## COMMENTS AND ADDENDA

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## Study of the Frequency-Locking Region of a Monomode Anisotropic Zeeman Laser

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The experimental study of polarization in a Zeeman laser with strong  $x-y$ -type anisotropies makes evident the rotations of the electric field vector greater than the limiting value of  $\frac{1}{4}\pi$ which is fixed by the self-consistent theory. The results are in agreement with the vectorial theory based on the resonance condition, which is necessary for the representation of such a type of quantum oscillator with nonreciprocal effects (including anisotropies). The self-consistent theory appears to provide a limit which becomes valid only for very weak anisotropies. Only the polarization of the so-called "Lamb's vector" can be described by the self-consistent theory and its rotation effectively attains  $\frac{1}{4}\pi$  whatever the value of the anisotropy.

In two recent  $\ar{ticles,}^{1,2}$  new polarization effects have been theoretically predicted by the resonance condition method, which gives results different from those given by the self-consistent theory in the description of lasers with  $x-y$ -type loss anisotropies. The experimental study of the polarization of a monomode laser subject to an axial magnetic field has been carried out on the lasers without Brewster windows. 3-6 The anisotropies of a laser with mirrors directly sealed to the active tube are essentially due to the mirrors. They are always very weak and we will call such lasers quasi-isotropic lasers. For a weak-coupled transition'  $(J=1-J=2)$ , the polarization of such a laser, being linear in zero field, rotates approximately by $3-5$  45°, with respect to the initial position when a weak axial field is applied to the laser. The limiting rotation is attained for a value  $H<sub>c</sub>$  of the magnetic field (of the order of <sup>1</sup> 6) called the critical field. Beyond this value the linear vibration decomposes into two (right and left) elliptical vibrations of slightly different frequencies.<sup> $7-10$ </sup> The region from 0 to  $H_c$  is called the locking region and is attributed to the presence of the anistropies of the cavity. The experimental results were interpreted in the framework of the self-consistent the- $\text{ory}^{7-11}$  where the losses are supposed to be distributed. This theory fixes the maximum rotation at exactly  $\frac{1}{4} \pi$ .

We have shown experimentally<sup>12</sup> that at a given point in an anisotropic Zeeman laser there exist two distinct electric field vectors, each corresponding to a direction of propagation of the wave in the cavity. This shows the necessity for describing such a laser by a vectorial model which

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FIG. 1. General scheme of the apparatus with the corresponding diagrams for the electric fields.

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FIG. 2. Maximum rotations of the different vectors vs anisotropy. Solid lines are the theoretical values given by the resonance condition method. Dashed line is the limit given by the self-consistent-field theory. Experimental points are plotted with error bars. The circles represent the maximum rotation of the "Lamb' s vector."

is in agreement with the theory based on the resonance condition. Particularly, this theory predicts the maximum rotation that can be attained for the critical field  $H_c$  for one of the vectors is not  $\frac{1}{4} \pi$ but may go from  $0^\circ$  to  $90^\circ$  depending on the resonator. On the other hand, it is also predicted<sup>2</sup> that there exists at a point of a laser a vector whose polarization obeys the self-consistent theory in 'that it attains a maximum rotation of  $\frac{1}{4}\pi$ . The object of the present addendum is to verify experimentally these predictions for the anisotropic lasers.

We have carried out our observations with an apparatus similar to the one described in a previous article.<sup>12</sup> This apparatus corresponds to case (a) of Ref. 2, whose notations are being used here  $(\epsilon_2$ =0), The general scheme is represented in Fig. 1. The choice of the 3.39- $\mu$  line of Ne<sup>20</sup> is justified by the important gain of this line which allows one to insert important known anisotropies, which is not the case in quasi-isotropic lasers. We use here a

thin plate of silicon (refractive index  $n=3.4$ ) which will permit us to obtain important differences between the predictions given by the self-consistent theory and those based upon the resonance condition. A thin plate of silica, inserted in the cavity at 6' (anisotropy negligible as compared to that of the silicon plate), enables us to analyze separately the polarization of the incident wave on the anisotropy and of the reflected wave constituting the mode (beams 1 and 1'). Moreover, the polarization of beam 2 representing the Lamb's vector of this laser can also be measured, The position of the mode in the cavity is controlled by a piezoelectric ceramic carrying one of the mirrors (the laser is centrally tuned). The rigidity of the whole apparatus limits the drift of the frequency during the time of measurement (about 2 MHz for 30 sec).

The experimental results are shown in Fig. 2. For various inclinations of the silicon plate with respect to the laser axis we have determined the maximum rotations  $\theta_{1m}$  and  $\theta'_{1m}$  of incident and reflected beams for the critical field  $H_c$ . We find that the experimental values obtained are in agreement with the theoretical results and diverge considerably in this example of cavity from the limiting value of  $45^\circ$  fixed by the self-consistent theory for all values of the anisotropy. It should be noted that the average geometric vector resulting from the composition of the vectors 1 and 1 'rotates effectively by  $\frac{1}{4}$   $\pi$  (it can be shown that its polari zation is represented by beam 3). We have also measured the maximum rotation of the Lamb's vector which is characterized by beam 2. We have plotted its maximum rotation in Fig. 2. Effectively this maximum rotation is  $\frac{1}{4}\pi$  and is the same for any value of the anisotropy. It is found that as the anisotropy 'diminishes, the divergences between the results given by the two theories diminish. The experimental results confirm that the self-consistent theory becomes suitable for very weak anisotropies. The experimental results thus obtained show the limit of validity of the self-consistent theory which appears to be a satisfactory approximation only for very weak anisotropies. This theory has the advantage of avoiding the boundary-value problem but it is not applicable in this case.

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#### PHYSICAL REVIEW A VOLUME 6, NUMBER 2 AUGUST 1972

# Rotation of the Plane of Polarization of a Pulse Undergoing Self-Induced Transparency

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The rotation of the plane of polarization during the propagation of a pulse undergoing selfinduced transparency is reinvestigated. The rotation per unit length, which is constant along the pulse, is shown to be exactly equal to the linear result obtained far out on the trailing edge of the hyperbolic-secant pulse. If the two circular components have their center frequencies shifted with respect to each other, it is pointed out that the rotation will not be stationary in the laboratory frame, but will propagate with a characteristic velocity. The existence of such propagation of a plane of polarization could be useful to test experimentally the existence of frequency pulling.

Faraday rotation during the propagation of an electromagnetic pulse undergoing self-induced transparency has been previously investigated. ' It was stated that one should expect a "giant" Faraday rotation accompanied by no energy loss for a system undergoing self-induced transparency.<sup>2</sup> The point of the present paper is to make three comments on the "giant" rotation. First, whereas the rotation does occur with no absorption or energy loss, a characteristic of any phenomenon associated with self-induced transparency, the rotation is indeed exactly equal to the classical (linear) value obtained far out on the tail of the hyperbolic-secant pulse. Second, the rotation decreases as the pulse is narrowed. Third, the rotation will not be stationary in the laboratory frame, but will propagate in the same direction as the pulse if the two circular components are frequency shifted with respect to each other.

The model that will be treated is shown in Fig. 1. It consists of ground and excited Kramers doublets, with degeneracies removed by a magnetic field. The right  $(+)$  and left  $(-)$  components of circular polarization couple to the levels as indicated. These components can be treated independently of each other if relaxation between the levels is neglected.

The rotation we consider is essentially equal to the difference between the shift in wave vector,  $\Delta k_+ - \Delta k_-,$  of the two circular components of the plane polarized wave after steady state has been achieved. McCall and Hahn<sup>2</sup> have pointed out that for asymmetric inhomogeneous linewidth broadening, the change in wave vector reduces to the ordinary result of linear-dispersion theory if the

pulse width is much greater than the inhomogeneous linewidth. Courtens and  $Szöke<sup>3</sup>$  have stated that the result of linear theory is obtained if the center frequency of the pulse is significantly detuned from the resonant absorption frequency.

It is relatively straightforward to demonstrate that the self-induced-transparency (SIT) expression for the change in wave vector is exactly equal to a particular result obtained from linear-dispersion theory for any pulse width and any detuning. First, the rotation per unit length, or the change in wave vector as calculated from the SIT equations, is independent of position. This results simply from the proportionality between the in phase component of polarization  $\mu(\Delta\omega, z, t)$  and the field amplitude  $\mathcal{S}(z, t)$ . Since the equation for the change in wave vector is given by  $v^2$ 

$$
\frac{\partial \phi(z)}{\partial z} = \Delta k = \mathcal{E}^{-1} \frac{2\pi \omega}{nc} \int_{-\infty}^{\infty} \mu(\Delta \omega, z, t) g(\Delta \omega) d(\Delta \omega), \tag{1}
$$

and since all spatial and temporal parts cancel on the right-hand side of Eq. (I), the change in wave vector or rotation is therefore a constant and can be



FIG. 1. Zeeman-split energy-level system.