with the $K\alpha$ transition should be observable in the Auger spectrum.

The various atomic rearrangement processes which are probably occurring in the excited target ions as discussed above are evidently influenced by strong electron correlation effects. This is suggested by the observation of similar $K\alpha$ spectra for Mg plus 21-MeV oxygen (present work), Si plus 30-MeV oxygen,[9] Al plus 5-MeV nitrogen,[7] and Al plus 18-MeV oxygen.[14] That is, the relative intensities of the peaks appear to be nearly independent of bombarding energy above 5 MeV.

†Work supported in part by the Robert A. Welch Foundation and the U.S. AEC.

*Present address: University of Texas Medical School, Dallas, Tex.

[1]P. Richard, I. L. Morgan, T. Furuta, and D. Burch, Phys. Rev. Letters 23, 1009 (1969).

[2]D. Burch and P. Richard, Phys. Rev. Letters 25, 983 (1970).

[3]P. H. Mokler, Phys. Rev. Letters 26, 811 (1971); R. C. Der *et al.* (unpublished).

[4]G. A. Bissinger and S. M. Shafroth, Bull. Am. Phys. Soc. 16, 125 (1971).

[5]H. J. Specht, Z. Physik 185, 30 (1965).

[6]M. E. Cunningham *et al.*, Phys. Rev. Letters 24, 931 (1970).

[7]A. R. Knudson, D. J. Nagel, P. G. Burkhalter, and K. T. Dunning, Phys. Rev. Letters 36, 1149 (1971).

[8]D. Burch, P. Richard, and R. L. Blake, Phys. Rev. Letters 26, 1355 (1971).

[9]D. G. McCrary and P. Richard, Phys. Rev. A (to be published).

[10]R. L. Blake, Ph.D. dissertation (University of Colorado, 1968) (unpublished).

[11]Siemens, Catalog No. C70144-A952-B40.

[12]J. A. Bearden, Rev. Mod. Phys. 39, 78 (1967); J. A. Bearden and A. F. Burr, *ibid.* 39, 125 (1967).

[13]V. Kunzl, Z. Physik 99, 481 (1936).

[14]M. Senglaub *et al.* (unpublished).

PHYSICAL REVIEW A VOLUME 6, NUMBER 1 JULY 1972

Coulomb T Matrix and the Proton-Hydrogen Charge Exchange

C. S. Shastry, A. K. Rajagopal, and J. Callaway
Department of Physics and Astronomy, Louisiana State University
Baton Rouge, Louisiana 70803

(Received 3 September 1971; revised manuscript received 6 March 1972)

The proton-hydrogen charge-exchange amplitude is evaluated using the first-order terms in the Faddeev expansion for the corresponding transition operator. The new Coulomb T matrix derived by us recently is employed in computing the cross sections. For high energies of the incident proton, the Coulomb T matrix approaches the Coulomb potential to order $v^{-1}\ln v$ in contrast to the situation for a short-range potential ($\sim v^{-2}$). In the extreme high-energy limit the results of the first-order Faddeev-Watson approximation approach those of the Jackson-Schiff approximation. Explicit numerical calculation in the energy region 100 keV to 3 MeV shows that the cross sections lie in between those of the Brinkman-Kramers and Jackson-Schiff (JS) results and they approach the JS results from above. This is in contradistinction to our previous work based on an incorrect form of the T matrix.

I. INTRODUCTION

Since the pioneering work of Faddeev[1] on three-particle scattering, there has been a revival of interest in the classical problems of the Coulomb T matrix[2-8] and the behavior of the proton-hydrogen charge-exchange amplitude.[9-15] Lively controversies exist in each case. This work concerns both problems and should reduce some of the confusion.

We begin with a brief survey of the relevant history. A more complete review of work prior to 1968 has been given by Bransden.[16] The high-energy limit of the proton-hydrogen charge exchange was first calculated by Brinkman and Kramers[17] (BK) who evaluated the transition amplitude in the first Born approximation but neglected the interaction between the protons. The neglect of this term is made plausible by a physical argument based on the impact parameter approximation.

Jackson and Schiff[18] (JS), who were aware of the impact-parameter argument, found that the inclusion of the proton-proton interaction reduced the cross section in the high-energy limit by a factor of 0.66 compared to the BK result.

Subsequently, second-order terms in the Born series were calculated by Drisko[19] and third-order terms were estimated. The result is

$$\sigma = \sigma_{BK}(0.319 + 5\pi v/2^{12}) ,$$

where σ_{BK} is the Brinkman-Kramers cross section and v is the speed of the incident proton in a.u. in the laboratory system. This calculation indicated that no matter how high the energy, the Born series does not converge to its first term. Other calcula-

tions have appeared which tend to justify and confirm Drisko's result.[20,21]

However, Born-type approaches suffer from a fundamental difficulty due to the divergence of the Born series for the Green function in a three-body problem.[22,23] Therefore the results of these calculations should not be considered conclusive.

Faddeev's work[1] on three-particle scattering seems to furnish a more satisfactory basis for calculations. This provides an expansion for the transition operators in terms of two-body T operators (except for a bare-potential term in the case of transition operators for rearrangement collision) corresponding to the two-particle pairs in the three-body system.

In order to utilize the Faddeev method in an atomic problem, one must construct an exact T matrix for the Coulomb potential. The four-dimensional hidden symmetry of the Coulomb Hamiltonian, which is different for positive and negative energies,[7,8] can be exploited to derive the required expressions for the Coulomb T matrix. These aspects have recently been examined by Rajagopal and Shastry,[8] and the expression for the Coulomb T matrix valid for various regions of energy and momenta were derived following the work of Perelomov and Popov[7] on the Coulomb Green's function.[8] Differences in the analytical forms of the Coulomb T matrix for $E<0$ and $E>0$ were also noted by Roberts and co-workers[24] using classical-path-integral techniques. In this paper we use the T matrix of Ref. 8 to calculate the proton-hydrogen charge-exchange cross sections.

Chen and co-workers[12-15] have also investigated the three-particle Coulomb systems using Nutt's representation. They analyze the contributions from the antibound states, asymptotic bound states, and the branch point singularities. Carpenter and Tuan[10] adopted a different approach to study the proton-hydrogen rearrangement collision. They started with the coupled equations of Faddeev and Lovelace[1,25] for transition operators. These were reduced to an ordinary integral equation by neglecting the proton-proton interaction in accord with the impact-parameter arguments mentioned earlier. Such neglect may not be justified. In this paper we find that the Born-approximation result of JS[15] is indeed the high-energy limit of the first-order approximation to the Faddeev-Watson series (FOFW).

There has been considerable controversy concerning the second-order contributions. It should be pointed out that the complete evaluation of the second-order FW series for this problem has not yet been accomplished. Drisko[19] showed that in the high-energy limit the contribution from the proton-proton potential in first order is cancelled by two of the second-order terms involving the proton-electron and proton-proton potentials; also a different (v^{-11}) velocity dependence was obtained. Chen *et al.*[13] assert that in the corresponding second-order FW series the terms $T_i G_0 T_j$ cancel their Born counterparts $V_i G_0 V_j$ exactly in the high-energy limit. The complete expression for the amplitude to second order is given below in Eq. (3.3). To the best of our knowledge, this has not yet been analyzed as rigorously as the first-order terms discussed in the present paper. In our opinion, the proof that the high-energy limit of the FOFW term is the JS result is interesting in itself. Furthermore, we have presented here an exact numerical evaluation of the FOFW contribution, which must be incorporated in any future, more complete analysis of the FW series.

The plan of the paper is as follows: In order to make the paper self-contained, in Sec. II we give expressions for the Coulomb T matrix for $E<0$ and $E>0$ which is the basis of the present calculation. In Sec. III, we express the charge-exchange amplitude in terms of certain integrals. In Sec. IV it is established that the high-energy limit of the FOFW approximation is the result obtained previously by JS. This is accomplished by proving that the Coulomb T matrix approaches the Coulomb potential at high energies to order $v^{-1}\ln v$. Explicit numerical results for FOFW cross sections for proton-hydrogen charge exchange are presented in Sec. V for an energy range from 100 keV to 3 MeV. For large but finite energies the cross sections from FOFW is shown to lie between those of BK and JS. The numerical results are in agreement with analytic conclusions of Sec. IV.

II. COULOMB T MATRIX

The Coulomb Hamiltonian has, in addition to the usual rotational symmetry, a hidden symmetry in a special four-dimensional space constructed by using certain functions of the energy and the momenta as coordinates. This transformation enables one to obtain the explicit expression for the Coulomb Green's function as a solution of the corresponding Lippmann-Schwinger equation. From this we obtain the following expression for the T matrix[8]:

$$T(\vec{p},\vec{p}',E)\equiv\langle\vec{p}|T(E+i\delta)|\vec{p}'\rangle$$

$$=\pm\frac{4p_0^3\eta}{2\pi m}\frac{1}{(p^2-p_0^2)(p'^2-p_0^2)}\left[\eta\frac{e^{-i\eta\chi}}{\sinh\chi}\frac{\Theta(\eta)}{1-e^{-2\pi\eta}}+\frac{1}{\pi}\sum_{l=1}^{\infty}S_l\frac{e^{-l\chi}}{\sinh\chi}\frac{l^2}{l^2+\eta^2}\right],\quad E>0$$

with

$$\Theta(\eta)=e^{-2\pi\eta},\quad s_l=1 \text{ for } p^2-p_0^2>0,\ p'^2-p_0^2>0,$$
$$\Theta(\eta)=1,\quad s_l=1 \text{ for } p^2-p_0^2<0,\ p'^2-p_0^2<0,$$
$$\Theta(\eta)=e^{-\pi\eta},\quad s_l=(-1)^l \text{ for } (p^2-p_0^2)(p'^2-p_0^2)<0,$$

where $p_0^2=2m|E|$, $\eta=z_1z_2e^2m/p_0$, and χ is defined by

$$2(1-\cosh\chi)=\frac{-4p_0^2|\vec{p}-\vec{p}'|^2}{(p^2-p_0^2)(p'^2-p_0^2)}, \tag{2.1}$$

$$T(\vec{p},\vec{p}',E)=\left(\frac{\eta p_0^3}{m\pi^2(p^2+p_0^2)(p'^2+p_0^2)}\right)\times\left(\frac{1}{1-\cos\chi}-\frac{\eta\pi\sin(\pi-\chi)\eta}{\sin\chi}+\frac{2\eta^2}{\sin\chi}\sum_{l=1}^{\infty}\frac{(-1)^l\sin(\pi-\chi)l}{\eta^2-l^2}\right),\quad E<0$$

where

$$2(1-\cos\chi)=\frac{4p_0^2|\vec{p}-\vec{p}'|^2}{(p^2+p_0^2)(p'^2+p_0^2)}. \tag{2.2}$$

When $E>0$, $e^{-i\eta\chi}$ does not have a definite on-shell limit. However, these oscillating parts can be associated with the asymptotic Coulomb wave functions in the momentum space. If the amplitude is defined with respect to these Coulomb distorted asymptotic states the on-shell limit gives the usual Coulomb amplitude. This is discussed in Ref. 8. The integral representation given by Nutt,[3] valid for $E<0$ can be shown to be equivalent to (2.2). The details can be found in Ref. 8, along with other representations for the Coulomb T matrix. The term $e^{-i\eta\chi}$, near the half on-shell or on-shell points varies as $\epsilon^{2i\eta}$ where

$$\epsilon=(p_0^2-p^2)(p_0^2-p'^2)/p_0^2|\vec{p}-\vec{p}'|^2.$$

Hence, there is no Taylor expansion of the T matrix as a power series in ϵ near this region. Indeed, it is the term containing $e^{-i\eta\chi}$ which generates the physical Coulomb amplitude with respect to the Coulomb distorted asymptotic states,[12,13] when one approaches the energy shell. Equation (2.1) is used in our study of the proton-hydrogen problem.

III. PROTON-HYDROGEN CHARGE EXCHANGE

We denote the incident proton and the proton in the target by 1 and 2, respectively; 3 denotes the electron; $\vec{k}_i$ and $\vec{k}_i'$ denote the momenta of the ith particle before and after scattering. The process of interest is

$$1+(2,3)\rightarrow 2+(1,3), \tag{3.1}$$

with (2, 3) and (1, 3) denoting the hydrogen atoms in the 1s state. Let V_i denote the potential between j and k, $(i,j,k=1,2,3;\ i\neq j\neq k)$. H_0 is the Hamiltonian for three free particles and the full Hamiltonian is $H=H_0+V_1+V_2+V_3$. The transition amplitude to be evaluated is

$$T_{fi}=\langle f|T_{21}|i\rangle \tag{3.2}$$

in which $\langle f|$ and $|i\rangle$ designate, the final and the initial states. The two-body T operator for the (1, 2) pair in the three-particle space is defined as T_3. The FW expansion for T_{21} is

$$T_{21}=V_1+T_3+T_1G_0^+T_2+T_1G_0^+T_3+T_3G_0^+T_2+\cdots. \tag{3.3}$$

The FOFW retains only the first two terms of (3, 3):

$$T_{21}=V_1+T_3. \tag{3.4}$$

Our object is to investigate the matrix element $\langle f|T_{21}|i\rangle$ for the proton-hydrogen charge-exchange problem and determine its high-energy limit. Assuming the hydrogen atom to be in its 1s state before and after scattering, we obtain the following expressions for the matrix elements $\langle f|V_3|i\rangle$, $\langle f|T_3|i\rangle$, and $\langle f|V_1|i\rangle$, respectively:

$$\langle f|V_3|i\rangle=64\pi\lambda^5\int_0^\infty k^2dk\int_{-1}^{+1}d\mu\int_0^{2\pi}d\phi\,\frac{1}{[(\vec{k}-\vec{A})^2+\lambda^2]^2}\,\frac{e^2}{2\pi^2k^2}\,\frac{1}{[(\vec{k}-\vec{B})^2+\lambda^2]^2}, \tag{3.5}$$

$$\langle f|T_3|i\rangle=64\pi\lambda^5\int_0^\infty k^2dk\int_{-1}^{+1}d\mu\int_0^{2\pi}d\phi\,\frac{1}{[(\vec{k}-\vec{A})^2+\lambda^2]^2}\left\langle\vec{k}_{12}\left|T_3\left(E-\frac{p_3^2}{2\mu_2}+i\delta\right)\right|\vec{k}_{12}'\right\rangle\frac{1}{[(\vec{k}-\vec{B})^2+\lambda^2]^2}, \tag{3.6}$$

$$\langle f|V_1|i\rangle=-32\pi\lambda^4e^2/(A^2+\lambda^2)^3. \tag{3.7}$$

Here

$$\lambda=\mu_{13}e^2=\mu_{23}e^2,$$
$$\vec{k}=\vec{k}_{12}-\vec{k}_{12}', \tag{3.8}$$

$$\vec{A}=\vec{p}+\frac{M}{m+M}\vec{p}',$$
$$\vec{B}=\vec{p}'+\frac{M}{m+M}\vec{p}, \tag{3.9}$$

$$\vec{k}_{jk}=\frac{m_k\vec{k}_j-m_j\vec{k}_k}{m_j+m_k},\quad \vec{k}_{jk}'=\frac{m_k\vec{k}_j'-m_j\vec{k}_k'}{m_j+m_k}, \tag{3.10}$$

$$m_1 = m_2 = M\,, \quad m_3 = m\,, \tag{3.11}$$

$$\mu_{ij} = m_i m_j/(m_i + m_j)\,, \tag{3.12}$$

$$\mu_k = m_k(m_i + m_j)/(m_i + m_j + m_k)\,. \tag{3.13}$$

$\vec{p}_1$ is the momentum of the incident proton with respect to the center of mass of (2, 3), $\vec{p}\,'_2$ is the momentum of 2 with respect to the center of mass of (1, 3) after the rearrangement scattering. The scattering angle Θ is given by

$$-\cos\Theta = \vec{p}_1 \cdot \vec{p}\,'_2/p^2 \tag{3.14}$$

and

$$p_1^2 = p_2'^2 = p^2\,. \tag{3.15}$$

Throughout this paper the polar angles of $\vec{p}_1$, $\vec{p}\,'_2$, and $\vec{k}$ are (0, 0), $(\pi - \Theta, 0)$, and (Θ, φ), respectively:

$$E - \frac{p_3^2}{2\mu_3} = E - \frac{(\vec{p}_1 + \vec{p}\,'_2 - \vec{k})^2}{2\mu_3} = \frac{\kappa_{12}^2}{2\mu_{12}} \tag{3.16}$$

$$= \frac{p_1^2}{2\mu_1} - \frac{p_1^2}{2\mu_3} - \frac{p_2'^2}{2\mu_3} - \frac{k^2}{2\mu_3} - \frac{\lambda^2}{2\mu_{23}} + \frac{[k(\cos\beta + \cos\gamma) - p_1\cos\alpha]\,p'_2}{\mu_3} \tag{3.17}$$

$$\simeq \frac{p^2}{2\mu_{12}} + O\left(\frac{m}{M}\right) \tag{3.18}$$

if $|\vec{k}| \simeq A$ and $m/M \ll 1$. $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are the cosines of the angles between $\vec{p}_1$ and $\vec{p}\,'_2$, $\vec{p}\,'_2$ and $\vec{k}$, and $\vec{k}$ and $\vec{p}_1$, respectively. The exact expression for $\langle \vec{k}\,'_{12}|\, T_3(E - p_3^2/2\mu_3 + i\delta)\,|\vec{k}_{12}\rangle$, Eq. (2.1), can be written as

$$\langle \vec{k}\,'_{12}|\, T_3(\kappa_{12}^2/2\mu_{12} + i\delta)\,|\vec{k}_{12}\rangle = \frac{z_1 z_2 e^2}{2\pi^2 k^2 (1+\epsilon)^{1/2}} \left(\frac{2\pi\eta e^{-i\eta\chi}}{e^{2\pi\eta} - 1} + 2\sum_{m=1}^{\infty} \frac{m^2 e^{-m\chi}}{m^2 + \eta^2} \right)\,, \tag{3.19}$$

where

$$\eta = z_1 z_2 e^2 \mu_{12}/\kappa_{12}\,, \tag{3.20}$$

$$\epsilon = \frac{\mu_{12}^2}{\mu_{13}\mu_{23}} \frac{(k_{31}'^2 + \lambda^2)(k_{23}^2 + \lambda^2)}{\kappa_{12}^2 k^2}\,, \tag{3.21}$$

$$e^{-\chi} = [2 + \epsilon - 2(1+\epsilon)^{1/2}]/\epsilon\,, \tag{3.22}$$

$$\vec{k}_{23} = \vec{B} - \vec{k}\,, \tag{3.23}$$

$$\vec{k}\,'_{31} = \vec{k} - \vec{A}\,. \tag{3.24}$$

IV. HIGH-ENERGY LIMIT

We study the high-energy limit of the T-matrix element given by (3.19). At high energies, the parameter $\eta = \mu_{12} e^2/\kappa_{12}$ becomes small ($\sim 1/v$). A result which is correct to order η^2 may be obtained by neglecting η^2 in the denominator of the sum, which becomes

$$\sum_1^{\infty} \frac{m^2 e^{-m\chi}}{m^2 + \eta^2} \simeq \sum_1^{\infty} e^{-m\chi} = \frac{1}{e^{\chi} - 1} = \tfrac{1}{2}[(1+\epsilon)^{1/2} - 1]\,. \tag{4.1}$$

We have used (3.22) in the last step. Moreover,

$$2\pi\eta/(e^{2\pi\eta} - 1) \simeq 1 - \pi\eta\,. \tag{4.2}$$

The function $e^{-\chi}$ varies smoothly between zero and unity as ϵ goes from zero to infinity. The limit of $\langle \vec{k}\,'_{12}|\, T_3\,|\vec{k}_{12}\rangle$ can be obtained as κ_{12} becomes large, all other quantities remaining finite. In this case,

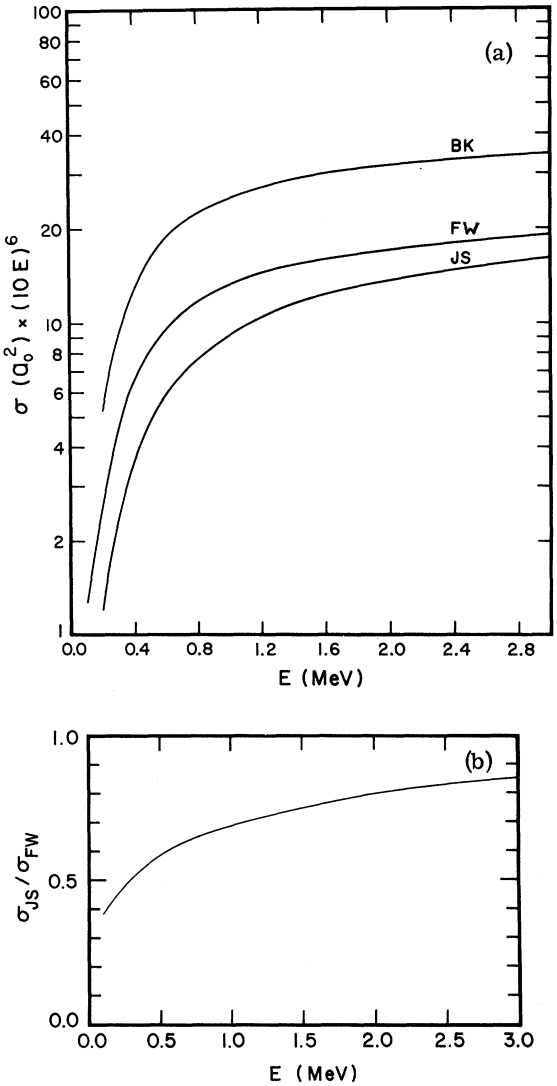

FIG. 1. (a) Proton-hydrogen charge-exchange cross sections in BK, JS, and FOFW approximation (FW) as a function of energy. To bring out clearly the difference between the results of the various calculations at high energies, we have plotted along the ordinate the actual cross section divided by the asymptotic BK energy dependence [$\sigma_{BK} \simeq 40/(10E)^6$]. (b) Ratio of proton-hydrogen charge-exchange cross section in FOFW approximation and *JS* approximation as a function of energy.

TABLE I. Proton-hydrogen charge-exchange cross sections in a.u. for BK, JS, and FOFW approximations.

	Cross sections in a.u.		
E(keV)	BK	JS	FOFW
100	1.256	0.230	0.601
200	0.0827	0.0188	0.0406
300	0.0131	0.0034	0.00652
400	0.00322	0.00092	0.00162
500	0.00103	0.00031	0.00053
600	0.000400	0.000126	0.00020
700	0.000176	0.000058	0.0000907
800	0.000085	0.000029	0.000044
900	0.000045	0.000016	0.0000234
1000	0.000025	0.0000092	0.0000132

$\epsilon \to 0$, and $\chi \sim \ln\epsilon/4$. But $\epsilon \sim \eta^2$. Thus the oscillatory term can be expanded

$$e^{-i\eta\chi} \simeq 1 - i\eta\chi \simeq 1 - 2i\eta \ln\eta + O(\eta) + \cdots . \qquad (4.3)$$

Note that $\eta \ln\eta$ vanishes as $\eta \to 0$. Equations (4.1)–(4.3) are inserted in Eq. (3.19), which becomes

$$\langle \vec{k}'_{12} | T_3 | \vec{k}_{12} \rangle \simeq \frac{z_1 z_2 e^2}{2\pi^2 k^2} \left(1 - \frac{\eta(\pi + i\chi)}{(1+\epsilon)^{1/2}} \right) . \qquad (4.4)$$

Equation (4.4) indicates that the high-energy limit of the Coulomb T matrix is simply given by the potential

$$\lim_{\kappa_{12} \to \infty} \langle \vec{k}'_{12} | T_3(\kappa_{12}) | \vec{k}_{12} \rangle = \langle \vec{k}'_{12} | V_3 | \vec{k}_{12} \rangle \qquad (4.5)$$

as one would expect. The approach to the asymptotic limit specified by (4.4) is, however, slower $[(1/v) \ln v]$ for the Coulomb potential than for a short-range potential $(1/v^2)$. See also Chen and Kramer.[15]

Equation (4.4) implies that the high-energy limit of the FOFW charge-exchange amplitude is the JS result. The integral of Eq. (3.6) is dominated by the region in which the wave function peaks. In this area, ϵ is small. The term involving χ in (4.4), which has a logarithmic dependence on η, gives rise to a term in the amplitude proportional to $\ln v$. The ratio of the FOFW amplitude to the JS amplitude is

$$\frac{T_{fi}^{\mathrm{FOFW}}}{T_{fi}^{\mathrm{JS}}} \simeq 1 - iO\left(\frac{1}{v} \ln v\right) + O\left(\frac{1}{v}\right) , \qquad (4.6)$$

where v is the incident proton velocity in the laboratory system. The difference between the FOFW and JS results goes to zero quite slowly.

In Sec. V, the FOFW amplitude is calculated by numerical integration without introducing the approximations used here in obtaining the high-energy limit.

V. NUMERICAL RESULTS AND DISCUSSION

The analysis described here shows that in the intermediate-energy region the contribution to the charge-exchange amplitude from the T matrix will be smaller than that from V_3. In the extreme high-energy limit the analysis given in Sec. IV shows that the FOFW approximation should be the same as the Born approximation to the leading order. To see these features numerically, we carried out the calculations from 100 keV to 3 MeV by numerically evaluating the triple integrals (3.5) and (3.6) without making any approximation. The results up to this energy clearly substantiate fully the main conclusions drawn from the analytical results for the high-energy limit previously obtained. Table I, and Figs. 1(a) and 1(b) summarize the results of the numerical calculations. Energies above 3 MeV were not considered because with the increase in energy the peak in the integrand becomes sharper and hence numerical accuracy in the computation becomes unsatisfactory. Twelve to fifteen terms were included from the series in the second term in the large parentheses of (3.19). This was found to be adequate in the energy region we considered. In the numerical analysis the T matrix was calculated exactly. The accuracy of the numerical integration was checked by computing the Born amplitude at the same time.

Our numerical results are in agreement with the results of similar calculations recently reported by Chen and Kramer.[15] Our analysis of the FOFW amplitude and the corresponding numerical results lead us to the following conclusions: In the keV region, off-shell effects are important and reduce the contribution from the proton-proton interaction. However in the extreme high-energy limit, the FOFW amplitude is the same as the Born amplitude to the leading order in $1/v$.

The high-energy limit of the proton-hydrogen charge exchange when the second- and higher-order terms are included is currently being investigated. This is important in view of the drastically different results obtained in the first- and second-order Born approximations to the proton-hydrogen charge-exchange amplitude.

[1] L. D. Faddeev, *Mathematical Aspects of the Three Body Problem in the Quantum Scattering Theory*, translated by Ch. Gutfreund (Israel Program for Scientific Translations, Jersualem, 1965); C. S. Shastry and A. K. Rajagopal (unpublished).

[2] J. Schwinger, J. Math. Phys. 5, 1606 (1964).

[3] G. L. Nutt, J. Math, Phys. 9, 796 (1968).

[4] M. R. C. McDowell and R. Richards, J. Phys. B 4, L105 (1971).

[5] J. Nuttall and R. W. Stagat, Phys. Rev. A 3, 1355

(1971).
[6]C. S. Shastry and A. K. Rajagopal, Phys. Rev. A 2, 781 (1970).
[7]A. M. Perelemov and V. S. Popov, Zh. Eksperim. i Teor. Fiz. 50, 179 (1966) [Sov. Phys. JETP 23, 118 (1966)]; see also M. Bander, C. Itzykson, Rev. Mod. Phys. 38, 330 (1966); 38, 346 (1966).
[8]A. K. Rajagopal and C. S. Shastry, J. Math. Phys. 12, 2387 (1971).
[9]C. S. Shastry, L. Kumar, and J. Callaway, Phys. Rev. A 1, 1137 (1970).
[10]C. Carpenter and T. F. Tuan, Phys. Rev. A 2, 1851 (1970).
[11]A. M. Brodskii, V. S. Potapov, and V. V. Tolmachev, Zh. Eksperim. i Teor. Fiz. 58, 264 (1970) [Sov. Phys. JETP 31, 144 (1970).
[12]J. C. Y. Chen and L. Hambro, J. Phys. B 4, 191 (1970).
[13]J. C. Y. Chen, A. C. Chen, and P. J. Kramer, Phys. Rev. A 4, 1982 (1971).
[14]A. C. Chen, J. C. Y. Chen, A. L. Sinfailam, and L. Hambro, Phys. Rev. A 4, 1998 (1971).
[15]J. C. Y. Chen and P. J. Kramer, Phys. Rev. Letters 24, 899 (1971); the observation made by us in Sec. IV of the present paper has also been independently made by in a detailed paper [C. S. Shastey, A. K. Rajagopal, and J. Callaway, Phys. Rev. A 5, 1207 (1972)].
[16]B. H. Bransden in *Lectures in Theoretical Physics*, edited by S. Geltman, K. T. Mahanthappa, and W. E. Brittin (Gordon and Breach, New York, 1969), Vol. XI-c, p. 139.
[17]H. C. Brinkman and H. A. Kramers, Proc. Acad. Sci. Amsterdam 33, 973 (1930).
[18]J. C. Jackson and H. Schiff, Phys. Rev. 89, 359 (1953).
[19]R. M. Drisko, Ph. D. thesis (Carnegie Institute of Technology, 1955) (unpublished).
[20]R. McCarrol and A. Salin, Proc. Roy. Soc. (London) A300, 202 (1967).
[21]K. Dettmann and G. Leibfried, Z. Physik 210, 43 (1968).
[22]R. Aaron, R. D. Amado, and B. W. Lee, Phys. Rev. 121, 319 (1961).
[23]M. M. Rubin, R. L. Sugar, and G. Tiktopoulos, Phys. Rev. 162, 1555 (1967).
[24]A. Norcliffe, I. C. Percival, and M. J. Roberts, J. Phys. B 2, 578, (1969); 2, 590 (1969); M. J. Roberts, Chem. Phys. Letters 6, 46 (1970); J. Phys. B 3, L115 (1970).
[25]C. Lovelace, Phys. Rev. 135, B1225 (1964).

PHYSICAL REVIEW A VOLUME 6, NUMBER 1 JULY 1972

m_J Mixing in Oriented $6\,^2P_{1/2}$ Cesium Atoms, Induced in Collisions with Noble Gases*

J. Guiry† and L. Krause
Department of Physics, University of Windsor, Windsor, Ontario, Canada
(Received 24 January 1972)

The cross sections for disorientation of magnetically oriented $6\,^2P_{1/2}$ cesium atoms, induced in collisions with noble gases, have been determined using Zeeman-scanning techniques at kilogauss magnetic fields. The $m_J=-\frac{1}{2}$, $m_I=-\frac{7}{2}$ hyperfine Zeeman substate was populated selectively by irradiating the cesium-vapor–noble-gas mixture with cesium-resonance radiation, and collisional transitions to the $m_J=+\frac{1}{2}$, $m_I=-\frac{7}{2}$ substate were monitored by measurements of the relative intensities of the σ^- and σ^+ components in resonance fluorescence observed in the direction parallel to the magnetic field. The following cross sections were obtained: Cs-He, 11.8 Å^2; Cs-Ne, 4.7 Å^2; Cs-Ar, 10.7 Å^2; Cs-Kr, 37.9 Å^2; Cs-Xe, 71.7 Å^2.

I. INTRODUCTION

Considerable attention has recently been devoted to theoretical and experimental aspects of depolarization of $^2P_{1/2}$ and $^2P_{3/2}$ alkali-metal atoms, induced in collisions with noble gases. Dyakonov and Perel,[1] Omont,[2] and Wang and Tomlinson[3] developed a general theoretical treatment of collisional relaxation of excited atoms, while Franz and coworkers[4,5] and Elbel and Naumann[6] derived a selection rule $m_J \nrightarrow -m_J$ for transitions between Zeeman substates in alkali metal atoms, which was not upheld in subsequent theoretical studies,[7–9] even though such intramultiplet transitions are strictly forbidden in the Born approximation. Most recently, Mies[10] derived cross sections of the order of 10 Å^2 for proton-induced collisional transitions $m_J=\frac{1}{2} \nrightarrow m_J=-\frac{1}{2}$ in fluorine, having formulated the scattering problem in terms of molecular continuum wave functions.

Experimental investigations of collisional depolarization were carried out at high magnetic fields by Krause and co-workers[11,12] who used Zeeman-scanning methods to study the $^2P_{1/2}$ and $^2P_{3/2}$ resonance states in potassium. Gallagher[13] studied the corresponding states of rubidium and cesium in Hanle experiments at zero magnetic field. Bulos and Happer[14] quoted depolarization cross sections for the $^2P_{1/2}$ states in rubidium and cesium, incorporating in their results corrections for the effect