merely because of the effect of an external potential.

Our aim in the present work is to show that it is possible to go one step forward and obtain a stable condensate with a larger number of particles (in fact with an arbitrarily high number), as a periodic copy of the mentioned soliton state. The idea is to use the fringe pattern obtained in the interference between two condensates to generate multiple copies of the one-dimensional soliton cloud. Due to the robust nature of solitons, the system of matter-wave fringes will give rise to a stationary state formed by parallel one-dimensional solitons. The critical number of particles present in each soliton fringe can be calculated by means of a perturbative method. The configuration proposed can be used as a reservoir of atomic solitons which could be used as a source for an atomic soliton laser. Additionally this configuration allows the stabilization of systems with a large number of particles as we will see later.

Our detailed plan is as follows. In Sec. II we describe the system and make an analytical description using multiscale expansions and a simplified stability analysis. In Sec. III we support our theoretical predictions with detailed numerical simulations. Finally In Sec. IV we discuss the experimental implications of our results as well as a summary of our conclusions [14].

II. ANALYTICAL RESULTS

A. System configuration and theoretical model

The usual theoretical model used to describe a system of weakly two-body interacting bosons of mass m, with a fixed mean number N, trapped in a parabolic potential $V(\vec{r})$ is a

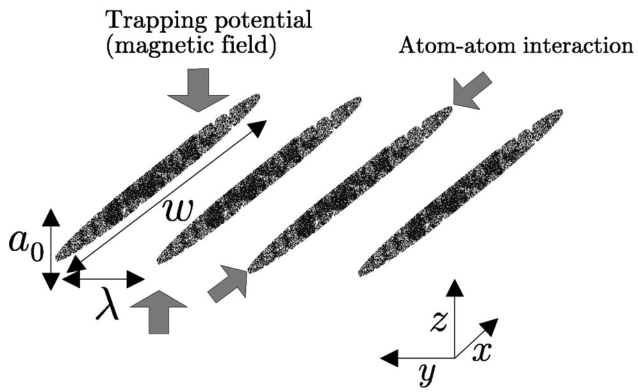

FIG. 1. The system studied. Shown are the effects which dominate the dynamics in each direction (see the text).

nonlinear Schrödinger equation which in this context is called the Gross-Pitaevskii (GP) equation,

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\vec{r})\Psi + U_0|\Psi|^2\Psi, \tag{1}$$

where $U_0 = 4\pi\hbar^2 a/m$ characterizes the two-body interaction, the normalization for Ψ is $N = \int |\Psi|^2 \, d^3\mathbf{r}$, and the trapping parabolic potential is given by

$$V(\vec{r}) = \frac{1}{2}\frac{\hbar^2}{m}\left(\frac{x^2}{a_x^4} + \frac{y^2}{a_y^4} + \frac{z^2}{a_z^4}\right),$$

where a_η, $(\eta = x,y,z)$ are constants describing the characteristic length of the trap. This equation is valid when the particle density $|\Psi|^2$ and temperature of the condensate are small enough and boson-boson interactions can be considered as a Dirac delta function. Recent theoretical work extends the applicability of the GPE to the high density limit [15]. On the other hand linearized stability analysis based on perturbative expansions on $1/\sqrt{N}$ seems to point that the validity of the equation is restricted to the cases where no exponential separation of nearby orbits appear as it happens for example in chaotic pulsations of the atom cloud [16,17].

Let us consider a system as shown in Fig. 1. The condensate is strongly trapped in z; in the y direction a weak or no trapping will be considered while x is a completely free direction. Our aim is to study the stability of a fringe system as the one shown in the figure. We will make the problem more precise in what follows. In the figure we have indicated the effects which are proven to be dominant in each direction provided the parameter ranges are appropriate. This hierarchy of efects will be clear after the subsequent analysis.

The fact that the potential in z provides a tight trapping means that the wave function along the z axis will essentially take the form of the fundamental gaussian mode ϕ_0 of characteristic width a_0 and the solution of Eq. (1) can be factorized as

$$\Psi(\vec{r},t) = \psi(x,y,t)\,\phi_0(z) = \psi(x,y,t)\,e^{-z^2/2a_0^2}, \tag{2}$$

being $a_0 \equiv a_z$, as can be easily seen using multiscale analysis [13]. The intuitive idea is that the trap along z is strong enough that nonlinear boson-boson interactions do not change the linear mode profile in the z direction, however it can be put on more rigorous ground as shown in our previous work.

B. Multiscale analysis

Multiplying Eq. (1) by ϕ_0^* and integrating over z we find the following time-dependent equation for the wave function on the trasverse x-y plane

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m}(\nabla_\perp^2 - a_0^{-2})\psi - \frac{U_0}{\sqrt{2}}|\psi|^2\psi = 0. \tag{3}$$

For the sake of simplicity we will first consider an infinitely extended system. We will study the evolution of initial data of the form

$$\psi_0(x,y,t) = u(x,t)\cos(ky), \tag{4}$$

where u is a spatial profile to be fixed later with a scale of variation much larger than $\lambda = 2\pi/k$. This function represents an initial state which corresponds to a fringe system with interfringe along y given by $\lambda = 2\pi/k$.

Our consideration of an infinitely extended system makes the theoretical analysis to be performed later simpler but the conclusions obtained are essentially valid when a weak trapping potential is included in y, the only difference being that instead of considering a cosine function (which is an eigenfunction of the free linear problem) we should use a Hermite function (which is the eigenfunction of the parabolic potential) as we will show later in Sec. III when presenting the numerical results. In practice our use of a cosine function means that we will consider a large number n of fringes but as will be shown later the results are valid when the number of fringes is quite small.

We will look for solutions $\psi(x,y,t)$ depending on a small parameter ϵ which will be related to the quotient of the two characteristic scales present in the initial data in the form

$$\psi = \epsilon\psi_1 + \epsilon^2\psi_2 + \ \ldots\ . \tag{5}$$

The physical idea behind the previous expansion is that the adimensional quantity $1/\epsilon$ plays the role of a scaling factor between the several spatio-temporal scales involved in the evolution of the cloud. If the scaling factor of the problem is high enough, the different scales involved can be considered as independent and thus, it is possible to define a set of variables $x_j = \epsilon^j x, y_j = \epsilon^j y, t_j = \epsilon^j t$ for the jth order of perturbation and make the expansion:

$$x \rightarrow x_0 + x_1 + x_2 + \ \ldots, \tag{6a}$$

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x_0} + \epsilon\frac{\partial}{\partial x_1} + \epsilon^2\frac{\partial}{\partial x_2} + \ \ldots, \tag{6b}$$

being x_0 the usual spatial dimension and the same holds for y and t. This perturbative method is usually called ''multiple scale analysis'' or ''two timing'' [18] and is widely used in nonlinear science [19].

For the sake of conciseness we will use the notation L_j to denote the expansion of the linear operator in equation (3) up to the jth perturbative order, then we can write the perturbative expansion of the linear operator up to the first order as:

$$L_0 \equiv i\hbar \frac{\partial}{\partial t_0} + \frac{\hbar^2}{2m}\left[\left(\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2}\right) - a_0^{-2}\right], \tag{7a}$$

$$L_1 \equiv i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{m}\left[\left(\frac{\partial^2}{\partial x_1 \partial x_0} + \frac{\partial^2}{\partial y_1 \partial y_0}\right)\right], \tag{7b}$$

$$L_2 \equiv i\hbar \frac{\partial}{\partial t_2} + \frac{\hbar^2}{2m}\left[2\left(\frac{\partial^2}{\partial x_2 \partial x_0} + \frac{\partial^2}{\partial y_2 \partial y_0}\right) + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right]. \tag{7c}$$

The calculations start by writing the wave function at $t=0$ as a function of the expanded coordinates x_i up to first order:

$$\psi(t=0) = u(x_1, x_2, \ldots,)\cos(ky_0), \tag{8}$$

Since we look for solutions with slowly varying x profile it is assumed that u depends only on the ''slow'' variables. Using Eq. (3) we find

$$L_0\psi_1 \equiv \left(i\hbar \frac{\partial}{\partial t_0} + \frac{\hbar^2}{2m}\nabla_0^2 - \frac{\hbar^2}{2ma_0^2}\right)\psi_1 = 0. \tag{9}$$

This is the equation for a wave propagating through a homogeneous linear medium. The solution compatible with the initial condition (8) has the form

$$\psi_1 = u_1(x_1, t_1, x_2, t_2, \ldots,)\cos(ky_0)e^{-i\omega t_0}, \tag{10}$$

Where u_1 represents a slowly varying envelope and $\omega = (k^2 + a_0^{-2})\hbar/2m$. The following step in the perturbative expansion yields to the following equation:

$$L_0\psi_2 + L_1\psi_1 = 0. \tag{11}$$

Substituting back (10) into Eq. (3) up to second order on the perturbative expansion, we obtain:

$$L_0\psi_2 = -L_1\psi_1 = -i\hbar \frac{\partial u_1}{\partial t_1}\cos(ky_0)e^{-i\omega t_0}. \tag{12}$$

The linear operator in the left-hand-side of Eqs. (9) and (12) is L_0. Thus, the right hand side term in Eq. (12) is singular because it resonates with a solution of the homogeneous equation (9). This kind of term is usually called *secular*. To prevent the appearance of nonphysical solutions we may either take u_1 to be zero or, more generally, demand that: $\partial u_1/\partial t_1 = 0$. This choice can be formalized in the framework of the Fredholm alternative theorem. Thus, $L_0\psi_2 = 0$, which yields to the following solution up to second order:

$$\psi_2 = u_2(x_1, t_1, x_2, t_2, \ldots,)\cos(ky_0)e^{-i\omega t_0}. \tag{13}$$

Up to this point, the physical meaning of the perturbative expansion is evident; it has been found the best linear approach to the problem. Following the perturbative treatment up to order three, nonlinear terms will appear, i.e.,

$$L_0\psi_3 + L_1\psi_2 + \left(L_2 - \frac{U_0}{\sqrt{2}}|\psi_1|^2\right)\psi_1 = 0, \tag{14}$$

and then

$$L_0\psi_3 = \left[-i\hbar\left(\frac{\partial u_2}{\partial t_1} + \frac{\partial u_1}{\partial t_2}\right) - \frac{\hbar^2}{2m}\frac{\partial^2 u_1}{\partial x_1^2} + \frac{3}{4\sqrt{2}}U_0|u_1|^2 u_1\right]\cos(ky_0)e^{-i\omega t_0} + \frac{U_0}{4\sqrt{2}}|u_1|^2 u_1 \cos(3ky_0)e^{-i\omega t_0}. \tag{15}$$

Here we have again a secular term which resonates at the spatial frequency k and another term oscillating at $3k$ which will be analyzed later. Since u_1 does not depend on t_1, we find that u_2 is as much, linearly dependent on t_1. However, to avoid unbounded growth of these solutions with t_1, u_2 should not depend on t_1. Under this condition and setting the resonant term to zero, the following equation is obtained:

$$i\hbar\frac{\partial u_1}{\partial t_2} + \frac{\hbar^2}{2m}\frac{\partial^2 u_1}{\partial x_1^2} - \frac{3}{4\sqrt{2}}U_0|u_1|^2 u_1 = 0, \tag{16}$$

which is the well known one dimensional nonlinear Schrödinger equation (NLSE) [20]. For the fundamental soliton state, it is straightforward that Eq. (16) admits a stationary solution of the form $u_1 = B\text{sech}(qx_1)e^{iWt_2}$,. Taking into account that $U_0 = 4\pi\hbar^2 a/m$ and $a<0$, the relationship between B and q is found to be $B^2/q^2 = -\sqrt{2}/(3\pi a)$. The phase parameter $W = 3\pi\hbar a B^2/4m$ is amplitude dependent, showing the typical self-phase modulation of these kind of solutions [21]. Defining $qx_1 = q\epsilon x_0 = x_0/w, \Omega = \epsilon^2 W$ and $A = \epsilon B$ the solution in terms of the x,y,z,t usual variables is

$$\Psi(\vec{r},t) = A\,\text{sech}\left(\frac{x}{w}\right)\cos(ky)e^{-z^2/2a_0^2}e^{i(\Omega-\omega)t}, \tag{17}$$

where $A^2w^2 = -\sqrt{2}/(3\pi a)$. The number of particles N_f per fringe (which corresponds to $\Delta y = \lambda/2$) can be obtained by integrating (17) on a fringe to obtain

$$N_f = \frac{1}{3\sqrt{2\pi}}\left(\frac{a_0\lambda}{|a|w}\right). \tag{18}$$

The important fact from the above discussion is that we have obtained the soliton profile for *one* fringe. As the global wave function is a periodic train of parallel solitons along the y axis, the total number N of particles in the condensate will be distributed over the soliton fringes and thus, the collapse threshold could be higher, depending on the total number of fringes of the interference pattern. The intuitive idea is that the enhanced dispersion provided by the oscillatory profile compensates the collapsing tendency along y. The validity of this idea has been experimentally tested in the frame of nonlinear optics, where interference techniques were used to generate spatial optical solitons [22].

C. Stability against collapse

Some questions about the stability of the approximate solutions remain to be explained. The first one concerns the global nature of the solution, i.e., whether blow up takes place in finite time. It is not easy to answer this question on theoretical grounds since we use a infinitely extended initial data which provides an infinite value for the energy. We can however compute the energy per fringe. Since a sufficient condition for collapse is that the energy is negative [9,8,24] it will give us a way to ensure collapse in certain situations:

$$E=\int\left[\frac{\hbar}{2m}|\nabla\Psi|^2+\frac{U_0}{2}|\Psi|^4+\frac{1}{2}V(\vec{r})|\Psi|^2\right]d^3\vec{r}$$

$$=\frac{\hbar^2}{m}\frac{(2\pi)^{3/2}a_0}{6w|a|\lambda}>0. \qquad (19)$$

This result means that the proposed configuration is a good candidate to avoid collapse. However, it is important to keep in mind that this is only a necessary condition to avoid collapse. So data satisfying this condition could in principle collapse. Concerning sharp estimates some works have been published for the free NLSE [23] and recent work addresed the trap case [24], however there are no better results and collapse has to be analyzed in practice numerically, a study we defer to a later section. In fact, as we will see later, data of the type (17) may collapse by the combination of a destabilization mechanism which first breaks the fringe system and then concentrates many particles in small regions of the space to get local negative energy densities.

D. Stability against transverse perturbations

The fact that the fringe system could be stable agaist collapse does not imply its dynamical stability. We will not perform here a detailed stability analysis, which would be quite complicated for our present purpouses but only a simple discussion to have a qualitative picture of the possible instabilites that could arise on top of our proposed configuration. To simplify the calculations, let us consider a region of the condensate far from the limits of the trap where the cloud can be taken as a planar wave of amplitude Ψ_0. Let us consider the evolution of the solution when a small spatially periodic perturbation with a wavelength λ_p is added to the condensate

$$\Psi=(\Psi_0+\varepsilon)e^{i\gamma\tau}, \qquad (20)$$

where $\varepsilon=\varepsilon_0 e^{i(y/2\pi\lambda_p+\Gamma t)}$. Substituting back in (1), linearizing on ε, we arrive to the following condition for Γ

$$\Gamma^2=\frac{k_p^2}{2m}\left(\frac{\hbar^2k_p^2}{2m}+2U_0|\Psi_0|^2\right). \qquad (21)$$

Thus taking into account that $U_0=4\pi\hbar^2/ma$, $a<0$ we find a critical value $\lambda_{cr}=\sqrt{-\pi/4a}/|\Psi_0|$ of the perturbation wavelength, such that the perturbation keeps oscillating with t for $\lambda_p<\lambda_{cr}$, as $\Gamma^2>0$. For $\lambda_p>\lambda_{cr}$, the amplitude of the perturbation grows exponentially as $\Gamma^2<0$. The maximum rate of growth takes place for a characteristic width $\lambda_{max}=\sqrt{2}\lambda_{cr}$. This behavior is called modulational instability and is well known in other areas of physics [25,27]. Experimental observations of this instability have been also recently reported [26] using a slightly different nonlinearity than the proposed in the present analysis.

Turning back to the solution derived in Eq. (17), we can take the amplitude of the previous plane wave to be that of the soliton state, i.e., $\Psi_0=A$. This yields to $\lambda_{cr}=\sqrt{3/8}\pi w$, close to the value of the soliton width calculated in (17). It may seem conterintuitive that the soliton solution will not be affected (up to the first orders of perturbation) by the characteristic lenght of the modulation. However, it is evident that for larger values of the perturbation wavelength (always below the critical value for blow up) more number of atoms will correspond to each fringe as is given by Eq. (18). In fact, the effect of modulational instability along the y axis is to achieve an effective one-dimensional structure for each fringe along the x axis. Thus, we can conclude that the fringe pattern resulting from the interference of two overlapping condensates will be stable against periodic perturbations if the interfringe is below the modulational instability critical length λ_{cr} and the spatial scales x-y in the plane transverse to the trappinng axis are much larger than the confinement size (i.e., $\omega, a_y \gg a_0$).

III. CONFINED SYSTEMS

The main result of our preceeding analysis is that the soliton-fringe system can be stabilized by choosing an appropriate wave function at least for some time (which can be longer than the condensate timelife) before the instabilities appear. However the analysis was done in the infinitely extended case which is neither realistic from the experimental point of view nor computationaly tractable. Let us then consider the more realistic system where a trapping potential along y is added. Though it is possible to apply again the multiscale technique, the computation is quite complicated and physical insight is lost. To study the changes with respect to the previous situation we will make numerical simulations of Eq. (3) taking as initial data the product of an hyperbolic secant along x by a Hermite function (the eigenfunction of the parabolic potential) in the y direction. We have analyzed the adimensional form of the equation, which is obtained defining the new variables: $x=a_0X, y=a_0Y, z=a_0Z, \tau=\nu t$ the constants, $Q_0=4\pi aN/a_0, w=a_0w_0, \lambda=a_0\lambda_0$ and renormalizing ψ by defining $\psi=\Psi\sqrt{a_0^3/N}$. Once the Z variable is eliminated using the first part of the multiscale expansion one finds $\psi(X,Y,Z)\simeq\chi(Z)\varphi(X,Y)$. Then eliminating a phase factor the 2D reduced equation for the wave function in X,Y obeys the following equation:

$$-\tfrac{1}{2}\nabla^2_{X,Y}\varphi+\tfrac{1}{2}\lambda_0^2Y^2\varphi+Q_0|\varphi|^2\varphi=i\frac{\partial\varphi}{\partial\tau}, \qquad (22)$$

and the solution is normalized to 1 ($\int|\varphi|^2dXdYdZ=1$).

The idea is the same as before; the interfringe system is provided by the Hermite function instead of the cosine function and the trapping in x is provided by the nonlinearity. If the number of particles per fringe is taken near the solitonic value (18) we obtain highly stable solutions until the particle number is increased above a collapsing threshold. To check whether the idea is extensible to the trapped system we have

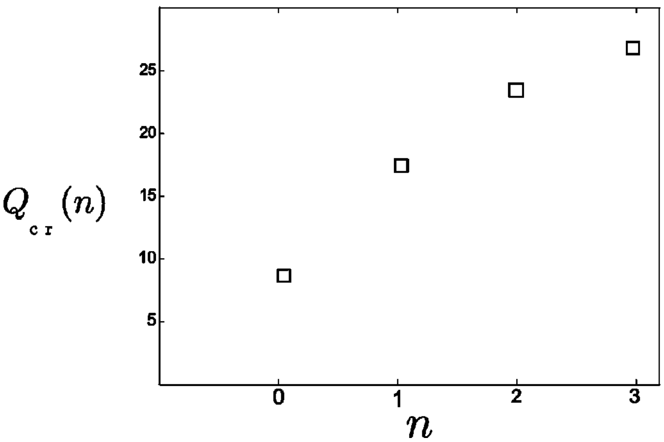

FIG. 2. Maximum number of particles as a function of the number of fringes (the order of the trasverse Hermite mode is used as the initial condition).

run numerical simulations of the evolution of Eq. (22) using as initial data the normalized wave function:

$$\Psi(\tau=0) = \left(\frac{\alpha}{w_0 2^{n+1}\pi n!}\right)^{1/2} \mathrm{sech}\left(\frac{X}{w_0}\right) e^{-\frac{Z^2}{2}} H_n(\alpha Y) e^{-\frac{\alpha^2 Y^2}{2}}, \tag{23}$$

where α measures the strength of the potential in y direction as compared to z ($\alpha=\sqrt{\lambda_0}$) and w_0 is not free if one wants solitonic solutions along X but given by the expression

$$w_0 = \frac{4\pi\sqrt{2}n!2^{n+1}}{3\alpha Q}.$$

Numerical simulations of Eq. (22) have been done using a symmetrized second order in time Fourier pseudospectral method on a grid with up to 1024×1024 points. Typical simulation times of the model where of the order of 100 (integration step $\Delta t=0.05$) which correspond to physical values of the orders of seconds, which are about the lifetimes of the condensates.

In Fig. 2 we show the computed collapse threshold for different number of fringes (order of the Hermite function considered). To compute these threshold values one has to keep in mind that collapse can stop at a scale which is smaller than the grid size and then near the collapse point a grid as large as possible must be used to improve the quality of the approximation. In practice to compute the threshold we start using a rough numerical grid and then increase slowly the value of Q_0 until collapse is observed. Then we refine the grid and continue increasing the norm until collapse is observed. We repeat this process until a saturation is observed for the larger grids used of 1024×1024 points.

Of course since our analytical prediction was made for the free space solutions there are deviations when trying to extend the idea to the confined system. In particular as the number of fringes increases it becomes more difficult to increase the critical number as can be seen in the saturation present for high particle numbers. The reason is that the Hermite mode has its absolute maxima localized in the last period of the oscillation and increases its amplitude with the mode number so that although the mean amplitude is clearly lower for the higher order Hermite modes the peak amplitude does not decay (Fig. 3).

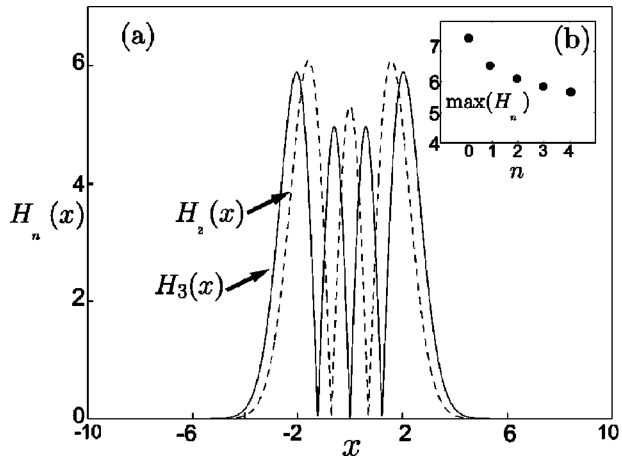

FIG. 3. (a) Two normalized Hermite modes showing the saturation of the peak value for increasing n. (b) The peak amplitude as a function of n.

We have analyzed Hermite modes with $m=0,1,2,3$. Simulation of higher order y modes corresponds to very extended systems in x since the higher the mode the lower the peak density value and thus the solitonic width is larger. This fact leads to very anisotropic systems which present computational difficulties. The optimum computational approach to this problem would be to use a robust multigrid method and work is ongoing [28].

The change of λ_0 is not relevant for collapsing properties of the wave packet as we have tested in our numerical simulations. In fact, we have observed that the collapsing singularity in these cases happens through compression of the wave function along x.

The dynamical stability has been observed by letting the initial data evolve for large times. Typical evolution of the spatial profiles after long times and of the normalized peak density for a condensate with three fringes (hermite mode H_2) are shown in Fig. 3 where it is seen that the condensate fringe system is very stable and only small pulsations around the stationary solution are observed in the dynamics.

IV. DISCUSSION AND CONCLUSIONS

Our proposed system is able to confine in a stable way a negative scattering length condensate. A related question concerns how to generate such state. Although it is not our intention to go through many experimental details we would like at least to point out some ideas.

A simple way to generate the fringe system could be to start with a positive scattering length condensate which is splitted and then joined again as it is done in the interference experiments [2,29]. This could be done by removing the trap along the x-y plane and driving the condensates along the y axis to make them collide, whereas the trap along z remains unaltered. Using Feschbach's resonances combined with an external field, it could be posible to switch the sign of the scattering length and then the soliton fringe system would be obtained provided the parameters of the system (trapping along z, interfringe) are tuned to select approximately the solitonic particle number per fringe. However there is not a great sensitivity to this parameter as we have confirmed in our numerical simulations.

When the particle number exceeds the soliton value, higher order states are excited or collapse is obtained. Concerning the dynamical stability for typical experimental values such as the ones used in [2] with $\lambda \approx 15\ \mu\mathrm{m} \ll \sigma \approx 1$ mm, σ being the soliton width the stability condition is satisfied.

As the maximum growth rate is reached for $\lambda_{max} = \pi\sqrt{3/4}\sigma$, we can derive another interesting question from the previous stability analysis, concerning cigar-shaped condensates. For this geometry, it has been experimentally shown [30] that optical dipole forces can be used to excite wave packets in a Bose-Einstein condensate. Thus, if this important result is combined with the above stability analysis, it can be derived that a periodically perturbed condensate will evolve into a train of soliton pulses of matter and thus, induced modulational instability could yield to stable trains of solitonic Bose-Einstein condensates. This could be another way to obtain coherent condensate states with high number of particles in the negative scattering case.

In conclusion, we have derived in the present work an analytical description of the interference pattern of two overlapping Bose-Einstein condensates with negative scattering length. The solutions obtained show that the interference pattern can be considered as a set of multiple copies of a single soliton state with fixed number of particles. Thus it possible to obtain, in the frame of current experiments, noncollapsing Bose-Einstein condensates with negative scattering length with higher number of particles than the critical particle number, provided the value of the interfringe λ is below a critical value. This could be a way to avoid the current bound imposed by the critical number in negative scattering length condensates.

We hope that this study will stimulate the investigation of the behavior of BEC's with negative scattering length and think that the soliton solutions here studied will be of practical applicability for experimentalists dealing with Bose-Einstein condensate engineering.

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